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Signature splitting in three-quasiparticle rotational bands

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An empirical rule, which is an extension of the rule for two-quasiparticle bands, is devised to check the favoured signature in three-quasiparticle bands of $A = 153$–187 mass region. Its applications, for testing the favored signature, confirmation of the spin/parity assignments, and confirmation of the configuration assignments to three-quasiparticle bands, are discussed. Violation of the rule for tilted rotational bands supports the fact that signature does not remain a good quantum number for tilted bands.

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An odd-even staggering in the rotational bands of odd-A, and odd-odd nuclei is one of the most significant characteristics linked to the signature quantum number (α). In this study based on our recent compilation [1], we present an empirical rule for verifying and predicting the favored signature in three quasiparticle (3qp) rotational bands. Its application for the spin/parity and the configuration assignments, particularly in those cases where experimental information for choosing between competing configurations is insufficient, has been discussed.

We know that the basis functions for one quasiparticle (1qp) bands of odd-A nuclei are given by [2]

$$\left|\Psi^{IMK}_{k_p}\right\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left(\left|D^I_{MK}\right\rangle|k_p\rangle + (-1)^{I+K}\left|D^I_{M-K}\right\rangle R_i|k_p\rangle\right), \tag{1}$$

where K is the projection of total angular momentum I on the symmetry axis, k_p is the projection of intrinsic (proton/neutron) angular momentum on the symmetry axis, and $R_i(= R_y(\pi))$ is a rotation operator which rotates the system by an angle π about an axis (y-axis) perpendicular to the symmetry axis.

The rule which governs the favored spin and the favored signature for 1qp bands is

$$I_f = |j| \bmod 2; \quad \alpha_f = \tfrac{1}{2}(-1)^{(j-\frac{1}{2})}, \tag{2}$$

where I_f is the favored spin, α_f is the favored signature and j is the particle angular momentum of the configuration. The origin of this rule is known to be the large decoupling in the $K = 1/2$ component of the bands; this coupling is transmitted by the Coriolis coupling to higher-K values [3].

Similarly, the wave function, the favored spin and the favored signature for two quasiparticle (2qp) bands in odd-odd nuclei are given by [4,5]

$$\left|\Psi^{IMK}_{k_pk_q}\right\rangle = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}}\left(\left|D^I_{MK}\right\rangle|k_pk_q\rangle + (-1)^{I+K}\left|D^I_{M-K}\right\rangle R_i|k_pk_q\rangle\right), \tag{3}$$

$$\begin{aligned} I_f &= |j_p + j_q| \bmod 2; \\ \alpha_f &= \tfrac{1}{2}(-1)^{(j_p-\frac{1}{2})} + \tfrac{1}{2}(-1)^{(j_q-\frac{1}{2})}, \end{aligned} \tag{4}$$

where k_p, k_q are the single particle wave functions and j_p, j_q are the spins of the odd particles. The origin of this rule can again be traced to the decoupling effects of the $K = 0$ band formed by combining $\Omega = 1/2$ odd-proton and $\Omega = 1/2$ odd-neutron. The signature splitting of the $K = 0$ band is then transmitted to higher-K bands by the higher order Coriolis coupling [6,7].

We propose that in case of 3qp bands, the wave function, the favored spin, and the favored signature may be given by

$$\left|\Psi^{IMK}_{k_pk_qk_r}\right\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left(\left|D^I_{MK}\right\rangle|k_pk_qk_r\rangle + (-1)^{I+K}\left|D^I_{M-K}\right\rangle R_i|k_pk_qk_r\rangle\right), \tag{5}$$

$$\begin{aligned} I_f &= |j_p + j_q + j_r| \bmod 2; \\ \alpha_f &= \tfrac{1}{2}(-1)^{(j_p-\frac{1}{2})} + \tfrac{1}{2}(-1)^{(j_q-\frac{1}{2})} + \tfrac{1}{2}(-1)^{(j_r-\frac{1}{2})}, \end{aligned} \tag{6}$$

where k_p, k_q, k_r are single particle wave functions and j_p, j_q, and j_r are angular momenta of the three particles. The observed signature splitting in the 3qp bands is basically due to the higher order Coriolis coupling [8].

It should be noted that j is generally not a good quantum number in deformed nuclei. However, most of these 3qp structures involve more than one, or, sometimes all three orbitals, which have high-j value. Such orbitals usually remain almost pure, and the corresponding j can be used in Eq. (6) given above. In the 3qp configurations having two like particles in same j and in the same orbital, favored signature is determined by a third particle only. Thus, the favored signature for a band based on the configuration $h_{11/2}(i_{13/2})^2$ will be $\alpha = -1/2$ and the favored spin will be $I_f = 3/2, 7/2, 9/2, \ldots$.

In our compilation [1], we have pointed out that there are 48 bands which show signature splitting and sometimes signature inversion. We have tested the rule given by Eq. (6) for all the 3qp bands exhibiting signature splitting. A pointwise application of this rule to all these cases is given below:

(1) Bands for which the experimentally observed favored signature matches with the theoretically favored signature are given in Table I. Those for which the experimentally observed favored signature does not match with the theoretically favored signature, a possible explanation for

TABLE I. Bands for which the experimentally favored signature matches with the theoretically favored signature.

S. No.	Nucleus	Band no.[a]	Configuration	α_f(exp)	α_f(th)	Key No. of reference[b]
1	^{153}Tb	2	$7/2[523]_p \otimes 3/2[651]_n \otimes 11/2[505]_n$	−1/2	−1/2	1998HA37
2	^{157}Ho	2	$7/2[523]_p \otimes 3/2[651]_n \otimes 3/2[521]_n$	+1/2	+1/2	1992RA17
3	^{157}Er	1	$7/2[523]_p \otimes 7/2[404]_p \otimes 3/2[651]_n$	−1/2	−1/2	1995GA13
4	^{159}Er	1	$7/2[523]_p \otimes 7/2[404]_p \otimes 3/2[651]_n$	−1/2	−1/2	1998SI03
5	^{163}Er	7	$7/2[523]_p \otimes 7/2[404]_p \otimes 5/2[523]_n$	−1/2	−1/2	1997HA23
6	^{165}Tm	1	$7/2[404]_p \otimes 5/2[642]_n \otimes 5/2[523]_n$	+1/2	+1/2	2001JE09
7	^{165}Tm	2	$7/2[523]_p \otimes 5/2[642]_n \otimes 5/2[523]_n$	+/2	+1/2	2001JE09
8	^{159}Lu	1	$7/2[523]_p \otimes 3/2[651]_n \otimes 3/2[521]_n$	+1/2	+1/2	1995MA46
9	^{165}Lu	1	$9/2[514]_p \otimes 5/2[642]_n \otimes 5/2[523]_n$	−1/2	−1/2	2004SC14
10	^{171}Lu	1	$7/2[404]_p \otimes 7/2[633]_n \otimes 1/2[521]_n$	−1/2	−1/2	1998BB02
11[c]	^{171}Lu	2	$1/2[541]_p \otimes 7/2[633]_n \otimes 1/2[521]_n$	+1/2	+1/2	1998BB02
12	^{169}Hf	2	$9/2[514]_p \otimes 1/2[411]_p \otimes 5/2[642]_n$	−1/2	−1/2	2001SC49
13[c]	^{171}Hf	1	$7/2[404]_p \otimes 5/2[402]_p \otimes 7/2[633]_n$	+1/2	+1/2	1997CU01
14	^{173}Ta	2	$1/2[541]_p \otimes 1/2[521]_n \otimes 7/2[633]_n$	+1/2	+1/2	1995CA27
15	^{175}Ta	3	$9/2[514]_p \otimes 7/2[633]_n \otimes 5/2[512]_n$	+1/2	+1/2	1996KO17
16	^{179}Re	4	$5/2[402]_p \otimes 7/2[514]_n \otimes 5/2[512]_n$	+1/2	+1/2	2002TH12
17	^{181}Re	1	$5/2[402]_p \otimes 9/2[624]_n \otimes 7/2[514]_n$	−1/2	−1/2	2000PE18
18	^{181}Re	4	$9/2[514]_p \otimes 9/2[624]_n \otimes 7/2[514]_n$	+1/2	+1/2	2000PE18
19	^{181}Os	3	$1/2[521]_n \otimes 9/2[624]_n \otimes 7/2[633]_n$	+1/2	+1/2	2003Cu03
20[c]	^{183}Os	1	$5/2[402]_p \otimes 1/2[541]_p \otimes 9/2[624]_n$	−1/2	−1/2	2001SH41
21	^{185}Pt	1	$1/2[541]_p \otimes 1/2[660]_p \otimes 9/2[624]_n$	−1/2	−1/2	1989Pi09

[a] Corresponds to the Band number of Ref. [1].

[b] The NSR key-numbers (www.nndc.bnl.gov) of the references are given in the Ref. column.

[c] Bands similar to the one shown in Fig. 1(a).

violation is given in Table II. In our compilation [1], we have seven bands as tilted rotational bands (t-bands). Out of these seven t-bands, only five bands exhibit signature splitting. Violation of rule in all these cases (as shown in Table II), supports the fact [9] that signature does not remain a good quantum number for tilted axis rotation.

(2) Out of these 48 cases, there are four cases for which configuration assignment is tentative and the rule helps in confirming the configuration assignment as shown in Table III. It should be noted that the validity of this rule for ^{163}Lu and ^{173}Ta (Table III) strengthens the explanations given in [10,11] for the configuration assignments. Hence we suggest that this rule will be helpful to experimentalists for the configuration assignments to 3qp bands.

(3) There are nine bands (included in Table I) which show signature splitting but have uncertain spin/parity in literature. Validity of the rule in these cases confirms the spin/parity assignments and hence strengthens the explanation given in literature for these assignments.

(4) In case of ^{163}Er, (Table II), the rule becomes valid by reducing the assigned spins by one unit. This reduction of

TABLE II. Bands for which the experimentally favored signature does not match with the theoretically favored signature and a possible explanation for violation is given.

S. No.	Nucleus	Band no.[a]	Configuration	α_f(exp)	α_f(th)	Explanation and key number of reference[b]
1	^{179}W	4	$9/2[624]_n \otimes 7/2[514]_n \otimes 7/2[633]_n$	−1/2	+1/2	1994WA05
2	^{181}Re	3	$5/2[402]_p \otimes 9/2[624]_n \otimes 7/2[633]_n$	+1/2	−1/2	1997PE15
3	^{181}Re	5	$9/2[514]_p \otimes 9/2[624]_n \otimes 7/2[633]_n$	−1/2	+1/2	1997PE15
4	^{183}Re	8	$9/2[514]_p \otimes 9/2[624]_n \otimes 11/2[615]_n$	−1/2	+1/2	1998HA51
5	^{181}Os	2	$7/2[514]_n \otimes 9/2[624]_n \otimes 7/2[633]_n$	−1/2	+1/2	2003CU03 All are t-bands.
6	^{163}Er	9	$5/2[642]_n \otimes 5/2[523]_n \otimes 3/2[521]_n$	+1/2	−1/2	Validity of the rule with reduction of spins by one unit is also in accordance with the explanation given in 1997Ha23.
7	^{155}Dy	1	$3/2[521]_n \otimes 1/2[660]_n \otimes 1/2[660]_n$	−1/2	+1/2	Signature inversion in the lower part of the band [see Fig. 1(b)]. 1994VL02

[a] Corresponds to the Band number of Ref. [1].

[b] The NSR key-numbers (www.nndc.bnl.gov) of the references are given in the reference column.

TABLE III. Bands having tentative configuration assignment in the literature and validity of the rule in these cases confirms the configuration assignments.

S. No.	Nucleus	Band no.[a]	Configuration	α_f(exp)	α_f(th)	Explanation and key number of reference[b]
1	^{163}Er	1	$5/2[642]_n \otimes 5/2[523]_n \otimes 3/2[521]_n$	$+1/2$	$+1/2$	1997HA23
2	^{163}Lu	1	$7/2[404]_p \otimes 5/2[642]_n \otimes 5/2[642]_n$	$-1/2$	$-1/2$	Validity of the rule for first configuration supports the explanation given in 2004JE03
			$5/2[402]_p \otimes 5/2[642]_n \otimes 5/2[642]_n$	$-1/2$	$+1/2$	
			$1/2[411]_p \otimes 5/2[642]_n \otimes 5/2[642]_n$	$-1/2$	$-1/2$	
3	^{169}Re	1	$9/2[514]_p \otimes 3/2[651]_n \otimes 3/2[521]_n$	$+1/2$	$+1/2$	2004ZH05
4	^{173}Ta	1	$5/2[402]_p \otimes 5/2[512]_n \otimes 7/2[633]_n$	$+1/2$	$+1/2$	validity of the rule for first configuration strengthens the explanation given in 1995CA27.
			$7/2[404]_p \otimes 7/2[633]_n \otimes 1/2[521]_n$	$+1/2$	$-1/2$	5/2[512] orbital has a contribution of ∼74% from $f_{7/2}$ and 23% from $h_{9/2}$ subshells.

[a] Corresponds to the Band number of Ref. [1].

[b] The NSR key-numbers (www.nndc.bnl.gov) of the references are given in the reference column.

spins by one unit is also consistent with the explanation given in [12] and hence this rule helps in confirmation of the spin and configuration assignments.

(5) There are some bands for which lower part of the band is not observed experimentally, but these bands exhibit signature splitting at higher spins. Validity of the rule in these cases suggests that there may not be signature inversion in lower part/unobserved part of a band. We have presented [see Fig. 1(a)] four cases for which staggering is present at higher spin values and lower part of band is also observed experimentally (Table I). Validity of the rule in these cases indicates that there should not be signature inversion in the lower part of the band, which is consistent with the experimental observation. Since the staggering behavior arises due to the Coriolis effects [6,7], therefore, absence of staggering in low spin region suggests that Coriolis effects are not significant in this region. In reverse situation, violation of the rule at higher spin values of these bands indicates that there may be a signature inversion in lower/unobserved part of the band. Accordingly, we suggest signature inversion in three cases ^{157}Tm (Band 2), ^{163}Lu (Band 2), and ^{165}Dy (Band 1). Here the band number corresponds to the band number of Ref. [1]. Out of these three cases, lower part of the band is experimentally observed only in ^{155}Dy, which is consistent with our explanation as shown in Fig. 1(b).

(6) Some Nilsson orbitals are not pure (e.g., $f_{7/2}$ and $h_{9/2}$ have appreciable mixing) and have appreciable contribution from subshells different from their parent. In order to extend the validity of this rule for these cases, single particle calculations are done by taking deformations from [13] and κ, μ parameters from [14]. The j value to be used in Eq. (6) should be reasonably pure but, in case of mixed orbital, we can use j value of a subshell which contributes substantially (exceeds by at least 50% from its mixing partner) to a particular Nilsson orbital. Validity of the rule for these cases is discussed in Table IV. Exact contribution from each sub-shell is calculated and the results are in accordance with the experimental results.

(7) There are 10 cases exhibiting signature splitting but it is not possible to test the favored signature with the help of this rule because of the following reasons:

(i) Configuration assignment to a 3qp band is not pure, i.e., mixture of more than one configuration.

TABLE IV. Bands having mixed orbitals and validity of the rule is tested by taking the j value of a subshell having substantial contribution.

S. No.	Nucleus	Band no.[a]	Configuration	α_f(exp)	α_f(th)	Explanation and key number of reference[b]
1	^{185}Os	1	$1/2[521]_n \otimes 7/2[503]_n \otimes 11/2[615]_n$	$-1/2$	$-1/2$	Contribution of $f_{7/2}$ and $h_{9/2}$ subshells in 7/2[503] orbital is ∼89% and 7%, respectively. 2004SH08
2	^{185}Os	2	$3/2[512]_n \otimes 9/2[624]_n \otimes 11/2[615]_n$	$-1/2$	$-1/2$	Contribution of $p_{3/2}$ and $f_{5/2}$ subshells in 3/2[512] orbital is ∼12% and ∼64%, respectively. 2004SH08
3	^{181}Ir	1	$1/2[541]_p \otimes 7/2[633]_n \otimes 7/2[514]_n$	$-1/2$	$-1/2$	Contribution of $f_{7/2}$ and $h_{9/2}$ subshells in 7/2[514] orbital is 7% and 92%, respectively. 1993DR02

[a] Corresponds to the Band number of Ref. [1].

[b] The NSR key-numbers (www.nndc.bnl.gov) of the references are given in the reference column.

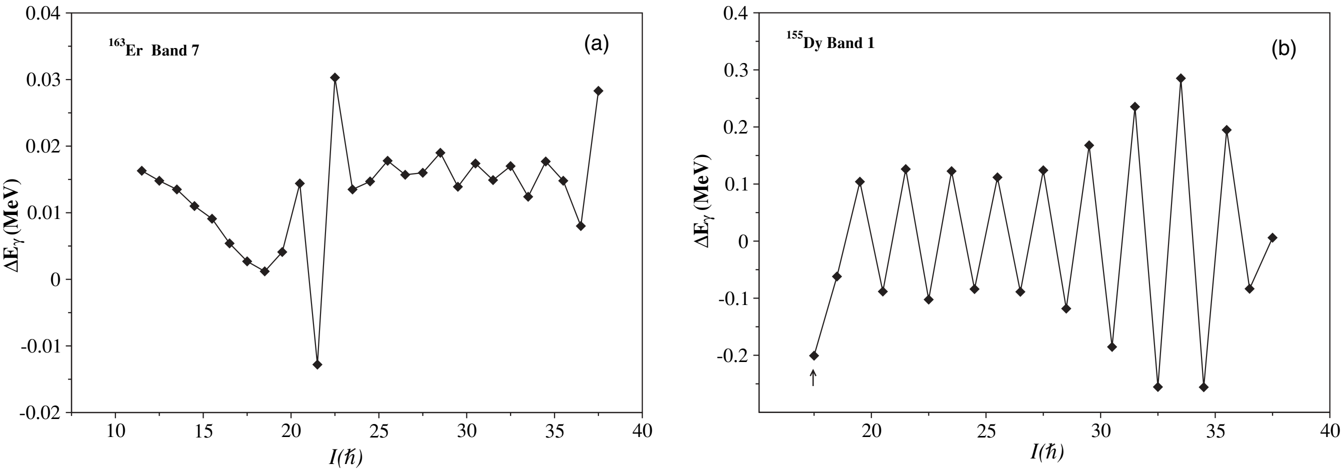

FIG. 1. (a) Bands for which the lower part is observed and do not exhibit signature splitting in the low spin region but rule is valid at higher spin values. (b) Band for which there is signature inversion in the unobserved part. Arrow indicates the point of inversion.

(ii) Configuration is not known in literature.
(iii) Staggering plot is not regular over a wide spin range.
(iv) Nilsson orbital taking part in the given configuration of a 3qp band is not pure and contributions from different subshells are almost same.

In summary, in this Brief Report we conclude that the rule given by Eq. (6) works reasonably well in all those cases where the configuration assignments are known and there is no significant mixing. This rule is useful for testing the favored signature, confirmation of the spin/parity assignment, confirmation of the configuration assignment, and prediction of signature inversion in the unobserved part of a band. Violation of this rule in tilted rotational bands confirms that signature does not remain a good quantum number under tilted rotation for 3qp bands.

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