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Nuclear superfluidity and cooling time of neutron star crusts

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We analyze the effect of neutron superfluidity on the cooling time of inner crust matter in neutron stars, in the case of a rapid cooling of the core. The specific heat of the inner crust, which determines the thermal response of the crust, is calculated in the framework of HFB approach at finite temperature. The calculations are performed with two paring forces chosen to simulate the pairing properties of uniform neutron matter corresponding to the BCS approximation and to many-body techniques including polarization effects. Using a simple model for the heat transport across the inner crust, it is shown that the two pairing scenarios mentioned above give very different values for the cooling time, i.e., of about 12 and 25 yr.

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I. INTRODUCTION

A newly-formed neutron star cools within minutes from a temperature of the order of 30 MeV to less than 1 MeV via neutrino emission. After this stage, the thermal evolution of the neutron stars can be strongly influenced by the onset of nuclear superfluidity [1,2]. This is especially the case for rapid cooling models. In these models, due to direct Urca or other exotic processes, the core cools down so rapidly that a temperature inversion develops between the core and the crust. The crust acts as an insulating blanket which keeps the surface relatively warm until the cooling wave reaches the surface. When this happens, the surface temperature drops precipitously to the temperature of the core. One of the relevant quantity in this cooling scenario is the cooling time, i.e., the time necessary for the cooling wave to arrive from the cold core to the surface of the star. The cooling time is primarily determined by the thermal response of the inner crust, formed by nuclear clusters immersed in a sea of unbound neutrons and ultrarelativistic electrons [3].

In the rapid cooling models, both the core cooling and the time needed for the core-crust thermalisation depend critically on nuclear superfluidity. Thus, on one hand, the onset of superfluidity in the core matter suppresses the neutrino cooling since the total energy of particles involved in the neutrino production must exceed the pairing gap. On the other hand, the superfluidity of inner crust matter is shortening significantly the cooling time. This happens due to the suppression of the heat capacity of the inner crust matter by the energy gap in the excitation spectrum of the superfluid neutron gas.

One of the first estimation of the cooling time was given by Brown *et al.* [4], who considered the possibility of a rapid cooling induced by the strangeness condensation. They calculated the heat diffusion time through the crust with a simple formula, i.e., $t_{\mathrm{diff}} = \frac{R_c^2 C_v}{\kappa}$, where R_c is the thickness of the crust, while C_v and κ are the specific heat and the thermal conductivity of the crust matter. The estimated cooling time was of the order of a few tens of years. Later on, using a direct Urca process as cooling mechanism, more realistic calculations of thermal evolution of neutron stars were performed [2]. The numerical simulations showed that the cooling time does not depend on the details of the rapid cooling mechanism but rather on the structure of the neutron star. Besides, it was also shown that the cooling time can be strongly reduced (by about a factor of three) if the neutron gas in the inner crust is in a superfluid phase.

In the calculations mentioned above, the effects of nuclear clusters on the superfluid and thermal properties of the neutron gas were disregarded. Since then, a few quantum calculations of the inner crust matter superfluidity, including the effects of the nuclear clusters, have been done [5–7]. Thus, using the Hartree-Fock-Bogoliubov (HFB) approach it was found that the presence of the nuclear clusters can modify significantly the heat capacity of the neutron gas. How the nuclear clusters could affect the cooling time of the inner crust was investigated by Pizzochero *et al.* [8]. Using a cooling model similar to the one employed by Brown *et al.* [4], it was concluded that the presence of the clusters, primarily in the outermost layers of the inner crust, could change the cooling time by amounts comparable with the cooling time itself. It was also found that the effect of the clusters on the cooling time depends rather strongly on the temperature and the pairing force used in calculating the specific heat of the inner crust matter.

The impact which the pairing force could have on the superfluid properties and the specific heat of the inner crust matter was recently analyzed in the framework of HFB approach at finite temperature (FT-HFB) [7]. Thus, it was shown that if the pairing force used in the FT-HFB equations is adjusted to describe two different scenarios for the neutron matter superfluidity, i.e., one corresponding to BCS calculations with the Gogny force and the other to Gorkov type calculations which take into account self-energy and screening effects [9], the results for the specific heat of the inner crust matter can change by several orders of magnitude. The scope of the present paper is to show what are the consequences of these changes in the specific heat upon the cooling time of inner crust matter. In the first part of the paper we shall extend the calculations of Ref. [7] to the low-density region of the inner crust, which was not treated before in the HFB

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approach, and analyze how the specific heat and the thermal diffusivity behave across the inner crust. Then, using the model of Refs. [4,8] for the heat transport, we shall discuss how the cooling time of the inner crust depends on neutron matter superfluidity.

II. THERMAL PROPERTIES OF THE INNER CRUST MATTER IN THE HFB APPROACH

The thermal response of the inner crust matter depends on thermal diffusivity, defined as the ratio of the thermal conductivity to the heat capacity. The heat capacity of the inner crust has contributions from the electrons, the neutrons and the lattice. The heat capacity of the electrons, considered as a uniform and relativistic gas, has the standard form [10] while the contribution of the lattice to the specific heat is usually neglected.

In the normal phase, the specific heat of the neutrons exceeds the specific heat of the electrons by about two orders of magnitude (see Fig. 1 below). However, the onset of the neutron superfluidity reduces drastically the neutron specific heat, which could become smaller than the electron specific heat in some regions of the inner crust. How the neutron specific heat is affected by the superfluidity as well as by the temperature and the presence of nuclear clusters was already studied in Ref. [7], but only for a few density regions of the inner crust. Here we extend this study to all relevant densities of the inner crust, starting from the neutron drip density up to about half the nuclear saturation density. This region of the inner crust is supposed to give the largest contribution to the cooling time of the crust [8].

In microscopic calculations the inner crust matter is divided in independent cells treated in the Wigner-Seitz approximation [11]. Up to baryonic densities of the order of half the nuclear saturation density, considered in this paper, each cell is supposed to contain in its center a spherical neutron-rich nucleus surrounded by unbound neutrons and immersed in a relativistic electron gas uniformly distributed inside the cell. The proton-to-neutron ratio and the dimension of the cell at a given baryonic density are determined from the beta equilibrium conditions. In the present study we use the cell structure determined in Ref. [11] by HF type calculations. The properties of the cells considered in this paper are displayed in Table I. Compared to Ref. [11], here we have not included the cell with $Z = 32$, which most probably belongs to the deformed pasta phase. For the cells listed in Table I we shall determine the specific heat by using the quasiparticle spectrum generated by the FT-HFB approach presented below.

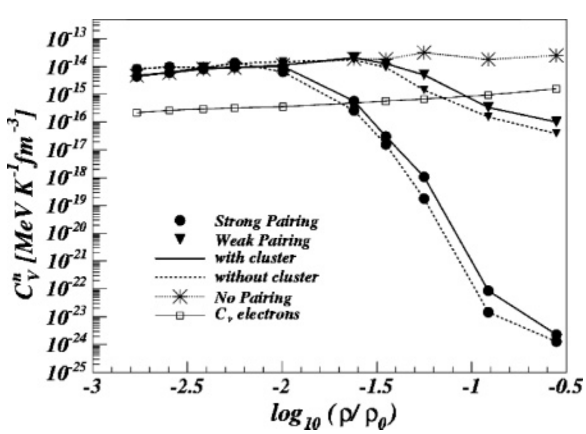

FIG. 1. Specific heat of the neutrons for the Wigner-Seitz cells listed in Table I. The results correspond to the strong and the weak pairing forces (see the text) and for the cells with (without) the nuclear clusters. The specific heat of the non-uniform cells obtained when the pairing correlations are switched off are indicated by star symbols. The square symbols show the specific heat of the electrons.

TABLE I. The Wigner-Seitz cells considered in the paper. The structure of the cells, i.e., the baryonic densities (ρ), the number of neutrons (N), the number of protons (Z) and the cell radii (R_{WS}) correspond to Ref. [11]. x_i are the thickness of the layers employed in Eq. (10).

N_{zone}	N	Z	R_{WS} [fm]	ρ [g cm^{-3}]	x_i [m]
10	140	40	54	4.7×10^{11}	12
9	160	40	49	6.7×10^{11}	12
8	210	40	46	1.0×10^{12}	15
7	280	40	44	1.5×10^{12}	21
6	460	40	42	2.7×10^{12}	40
5	900	50	39	6.2×10^{12}	45
4	1050	50	36	9.7×10^{12}	43
3	1300	50	33	1.5×10^{13}	87
2	1750	50	28	3.3×10^{13}	156
1	1460	40	20	7.8×10^{13}	187

A. The HFB approach at finite temperature

The FT-HFB approach for the inner crust matter was presented in details in Ref. [7]. For the sake of completeness, here we recall the main steps.

Assuming spherical symmetry for the Wigner-Seitz cell, the radial FT-HFB equations have the form:

$$\begin{pmatrix} h_T(r) - \lambda & \Delta_T(r) \\ \Delta_T(r) & -h_T(r) + \lambda \end{pmatrix} \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix}, \tag{1}$$

where E_i is the quasiparticle energy, λ is the chemical potential, $h_T(r)$ is the thermal averaged mean field hamiltonian and $\Delta_T(r)$ is the thermal averaged pairing field. The latter depends on the average pairing density κ_T given by

$$\kappa_T(r) = \frac{1}{4\pi} \sum_i (2j_i + 1) U_i^*(r) V_i(r)(1 - 2f_i), \tag{2}$$

where $f_i = [1 + \exp(E_i/k_B T)]^{-1}$ is the Fermi distribution, k_B is the Boltzmann constant, and T is the temperature. In a self-consistent calculation based on a Skyrme-type force, as used here, $h_T(r)$ depends on the thermal averaged particle density

$$\rho_T(r) = \frac{1}{4\pi}\sum_i (2j_i+1)[V_i^*(r)V_i(r)(1-f_i) + U_i^*(r)U_i(r)f_i], \quad (3)$$

as well as on thermal averaged kinetic energy density and spin density. The expressions of the last two densities are given in Ref. [7].

In the calculations presented here the mean field hamiltonian is calculated with a Skyrme type force while for the thermal averaged pairing field we use a density dependent contact force of the following form [14]:

$$V(\mathbf{r}-\mathbf{r}') = V_0\left[1-\eta\left(\frac{\rho(r)}{\rho_0}\right)^{\alpha}\right]\delta(\mathbf{r}-\mathbf{r}') \equiv V_{\text{eff}}(\rho(r))\delta(\mathbf{r}-\mathbf{r}'), \quad (4)$$

where $\rho(r)$ is the baryonic density and $\rho_0 = 0.16\ \text{fm}^{-3}$. With this force the thermal averaged pairing field is local and given by

$$\Delta_T(r) = \frac{V_{\text{eff}}(\rho(r))}{2}\kappa_T(r), \quad (5)$$

where $\kappa_T(r)$ is the thermal averaged pairing density.

To generate in the outer region of the Wigner-Seitz cell a constant density corresponding to the neutron gas, the FT-HFB equations are solved by imposing Dirichlet-Neumann boundary conditions at the edge of the cell [11], i.e., all wave functions of even parity vanish and the derivatives of odd-parity wave functions vanish. Apart from that, the self-consistent solutions of the HF-HFB equations are found in the same manner as for finite nuclei.

The calculation scheme outlined above is employed to study how the specific heat of the neutrons is behaving in various regions of the inner crust. In order to do that, one has to choose the two-body interactions in the FT-HFB calculations. These interactions should provide a reasonable description of both the nuclear clusters and the neutron gas, which are the baryonic components of the inner crust matter. For the calculation of the mean field we shall use the Skyrme force SLy4 [13], which was fixed to describe properly the mean field properties of neutron-rich nuclei and infinite neutron matter.

The choice of the pairing force is more problematic since at present it is not yet clear what is the strength of pairing correlations in neutron matter. Thus, on one hand, the BCS calculations with bare forces give a maximum gap in neutron matter of about 3 MeV [12]. A maximum gap of about 3 MeV one gets also with the Gogny force [15], which is commonly used to describe the pairing properties in finite nuclei. On the other hand, if one goes beyond the BCS approximation and takes into account the in-medium effects, the maximum gap is suppressed. The suppression depends on the many-body approximations used in the calculations [12]. In order to analyze how the uncertainty on the pairing gap in neutron matter could reflect upon the thermal response of the inner crust, we shall do calculations with two zero range pairing interactions which simulate the pairing gap in nuclear matter obtained either with the Gogny force, or with models which take into account the in-medium effects. For the latter we consider a maximum gap of 1 MeV, as indicated by recent calculations [9]. In Ref. [7] the requirements mentioned above were approximatively satisfied by using two zero range pairing forces [Eq. (4)] having the same parameters for the density dependent term, i.e., $\eta = 0.7$, $\alpha = 0.45$, and two different strengths, i.e., $V_0 = \{-430.0, -330.0\}$ MeV fm^{-3}. These values of the strengths were obtained by solving the FT-HFB equations with a cut-off energy equal to 60 MeV. Since with a 60 MeV cutoff we have numerical problems in solving the FT-HFB equations for large Wigner-Seitz cells, here we shall keep this cutoff and the corresponding strengths only for the first two cells while for the other cells we shall take a smaller cutoff, equal to 20 MeV. This cutoff is introduced smoothly, i.e., by an exponential factor $e^{-E_i^2/100}$ acting for quasiparticle energies $E_i > 20$ MeV. With this smooth energy cutoff we shall use the strengths values $V_0 = \{-570.0, -430.0\}$ MeV fm^{-3}. The pairing force corresponding to the first (second) value of the strength will be called below the strong (weak) pairing force.

B. Specific heat

The quasiparticle spectrum determined by solving the FT-HFB equations is used to calculate the specific heat of the neutrons inside the Wigner-Seitz cell, i.e.,

$$C_V = \frac{T}{V}\frac{\partial S}{\partial T}, \quad (6)$$

where V is the volume of the Wigner-Seitz cell and S is the entropy:

$$S = -k_B\sum_i (2j_i+1)(f_i \ln f_i + (1-f_i)\ln(1-f_i)). \quad (7)$$

The results obtained for the cells listed in Table I are shown in Fig. 1. In the same figure is also shown the specific heat of the electrons, given by [10]

$$C_V^{(e)} = \frac{k_B(3\pi)^{2/3}}{3\hbar c}\left(\frac{Z}{V}\right)^{2/3} T. \quad (8)$$

The specific heats are calculated for a temperature of $T = 0.1$ MeV, which is a typical temperature for the inner crust matter at the cooling stage analysed here (see the discussion below). From Fig. 1 we can see that if the neutrons are in the normal phase, their specific heat is greater than the specific heat of the electrons in all the Wigner-Seitz cells. When the neutron superfluidity is turned on, the specific heat of the neutrons is suppressed due to the pairing gap in the excitation spectrum. Since the suppression depends exponentially on the pairing gap, the results obtained with the strong and the weak pairing forces are very different, as seen for the WS cells 1–5. For the second WS cell, in which the pairing gap in the neutron gas region has the maximum value, the specific heat obtained with the two pairing forces differs by about 7 orders of magnitude. In the WS cells 7–10 the neutron gas is in the normal phase at the temperature $T = 100$ keV. Therefore both pairing forces give the same results for the specific heat.

In Fig. 1 are shown also the values of the specific heat obtained when the nuclear clusters are disregarded. For obtaining these values we have just removed the protons from the cells and perform the FT-HFB calculations in the same conditions as for the cluster+neutron gas. It can be seen that in some cases (see the results for the cells 2–4) the nuclear clusters could have a sizable influence upon the specific heat. However, the influence of the nuclear clusters are relatively small compared to the effect coming form the uncertainty of the pairing force.

C. Thermal diffusivity

The specific heat enters in the heat transport through the thermal diffusivity, defined by $D = \frac{\kappa}{C_V}$, where κ is the thermal conductivity. In the inner crust, the latter is primarily determined by the electrons. The dependence of thermal conductivity on density and temperature was parametrized by Lattimer *et al.* [2], based on the calculations of Itoh *et al.* [16]. For a temperature above 10^8 K analyzed here, the conductivity is nearly independent of the temperature and is given by $\kappa = C(\rho/\rho_0)^{2/3}$, where $C = 10^{21}$ ergs cm^{-1} s^{-1}. With the conductivity given by this expression and the specific heat calculated in the FT-HFB approach one gets the thermal diffusivity shown in Fig. 2. As expected from the behavior of the specific heat, the diffusivity is much smaller for the weak pairing force, except for the last four WS cells. For both pairing forces one can see that the diffusivity is much smaller in the outermost layers of the inner crust. As seen below, these layers have an important contribution to the cooling time of the inner crust.

III. COOLING TIME OF THE INNER CRUST MATTER

In order to calculate the cooling time, i.e., the time needed for the cooling wave to propagate from the cold core to the surface, one should integrate the heat equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\kappa\frac{\partial T}{\partial r}\right] = C_V\frac{\partial T}{\partial t}. \tag{9}$$

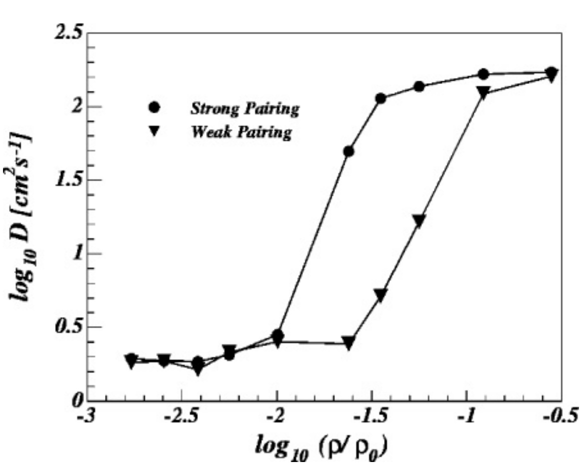

FIG. 2. Thermal diffusivity (neutrons plus electrons) corresponding to the Wigner-Seitz cells listed in Table I. The notations are the same as in Fig. 1.

Since the specific heat and the conductivity depend on density and temperature profile of the crust, the solution of the heat equation is not trivial. Here we use a simple model employed in Refs. [4,8]. The model is based on the following assumptions: (a) the spherical geometry for the heat transport is approximated by a planar geometry, i.e., one considers the heat diffusion through a one-dimensional piece of matter. This approximation is supported by the small thickness of the inner crust compared to the size of the core; (b) the inner crust is divided in layers of constant thermal diffusivity. The diffusion time through a layer of thickness x_i and diffusivity D_i is calculated by the relation $t_i = \gamma\frac{x_i^2}{D_i}$ [10], where the factor γ, which depends on the boundary conditions of the problem, is taken equal to $4/pi^2$ [8]; (c) the total diffusion time across the crust is obtained by summing up the contributions of the layers, i.e.,

$$t_{\rm diff} = \gamma\sum_i\frac{x_i^2}{D_i}. \tag{10}$$

In the equation above the thermal diffusivity depends on density and temperature, $D_i = D(\rho(R_i), T(R_i))$, where R_i is the position of the layer i. In the calculations we divide the inner crust into ten layers, corresponding to the ten cells listed in Table I. The position corresponding to each cell can be found by solving the Tolman-Oppenheimer-Volkov (TOV) equations, which provides the density profile of the star. In the present calculations we use the solution of TOV equations corresponding to the following equations of state [17]: Baym-Pethick-Sutherland [18] for the outer crust, Negele-Vautherin [11] for the inner crust, and Glendenning-Moszkowski [19] for the core. From the solution of the TOV equations one extracts the radii R_i corresponding to the densities of the cells given in Table I. Then, doing a linear interpolation, we determine the size x_i of the layers considered for each cell. The results are shown in Table I.

The diffusivity depends also on the temperature profile. Numerical simulations indicate that before the core-crust thermalisation the temperature is increasing from about $T = 0.1$ MeV to about $T = 0.2$–0.3 MeV when one goes from the outer part to the inner part of the crust. Since the inner part zones of the inner crust have large diffusivities, they contribute less to the cooling time compared to the outermost zones. Therefore, following Ref. [8], we shall consider for all layers a flat temperature profile equal to $T = 0.1$ MeV. The diffusion time across the inner crust obtained for this value of the temperature is shown in Fig. 3. The most striking thing we can notice is the critical dependence of the cooling time on the pairing force. Thus, for a strong pairing force the cooling time is about 12 yr. The largest contributions come from the outermost zones, as noticed also in Ref. [8]. Concerning the effect of the clusters, one can see that is rather small for this temperature. In the case of the weak pairing force, the cooling time is increasing by about a factor two compared to the strong force. Moreover, if the neutron superfluidity is ignored completely, the cooling time is increasing to about 90 yr. These dramatic changes show how important is the precise knowledge of the neutron matter superfluidity for the cooling time of the inner crust.

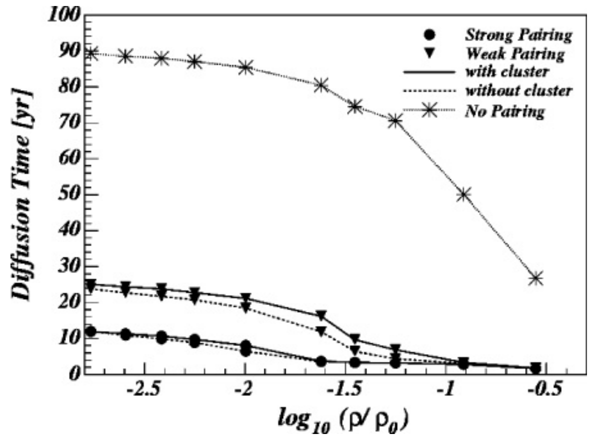

FIG. 3. The diffusion time across the inner crust. The notations are the same as in Fig. 1.

A similar strong dependence of the cooling time on the pairing scenarios we have obtained by using two other zero range forces with the parameters fixed following a different protocol, i.e., a unique strength, $V_0 = -648$ MeV fm^{-3}, and two sets of parameters for the term depended on density, $\eta = \{0.95, 0.87\}$ and $\alpha = \{0.45, 0.2\}$. The value of V_0 was taken so that to get, for a smooth cut-off energy equal to 20 MeV, the experimental value for the scattering length of two free neutrons. The cooling times obtained with these two zero range pairing forces are equal to about 9.1 and 33.8 yr, respectively.

The cooling times calculated in this section are based on the assumption of a flat temperature across the inner crust. This is a rather drastic approximation, especially for the scenario of a weak pairing force when, as seen in Fig. 3, all regions of the inner crust contribute significantly to the total diffusion time. More realistic calculations of the cooling time should be based on dynamical solutions of the heat equations (9).

IV. SUMMARY AND CONCLUSIONS

We have estimated the cooling time of the inner crust matter using the specific heats calculated in the framework of HFB approach at finite temperature. In order to study the effects of the neutron superfluidity on thermal properties of the inner crust, we have employed two paring forces. They have been fixed to reproduce the pairing properties of infinite neutron matter given either by a Gogny force or by microscopic calculations which take into account polarization effects. For the latter we considered a maximum pairing gap in neutron matter equal to 1 MeV. With the two pairing forces we have studied what are the effects of neutron superfluidity on the specific heat and the heat diffusion of inner crust matter. It is shown that the heat diffusion predicted by the two pairing forces are rather different, especially in the higher density part of the inner crust. These differences in the heat diffusion have a big influence upon the cooling time. Thus, if one shifts from one pairing force to the other the cooling time is changing by a factor of two. This show how large could be the window in which the cooling time may vary due to the present lack of knowledge of neutron matter superfluidity.

The neutron superfluidity affects the cooling time through the specific heat, calculated here with the noncollective quasiparticle spectrum provided by the FT-HFB equations. However, the excitation spectrum of the inner crust baryonic matter presents also low-lying collective modes [20]. Since these modes give an important contribution to the specific heat [21], they may also affect significantly the cooling time of the inner crust. This issue will be addressed in a future study.

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