

Irreducible tensor approach to spin observables in the photoproduction of mesons with arbitrary spin-parity s^π

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A theoretical formalism leading to elegant derivation of formulas for all spin observables is outlined for photoproduction of mesons with arbitrary spin-parity s^π . The salient features of this formalism, based on irreducible tensor techniques, are (i) the number of independent irreducible tensor amplitudes is $4(2s + 1)$, (ii) a single compact formula is sufficient to express these amplitudes in terms of allowed electric and magnetic multipole amplitudes, and (iii) all the spin observables, including beam analyzing powers as well as the differential cross section, are expressible in terms of bilinear irreducible tensors of rank 0 to $2(s + 1)$. The relationship between the irreducible tensor amplitudes and the helicity amplitudes is elucidated in general and explicit expressions for the helicity amplitudes are given in terms of the irreducible tensor amplitudes in the particular cases of pseudoscalar and vector meson photoproduction. The connection between the irreducible tensor amplitudes introduced here and the well-known Chew-Goldberger-Low-Nambu amplitudes for photoproduction of pseudoscalar mesons is also established.

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I. INTRODUCTION

Photoproduction of heavy mesons on a nucleon is a topic of considerable current interest. Meson photoproduction has excited attention for more than five decades since the pion was discovered. In the context of developing a relativistic dispersion relation approach to pion photoproduction, Chew, Goldberger, Low, and Nambu (CGLN) [1] have expressed the reaction amplitude \mathcal{F} in terms of four invariants, viz., $i\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\epsilon}}$, $(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}})[\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\epsilon}})]$, $i(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{k}})(\hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{\epsilon}})$, and $i(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}})(\hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{\epsilon}})$ and their respective coefficients, \mathcal{F}_1 , \mathcal{F}_2 , \mathcal{F}_3 , and \mathcal{F}_4 . The $\boldsymbol{\sigma}$ denote Pauli spin matrices of the nucleon, the vectors \boldsymbol{q} and \boldsymbol{k} denote the meson and the photon momenta in the center-of-mass (c.m.) frame, and $\hat{\boldsymbol{\epsilon}}$ denotes the photon polarization. The \mathcal{F}_i , $i = 1, \dots, 4$ are functions of the c.m. energy W at which the reaction takes place and the angle θ between \boldsymbol{q} and \boldsymbol{k} . Following the earlier work of Watson [2] and denoting the isospin index of the photoproduced pion by β , each of these amplitudes \mathcal{F}_i were expressed in terms of three independent nucleon isospin combinations $\mathcal{I}_\beta^{(+)}$, $\mathcal{I}_\beta^{(-)}$, and $\mathcal{I}_\beta^{(0)}$ and their respective coefficients $\mathcal{F}_i^{(+)}$, $\mathcal{F}_i^{(-)}$, and $\mathcal{F}_i^{(0)}$. Explicit formulae for \mathcal{F}_i have been given [1] in terms of the first two derivatives of Legendre polynomials in $\cos\theta$ and energy-dependent “magnetic” and “electric” multipole amplitudes denoted, respectively, by $M_{l\pm}$ and $E_{l\pm}$, where the suffix \pm indicates that the total angular momentum $j = l \pm \frac{1}{2}$, if l denotes the orbital angular momentum of the emitted meson. This traditional formalism has been reviewed along with the extension [3] to electroproduction by Berends, Donnachie, and Weaver [4] in a set of three articles. The

connection with the helicity formalism [5] for photoproduction as well as formulae for the differential cross-section and spin observables in terms of the CGLN amplitudes are also found in Ref. [4] along with numerical values for the amplitudes up to 500 MeV. The dominance of the first resonance viz., $\Delta_{33}(1232)$ is quite conspicuous in this energy region and as such attention has lately been focused [6,7] on details such as the quadrupole deformation of this resonance and ratio of E_{1+} over M_{1+} . In view of the necessity of neutron multipoles [8] to determine $\mathcal{F}^{(+)}$, $\mathcal{F}^{(-)}$, and $\mathcal{F}^{(0)}$ and their importance to decide questions of time reversal violation [9] or the possible existence of isotensor term [10], a careful discussion of target asymmetry and effective neutron polarization was presented [11] as also detailed theoretical analyses to extract the neutron multipoles more precisely from experiments on hydrogen isotopes [12]. Pion photoproduction has been studied extensively and reviewed by several groups [13]. The current database as well as numerical values for the electric and magnetic multipole amplitudes derived from accumulated data can be found on the Center for Nuclear Studies (CNS) Web site [14]. Observables have been defined in terms of helicity and transversity amplitudes [15]. The helicity amplitudes [16] can also be constructed from the multipole amplitudes [14] using relations found in Ref. [17]. As photon energy is increased, the nucleon resonances $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, and $S_{11}(1650)$ start contributing along with the first and higher Δ resonances [18].

In contrast to the isovector pion, the η meson is isoscalar. Consequently, the Δ resonances do not contribute to photoproduction of η and as such it is ideally suited to study the nucleon

resonances. The enigmatic [6,19] Roper resonance $P_{11}(1440)$ lies below η production threshold, so that only its high-energy tail can contribute. Close to threshold, the contributions of $P_{11}(1440)$ and $D_{13}(1520)$ with larger l values are suppressed compared to the $S_{11}(1535)$ and $S_{11}(1650)$. However, the decay pattern of the first and second S wave nucleon resonances were found to be quite different and are not easily explained in terms of nucleon models. In fact, a variety of nucleon models predict a much richer nucleon excitation spectrum than what has been identified in πN scattering. Considerable interest has, therefore, been evinced in looking for the so-called missing resonances [20]. Reference may be made to theoretical studies [21] based on effective Lagrangian approach. Excellent reviews [22] exist on these developments and the problem of missing resonances in photoproduction reactions. As η is also a pseudoscalar like the pion, the spin structure of the amplitudes for photoproduction of η is of the same form as for pion photoproduction and the corresponding multipole amplitudes can also be found in Ref. [14].

The associated strangeness photo production has been studied [23] over the past several decades employing different approaches. In contrast to photoproduction of pions and eta, kaon photoproduction can involve nucleon as well as hyperon resonances like $S_{01}(1405)$, $S_{01}(1670)$, $P_{01}(1810)$ and some models include nucleon resonances with spin greater than $3/2$. It was remarked [24] recently that $S_{11}(1650)$, $P_{13}(1720)$, $D_{13}(1700)$, $D_{13}(2080)$, $F_{15}(1680)$, and $F_{15}(2000)$ are required to fit one set of experimental data, whereas $P_{13}(1900)$, $D_{13}(2080)$, $D_{15}(1675)$, $F_{15}(1680)$, and $F_{17}(1990)$ resonances are required to fit another set. Although both the sets do not exhibit the need for $P_{11}(1710)$ and are agreed that the second peak in the cross sections at $W \sim 1900$ MeV originates from the $D_{13}(2080)$ resonance [18] whose mass lies between 1911 and 1936 MeV, fitting to all data simultaneously changes the conclusion and results in a model that is inconsistent to all data sets. It is clear that the delta resonances can also contribute in addition to the nucleon resonances in the case of $\gamma p \rightarrow \Sigma K$. The beam analyzing powers [25] and beam-recoil observables [26] have also been measured very recently. In hyperon photoproduction, the hyperons have spin-parity $\frac{1}{2}^+$ just like the nucleons, whereas kaons are pseudoscalar like the pion. Consequently, the spin structure of the amplitudes for hyperon and pion photoproduction are alike [27], although the isospin considerations are different as with the case of photoproduction of η .

As photon energy is increased further, the thresholds are reached for ρ , η' , ω , and ϕ , of which η' is pseudoscalar and ρ , ω , ϕ are vector mesons. Experimental studies on photoproduction of ρ and ω date back to the early 1960s and it has been observed that the reaction is governed by diffraction or pomeron exchange at higher energies. The inherent advantages in employing linearly polarized photons [28] have been noted and a detailed formalism to analyze production of vector mesons with polarized photons has also been presented [29], employing 12 complex helicity amplitudes. A historical perspective and a detailed account of the theoretical ideas that motivated early studies on vector meson photoproduction may be found in Ref. [30]. In the case of photoproduction of a vector meson, the meson polarization itself is an interesting

observable that has been studied experimentally in recent years [31] and analyzed using the formalism of Ref. [29]. All the polarization observables were expressed as bilinear [32] products of the 12 helicity amplitudes and their behavior near threshold [33] has been examined. It was shown [34] that a two-meson decay of a ρ or ϕ does not determine its vector polarization. More recently [35], it was shown that the three-meson decay of ω also does not determine its vector polarization. However, the $\pi_0\gamma$ decay mode of $\bar{\omega}$ with a smaller branching ratio of 8.92% may be utilized to determine its vector as well as tensor polarization [36]. The isovector ρ has a large width of 150.2 MeV, as compared to the widths of 8.44 and 4.458 MeV of the isoscalar ω and ϕ , respectively, and of 1.18×10^{-3} MeV and 0.2 MeV, respectively, of η and η' . There are hardly any experimentally known resonances which decay into $N\omega$, except $N(1710)$ which has a branching ratio [18] of $\sim 13\%$. However, there are theoretical expectations that missing resonances may couple more strongly or even exclusively to the $N\omega$ channel in comparison with the $N\pi$ channel. Hence several theoretical studies have been carried out [37] on the contribution of nucleon resonances in ω photoproduction. Another interesting aspect of vector meson photoproduction is the ϕ/ω ratio [38] in the context of the violation of OZI rule [39]. This energy region above the threshold for photoproduction of η , ρ , ω , η' , and ϕ is under intense study at present with the advent of the new generation of electron accelerators like CEBAF at JLab, ELSA at Bonn, ESRF at Grenoble, MAMI at Mainz and Spring8 at Osaka, which are equipped with tagged photon facilities where the incident photon energies are known event by event within the resolution of the tagging detector. While photon beams are produced by bremsstrahlung at JLab (CLAS), ELSA (SAPHIR), and MAMI, laser backscattering is employed at BNL (LEGS), ESRF (GRAAL), and Spring8 (LEPS). Availability of linearly and circularly polarized photons as well as polarized targets enable measurements of beam and target analyzing powers, in addition to the differential cross section. Such measurements are useful to determine resonance parameters. In future one may look forward to photoproduction of higher spin mesons [18], like $f_2(1270)$ or $a_2(1320)$ or $f'_2(1525)$ with spin-parity 2^+ , $\pi_2(1670)$ with spin-parity 2^- , $\omega_3(1670)$ or $\rho_3(1690)$ with spin-parity 3^- and $a_4(2040)$ or $f_4(2050)$ with spin-parity 4^+ . An exotic baryon state with mass $M = 1555 \pm 10$ MeV and strangeness $+1$ was observed [40] in photoproduction from the proton. More recently, a theoretical analysis has been carried out on meson spectra in a generalized constituent quark model [41] that pays attention also to glueballs, hybrids, or multiquark states. In view of these developments, it is felt that a supportive theoretical formalism for the spin structure of the amplitudes and their expansion in terms of “electric” and “magnetic” multipoles is needed to analyze measurements of spin observables in photoproduction of mesons with arbitrary spin-parity s^π .

The purpose of the present article is to answer this need. Employing irreducible tensor operator techniques [42] we derive formulas for all spin observables, including beam analyzing powers, associated with photoproduction of mesons with isospin I_m and arbitrary spin-parity s^π after outlining the basic theoretical formalism in the next section.

II. THEORETICAL FORMALISM

Let \mathbf{q} and \mathbf{k} denote the meson and the photon momenta in c.m. frame for photoproduction of arbitrary spin mesons at center-of-mass (c.m.) energy W . If $\hat{\mathbf{u}}_x, \hat{\mathbf{u}}_y$ denote unit vectors of a right-handed Cartesian coordinate system with \mathbf{k} along the z axis, the left and right circular polarized states of the photon may be defined, following Rose [43], through

$$\hat{\mathbf{u}}_\mu = \frac{1}{\sqrt{2}}(\hat{\mathbf{u}}_x + i\mu\hat{\mathbf{u}}_y) = -\mu\hat{\boldsymbol{\xi}}_\mu, \quad \mu = \pm 1. \quad (1)$$

When the reaction is initiated by photons polarized in a state $\hat{\mathbf{u}}_\mu$, the differential cross section in c.m. frame may be written as

$$\frac{d\sigma(\mu)}{d\Omega} = \frac{q}{k} \sum_{av} \sum_{av} |\langle f | \mathcal{F}(\mu) | i \rangle|^2, \quad (2)$$

where $|i\rangle$ and $|f\rangle$ denote, respectively, the initial and final hadron spin states, \mathbf{q} and \mathbf{k} have polar coordinates (q, θ, φ) and $(k, 0, 0)$, respectively, and \sum denotes summation with respect to final and \sum_{av} the average with respect to initial hadron spin states.

When a meson with spin s is photoproduced on a nucleon, the channel spin s_f in the final state could assume either of the values $|s - \frac{1}{2}|$ and $(s + \frac{1}{2})$ and the transition to the final hadron system with spin s_f takes place from the initial state of the hadron with spin $s_i = \frac{1}{2}$. We may, therefore write the reaction amplitude $\mathcal{F}(\mu)$ in the form

$$\mathcal{F}(\mu) = \sum_{s_f=|s-\frac{1}{2}|}^{(s+\frac{1}{2})} \sum_{\lambda=|s_f-\frac{1}{2}|}^{(s_f+\frac{1}{2})} \left(S^\lambda \left(s_f, \frac{1}{2} \right) \cdot \mathcal{F}^\lambda(s_f, \mu) \right) \quad (3)$$

in terms of irreducible tensor operators $S_{m_\lambda}^\lambda(s_f, s_i)$ of rank λ in hadron spin space defined in Ref. [42]. To identify the irreducible tensor amplitudes $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)$, we may evaluate $\langle f | \mathcal{F}(\mu) | i \rangle$ by writing it explicitly as

$$\begin{aligned} & \left\langle \left(s \frac{1}{2} \right) s_f m_f; \mathbf{q} \left| \mathcal{F} \right| \mathbf{k} \hat{\mathbf{u}}_\mu; \frac{1}{2} m_i \right\rangle \\ &= 4\pi(2\pi)^{\frac{1}{2}} \sum_{l=0}^{\infty} (-i)^l \sum_{L=1}^{\infty} i^L [L] \sum_{j=L-\frac{1}{2}}^{L+\frac{1}{2}} C \left(L \frac{1}{2} j; \mu m_i m \right) \\ & \times C(l s_f j; m_l m_f m) Y_{lm_l}(\theta, \varphi) (i\mu)^{f+(L,l)} \mathcal{F}_{ls_f;L}^j, \quad (4) \end{aligned}$$

using the standard multipole expansion [43] for the photon in the initial state and partial wave expansion for the meson in the final state. We denote $\sqrt{(2L+1)}$ by $[L]$. The rest of the notations follow Rose [43]. Using

$$f_\pm(L, l) = \frac{1}{2} [1 \pm \pi(-1)^{L-l}] \quad (5)$$

and parity conservation, we may express

$$\begin{aligned} \mathcal{F}_{ls_f;L}^j &\equiv \langle (l(s\frac{1}{2})s_f)j \| \mathcal{F} \| (L\frac{1}{2})j \rangle \\ &= \mathcal{M}_{ls_f;L}^j f_-(L, l) + \mathcal{E}_{ls_f;L}^j f_+(L, l) \quad (6) \end{aligned}$$

in terms of ‘‘magnetic’’ and ‘‘electric’’ multipole amplitudes denoted by $\mathcal{M}_{ls_f;L}^j$ and $\mathcal{E}_{ls_f;L}^j$, respectively. We rewrite the

two Clebsch-Gordan coefficients in Eq. (4) as

$$\begin{aligned} & C \left(L \frac{1}{2} j; \mu m_i m \right) C(l s_f j; m_l m_f m) \\ &= \sum_\lambda W \left(L \frac{1}{2} l s_f; j \lambda \right) [j]^2 [s_f]^{-1} (-1)^{L+l+\frac{1}{2}-j} (-1)^{m_\lambda} \\ & \times C(l L \lambda; m_l - \mu m_\lambda) (-1)^\mu C \left(\frac{1}{2} \lambda s_f; m_i - m_\lambda m_f \right) [\lambda] \quad (7) \end{aligned}$$

and replace

$$C \left(\frac{1}{2} \lambda s_f; m_i - m_\lambda m_f \right) [\lambda] = \langle s_f m_f | S_{-m_\lambda}^\lambda(s_f, \frac{1}{2}) | \frac{1}{2} m_i \rangle. \quad (8)$$

Comparing the resulting expression with $\langle f | \mathcal{F}(\mu) | i \rangle$ and using Eq. (3) we obtain the elegant and compact formula for the irreducible tensor amplitudes

$$\begin{aligned} \mathcal{F}_{m_\lambda}^\lambda(s_f, \mu) &= 4\pi(2\pi)^{\frac{1}{2}} \sum_{l=0}^{\infty} \sum_{L=1}^{\infty} (i)^{L-l} \sum_j (-1)^{L+l+\frac{1}{2}-j} \\ & \times [j]^2 [L] [s_f]^{-1} W \left(L \frac{1}{2} l s_f; j \lambda \right) \\ & \times (i\mu)^{f+(L,l)} \mathcal{F}_{ls_f;L}^j (-1)^\mu \\ & \times C(l L \lambda; m_l - \mu m_\lambda) Y_{lm_l}(\theta, \varphi), \quad (9) \end{aligned}$$

in terms of partial wave multipole amplitudes $\mathcal{F}_{ls_f;L}^j$ given by Eq. (6) for photoproduction of mesons with arbitrary spin-parity s^π .

Isospin considerations lead to

$$\begin{aligned} \mathcal{F}_{ls_f;L}^j &= \sum_{I=|I_m-\frac{1}{2}|}^{(I_m+\frac{1}{2})} C \left(\frac{1}{2} I_m I; \nu_f \nu_m \nu \right) \\ & \times \sum_{I_\gamma=0}^1 C \left(\frac{1}{2} I_\gamma I; \nu_i 0 \nu \right) \mathcal{F}_{ls_f;L}^{I_\gamma I j}, \quad (10) \end{aligned}$$

where ν_i, ν_f, ν_m denote, respectively, the isospin projection quantum numbers of the nucleon in the initial and final states and the meson which is photoproduced. Dropping the indices j, l, s_f, L common to both sides of Eq. (10), the $\mathcal{F}^{I_\gamma I}$ thus defined for pion photoproduction may readily be related to the $\mathcal{F}^{(\pm)}, \mathcal{F}^{(0)}$ of CGLN [1] as shown in Appendix A. The isospin indices I_γ, I may also be attached using Eq. (10) to the ‘‘magnetic’’ and ‘‘electric’’ multipole amplitudes defined by Eq. (6). If one is looking for the possible existence of an isotensor component [10], the summation over I_γ may readily be extended in Eq. (10) to include $I_\gamma = 2$. The advantage in having the index I along with j in this formalism is that it facilitates ready identification with the isospin and spin quantum numbers of the resonances that contribute in the intermediate state to photoproduction of mesons with arbitrary spin-parity s^π .

It may be noted that the spherical harmonics Y_{lm_l} in Eq. (9) contain the azimuthal angle φ along with the polar angle θ of the momentum \mathbf{q} of the meson. This facilitates the analysis of experiments on photo meson production employing linearly

polarized photons where the state of linear polarization of the beam is chosen to be along the x axis.

A. Irreducible tensor amplitudes in the Madison frame

In discussing meson photoproduction with a two-body final state, it is quite often convenient to choose the reaction plane containing \mathbf{k} and \mathbf{q} as the zx plane in which case the azimuthal angle $\varphi = 0$. In fact this choice of the Cartesian coordinate system has generally been recommended by the Madison convention [44]. We may therefore refer to this Cartesian coordinate system as the Madison frame (MF). The irreducible tensor amplitudes $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)$ in MF are then given by Eq. (9) with $\varphi = 0$, in which case they satisfy

$$\mathcal{F}_{-m_\lambda}^\lambda(s_f, -\mu) = \pi(-1)^{\lambda-m_\lambda} \mathcal{F}_{m_\lambda}^\lambda(s_f, \mu). \quad (11)$$

In view of the above, the total number

$$N_{\text{tot}} = \sum_{\mu=-1,1} \sum_{s_f=|s-\frac{1}{2}}^{(s+\frac{1}{2})} \sum_{\lambda=|s_f-\frac{1}{2}}^{(s_f+\frac{1}{2})} (2\lambda+1) = 8(2s+1), \quad (12)$$

of irreducible tensor amplitudes reduces to $4(2s+1)$ independent irreducible tensor amplitudes. This number is exactly in agreement with 4 [1] for $s = 0$ and 12 [29] for $s = 1$ arrived at by using different arguments in those particular cases. In the case of electroproduction [45], the longitudinal polarization state with $\mu = 0$ contributes additional $2(2s+1)$ independent amplitudes and consequently the total number of independent amplitudes for electroproduction turns out to be $6(2s+1)$. It may be noted that $\mathcal{M}_{s_f;L}^j$, $\mathcal{E}_{s_f;L}^j$ and $\mathcal{F}_{s_f;L}^j$ in Ref. [45] are $4\pi \frac{3}{2} i^{l+L} (-1)^{L+\frac{1}{2}-j} [L][j]^2 [s_f][s]^{-1}$ times the $\mathcal{M}_{s_f;L}^j$, $\mathcal{E}_{s_f;L}^j$ and $\mathcal{F}_{s_f;L}^j$ given by Eq. (6) and that the irreducible tensor amplitudes $\mathcal{F}_{m_\lambda}^\lambda(n, \mu)$, $n = 0, 1$ introduced in Ref. [45] may also be expressed in terms of the $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)$ through

$$\mathcal{F}_{m_\lambda}^\lambda(n, \mu) = \frac{1}{\sqrt{2}} [n][s]^{-1} \sum_{s_f} (2s_f+1) \times W\left(\lambda \frac{1}{2} s \frac{1}{2}; s_f n\right) \mathcal{F}_{m_\lambda}^\lambda(s_f, \mu), \quad (13)$$

or conversely

$$\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu) = \sqrt{2} [s][s_f]^{-1} \sum_n W\left(\lambda \frac{1}{2} s \frac{1}{2}; s_f n\right) \mathcal{F}_{m_\lambda}^\lambda(n, \mu). \quad (14)$$

It may be noted that the irreducible tensor operators $S_{m_n}^n(\frac{1}{2}, \frac{1}{2})$ of rank n in Ref. [45] are identified as

$$\begin{aligned} S_0^0\left(\frac{1}{2}, \frac{1}{2}\right) &= 1 \\ S_{\pm 1}^1\left(\frac{1}{2}, \frac{1}{2}\right) &= \mp \frac{1}{\sqrt{2}} (\sigma_x \pm i\sigma_y) \\ S_0^1\left(\frac{1}{2}, \frac{1}{2}\right) &= \sigma_z, \end{aligned} \quad (15)$$

in terms of Pauli matrices. It may perhaps be mentioned that in the case of kaon photoproduction, the rows and columns of these matrices are to be labeled by the spin states of the hyperon and nucleon, respectively. In the particular case of photoproduction of pseudoscalar mesons, the connection between our amplitudes given by Eq. (9) with $\phi = 0$ in the Madison frame [44], and those of CGLN is established in Appendix A.

B. Irreducible tensor amplitudes in the Transverse frame

The Transverse frame (TF) may be defined as the right-handed Cartesian coordinate system with the z axis chosen along $\mathbf{k} \times \mathbf{q}$, i.e., transverse to the reaction plane and with the x axis chosen along \mathbf{k} . The explicit form for $\langle f | \mathcal{F}(\mu) | i \rangle$, in this frame given by

$$\begin{aligned} & \left\langle \left(s \frac{1}{2} \right) s_f m_f; \mathbf{q} \left| \mathcal{F} \right| \mathbf{k} \hat{\mathbf{u}}_\mu; \frac{1}{2} m_i \right\rangle \\ &= 4\pi (2\pi)^{\frac{1}{2}} \sum_{l=0}^{\infty} (-i)^l \sum_{L=1}^{\infty} i^L [L] \sum_{j=L-\frac{1}{2}}^{L+\frac{1}{2}} (i\mu)^{f_+(L,l)} \mathcal{F}_{ls_f;L}^j \\ & \times \sum_{M=-L}^L C\left(L \frac{1}{2} j; M m_i m\right) C(ls_f j; m_i m_f m) \\ & \times D_{M\mu}^L\left(0, \frac{\pi}{2}, 0\right) Y_{lm_i}\left(\frac{\pi}{2}, \theta\right), \end{aligned} \quad (16)$$

instead of Eq. (4), so that the irreducible tensor amplitudes in TF are given by

$$\begin{aligned} \mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)_{TF} &= 4\pi (2\pi)^{\frac{1}{2}} \sum_{l=0}^{\infty} \sum_{L=1}^{\infty} (i)^{L-l} \sum_j (-1)^{L+l+\frac{1}{2}-j} \\ & \times [j]^2 [L][s_f]^{-1} W\left(L \frac{1}{2} l s_f; j \lambda\right) \\ & \times \mathcal{F}_{ls_f;L}^j \mathcal{A}_{m_\lambda}^\lambda(\mu, \theta), \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathcal{A}_{m_\lambda}^\lambda(\mu, \theta) &= (i\mu)^{f_+(L,l)} \sum_M (-1)^M C(lL\lambda; m_l - M m_\lambda) \\ & \times Y_{lm_l}\left(\frac{\pi}{2}, \theta\right) d_{M\mu}^L\left(\frac{\pi}{2}\right), \end{aligned} \quad (18)$$

which now defines the angular distribution of the meson as a function of θ in the TF.

The amplitudes (17) are related to $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)$ in the Madison frame through

$$\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)_{TF} = \sum_{m'_\lambda=-\lambda}^{+\lambda} D_{m'_\lambda m_\lambda}^\lambda\left(\frac{\pi}{2}, \frac{\pi}{2}, \pi\right) \mathcal{F}_{m'_\lambda}^\lambda(s_f, \mu). \quad (19)$$

In fact, the irreducible tensor amplitudes $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)_{AF}$ in any frame (AF) may be expressed in terms of the $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)_{GF}$ in any given frame (GF) through

$$\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)_{AF} = \sum_{m'_\lambda} D_{m'_\lambda m_\lambda}^\lambda(\alpha, \beta, \gamma) \mathcal{F}_{m'_\lambda}^\lambda(s_f, \mu)_{GF} \quad (20)$$

if (α, β, γ) denote the Euler angles [43] which characterize rotation connecting AF from GF. The relationship between the irreducible tensor amplitudes and helicity amplitudes is elucidated in Appendix B.

It is sometimes convenient to chose a frame whose z axis is along \mathbf{q} , y axis is along $(\mathbf{k} \times \mathbf{q})$, and x axis is along $(\mathbf{k} \times \mathbf{q}) \times \mathbf{q}$, in which case $\alpha = 0, \beta = \theta$, and $\gamma = 0$, when GF is identified with MF. The spin states $|sm_s\rangle$ of the meson defined with respect to this frame remain the same, when we make a Lorentz transformation along \mathbf{q} to reach the meson rest frame. This Lorentz transformation is needed en route to Gottfried-Jackson frame [46].

We will express the differential cross section and all spin observables in the next section in terms of bilinear irreducible tensors $B_{m_\Lambda}^\Lambda$ of rank Λ constructed out of the $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)$. It may be noted that the $B_{m_\Lambda}^\Lambda$ are related to observables measurable experimentally, which may conveniently be obtained in any required frame by choosing irreducible tensor amplitudes in the appropriate frame.

III. DIFFERENTIAL CROSS-SECTION AND SPIN OBSERVABLES

The unpolarized differential cross section, in c.m. frame, is given by

$$\frac{d\sigma_0}{d\Omega} = \frac{1}{2} \sum_{\mu=-1,1} \frac{d\sigma(\mu)}{d\Omega}, \quad (21)$$

where $d\sigma(\mu)/d\Omega$ defined by Eq. (2) is readily given, on using Eqs. (3) and (8) by

$$\frac{d\sigma(\mu)}{d\Omega} = \frac{q}{2k} \sum_{s_f} (2s_f + 1) \sum_{m_\lambda} |\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)|^2. \quad (22)$$

A. Beam analyzing powers

The beam analyzing power Σ_3 with respect to left and right circular polarized states is readily given by

$$\frac{d\sigma_0}{d\Omega} \Sigma_3 = \frac{d\sigma(+1)}{d\Omega} - \frac{d\sigma(-1)}{d\Omega}, \quad (23)$$

where $d\sigma_0/d\Omega$ and $d\sigma(\mu)/d\Omega$ for $\mu = \pm 1$ are known from Eqs. (21) and (22). It is clear from Eq. (1) that

$$\hat{\mathbf{u}}_x = \frac{1}{\sqrt{2}}(\hat{\mathbf{u}}_{+1} + \hat{\mathbf{u}}_{-1}); \quad \hat{\mathbf{u}}_y = \frac{-i}{\sqrt{2}}(\hat{\mathbf{u}}_{+1} - \hat{\mathbf{u}}_{-1}), \quad (24)$$

so that photons linearly polarized along an azimuthal angle α are represented by

$$\begin{aligned} \hat{\mathbf{u}}_\alpha &= \hat{\mathbf{u}}_x \cos \alpha + \hat{\mathbf{u}}_y \sin \alpha \\ &= \frac{1}{\sqrt{2}}[\hat{\mathbf{u}}_{+1} e^{-i\alpha} + \hat{\mathbf{u}}_{-1} e^{i\alpha}]. \end{aligned} \quad (25)$$

The differential cross section, when the beam is linearly polarized along α is then given by

$$\begin{aligned} \frac{d\sigma(\alpha)}{d\Omega} &= \frac{q}{4k} \sum_{s_f} (2s_f + 1) \sum_{\lambda} \sum_{m_\lambda} \\ &\times |\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) e^{-i\alpha} + \mathcal{F}_{m_\lambda}^\lambda(s_f, -1) e^{i\alpha}|^2 \end{aligned} \quad (26)$$

using which we may readily define the beam analyzing powers Σ_1 and Σ_2 , with respect to linearly polarized states, through

$$\frac{d\sigma_0}{d\Omega} \Sigma_1 = \frac{d\sigma(\alpha = 0)}{d\Omega} - \frac{d\sigma(\alpha = \pi/2)}{d\Omega} \quad (27)$$

$$\begin{aligned} &= \frac{q}{k} \sum_{s_f} [s_f]^2 \sum_{\lambda} \sum_{m_\lambda} \\ &\times \Re[\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) \mathcal{F}_{m_\lambda}^\lambda(s_f, -1)^*], \end{aligned} \quad (28)$$

$$\frac{d\sigma_0}{d\Omega} \Sigma_2 = \frac{d\sigma(\alpha = \pi/4)}{d\Omega} - \frac{d\sigma(\alpha = 3\pi/4)}{d\Omega} \quad (29)$$

$$\begin{aligned} &= \frac{q}{k} \sum_{s_f} [s_f]^2 \sum_{\lambda} \sum_{m_\lambda} \\ &\times \Im[\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) \mathcal{F}_{m_\lambda}^\lambda(s_f, -1)^*], \end{aligned} \quad (30)$$

where $\mathcal{F}_{m_\lambda}^\lambda(s_f, \mu)^*$ denotes the complex conjugate of Eq. (9).

We may note that the analyzing powers $\Sigma_1, \Sigma_2, \Sigma_3$ correspond, respectively, to the well-known Stokes parameters s_1, s_2, s_3 in terms of which the state of polarization of the beam may be described by the density matrix

$$\rho^\gamma = \frac{1}{2} [1 + \sigma_1^\gamma s_1 + \sigma_2^\gamma s_2 + \sigma_3^\gamma s_3], \quad (31)$$

where $\sigma_1^\gamma, \sigma_2^\gamma, \sigma_3^\gamma$ denote Pauli matrices, whose rows and columns are labeled by $\hat{\mathbf{u}}_{\pm 1}$. We may also note that any arbitrary state of polarization of radiation, represented by a point on the Poincare sphere with polar coordinates (θ_P, φ_P) , may be represented by

$$\hat{\boldsymbol{\epsilon}}(\alpha, \beta) = \hat{\mathbf{u}}_\alpha \cos \beta + i \hat{\mathbf{u}}_{\alpha+\pi/2} \sin \beta, \quad (32)$$

with $0 \leq \alpha = \varphi_P/2 < \pi$ and $-\pi/4 \leq \beta = \pi/4 - \theta_P/2 \leq \pi/4$, corresponding to which the differential cross section is given by

$$\begin{aligned} \frac{d\sigma(\alpha, \beta)}{d\Omega} &= \frac{q}{4k} \sum_{s_f} [s_f]^2 \sum_{\lambda} \sum_{m_\lambda} |\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) \\ &\times (\cos \beta + \sin \beta) e^{-i\alpha} + \mathcal{F}_{m_\lambda}^\lambda(s_f, -1) \\ &\times (\cos \beta - \sin \beta) e^{i\alpha}|^2 \\ &= \frac{q}{4k} \sum_{s_f} [s_f]^2 \sum_{\lambda} \sum_{m_\lambda} [(1 + \sin 2\beta) |\mathcal{F}_{m_\lambda}^\lambda(s_f, 1)|^2 \\ &+ (1 - \sin 2\beta) |\mathcal{F}_{m_\lambda}^\lambda(s_f, -1)|^2 \\ &+ 2 \cos 2\beta \sin 2\alpha \Re[\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) \mathcal{F}_{m_\lambda}^\lambda(s_f, -1)^*] \\ &+ 2 \cos 2\beta \sin 2\alpha \Im[\mathcal{F}_{m_\lambda}^\lambda(s_f, 1) \mathcal{F}_{m_\lambda}^\lambda(s_f, -1)^*]], \end{aligned} \quad (33)$$

which readily specializes to give Eqs. (21), (22), and (26).

B. General expression for hadron spin observables

The state of polarization of the target is conveniently specified in terms of the initial spin density matrix

$$\rho^i = \frac{1}{2} [1 + \boldsymbol{\sigma} \cdot \mathbf{P}], \quad (35)$$

where $\boldsymbol{\sigma}$ denote Pauli spin matrices for the nucleon and \mathbf{P} its polarization. Using Eqs. (31) and (35), the state of hadron

polarization in the final state is, in general, described by the final spin density matrix

$$\rho^f = \sum_{\mu, \mu' = -1, 1} \mathcal{F}(\mu) \rho_{\mu\mu'}^f \mathcal{F}^\dagger(\mu') \quad (36)$$

where $\rho_{\mu\mu'}^f$ denote elements of Eq. (31) and $\mathcal{F}^\dagger(\mu)$, which denotes the hermitian conjugate of Eq. (3) with respect to hadron spin states may be written as

$$\begin{aligned} \mathcal{F}^\dagger(\mu) &= \sum_{s_f} \sum_{\lambda} (-1)^{\frac{1}{2} - s_f} [s_f][s_i]^{-1} \\ &\times (S^\lambda(s_i, s_f) \cdot \mathcal{F}^{\dagger\lambda}(s_f, \mu)), \end{aligned} \quad (37)$$

where the notation

$$\mathcal{F}_{m_\lambda}^{\dagger\lambda}(s_f, \mu) = (-1)^{m_\lambda} \mathcal{F}_{-m_\lambda}^\lambda(s_f, \mu)^* \quad (38)$$

is used. The polarized differential cross section, in general, is given by

$$\frac{d\sigma}{d\Omega} = \frac{q}{k} \text{Tr} \rho^f, \quad (39)$$

where Tr denotes the trace or spur. The results of Sec. III A are simply particular cases of Eq. (39) with $\mathbf{P} = 0$. Therefore, all the analyzing powers and initial spin correlations may be defined in terms of Eq. (39), whereas comprehensive information with regard to spin transfers as well as final state polarizations and spin correlations are contained in Eq. (36). To proceed further and to derive explicit formulas for the spin observables in terms of the irreducible tensor amplitudes Eq. (9), we introduce the notations

$$\rho^i = \frac{1}{2} \sum_{\lambda_i=0} \left(S^{\lambda_i} \left(\frac{1}{2}, \frac{1}{2} \right) \cdot P^{\lambda_i} \right), \quad (40)$$

where $P_0^0 = 1$; $P_0^1 = P_z$; $P_{\pm 1}^1 = \mp \frac{1}{\sqrt{2}} [P_x \pm iP_y]$ and

$$\begin{aligned} \rho^f(\mu, \mu') &= \mathcal{F}(\mu) \rho_i \mathcal{F}^\dagger(\mu') \\ &= \sum_{s_f, s_f'} \sum_{\lambda_f = |s_f - s_f'|}^{s_f + s_f'} (S^{\lambda_f}(s_f, s_f') \cdot t^{\lambda_f}). \end{aligned} \quad (41)$$

Using the known [42] property

$$\begin{aligned} (S^{\lambda_2}(s_3, s_2) \otimes S^{\lambda_1}(s_2, s_1))_{m_\lambda}^\lambda \\ = (-1)^{\lambda_1 + \lambda_2 - \lambda} [\lambda_1][\lambda_2][s_2] W(s_1 \lambda_1 s_3 \lambda_2; s_2 \lambda) \\ \times S_{m_\lambda}^\lambda(s_3, s_1), \end{aligned} \quad (42)$$

of the irreducible tensor operators and standard Racah algebra, we have

$$t_{\mu_f}^{\lambda_f} = \sum_{\lambda_i, \Lambda, \lambda, \lambda'} G(P^{\lambda_i} \otimes B^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'})_{\mu_f}^{\lambda_f}, \quad (43)$$

where $B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'}$ denote the bilinear irreducible tensors

$$B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'} = (\mathcal{F}^\lambda(s_f, \mu) \otimes \mathcal{F}^{\dagger\lambda'}(s_f', \mu'))_{m_\Lambda}^\Lambda \quad (44)$$

of rank Λ and the geometrical factors G are given by

$$\begin{aligned} G &= \frac{1}{\sqrt{2}} (-1)^{s_f - \frac{1}{2}} [s_f'] [\lambda_i] [\lambda] [\lambda'] [\Lambda] \\ &\times \sum_{\lambda'' = |\lambda_i - \lambda'|}^{\lambda_i + \lambda'} (-1)^{\lambda_f - \lambda' - \lambda'' + \Lambda} [\lambda'']^2 \\ &\times W\left(s_f' \lambda' \frac{1}{2} \lambda_i; \frac{1}{2} \lambda''\right) W\left(s_f' \lambda'' s_f \lambda; \frac{1}{2} \lambda_f\right) \\ &\times W(\lambda_i \lambda' \lambda_f \lambda; \lambda'' \Lambda). \end{aligned} \quad (45)$$

We may thus express ρ^f in terms of its elements

$$\rho_{s_f m_f; s_f' m_f'}^f = \sum_{\lambda_f} (-1)^{\mu_f} C(s_f' \lambda_f s_f; m_f' - \mu_f m_f) [\lambda_f] T_{\mu_f}^{\lambda_f}, \quad (46)$$

where the $T_{\mu_f}^{\lambda_f}$ are given in general by

$$T_{\mu_f}^{\lambda_f} = \sum_{\mu, \mu' = -1, 1} t_{\mu_f}^{\lambda_f} \rho_{\mu\mu'}^f, \quad (47)$$

in terms of $t_{\mu_f}^{\lambda_f}$ given by Eq. (43) and ρ^f specified by Eq. (31).

If the target is unpolarized, the $T_{\mu_f}^{\lambda_f}$ reduce to

$$\begin{aligned} T_{\mu_f}^{\lambda_f} &= \frac{1}{2} \sum_{\lambda, \lambda'} (-1)^{s_f' - \frac{1}{2}} [\lambda] [\lambda'] [s_f'] \\ &\times W\left(s_f' \lambda' s_f \lambda; \frac{1}{2} \lambda_f\right) B_{\mu_f}^{\lambda_f}(\lambda s_f; \lambda' s_f'), \end{aligned} \quad (48)$$

where $B_{\mu_f}^{\lambda_f}(\lambda s_f; \lambda' s_f')$ are given in terms of Ref. (44) through

$$B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f') = \sum_{\mu, \mu' = -1, 1} B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'} \rho_{\mu\mu'}^f, \quad (49)$$

with $\Lambda = \lambda_f$ and $m_\Lambda = \mu_f$.

If the beam is also unpolarized, we may replace $\rho_{\mu\mu'}^f$ by $\frac{1}{2} \delta_{\mu\mu'}$, so that we have

$$B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_0 = \frac{1}{2} \sum_{\mu = -1, 1} B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu}. \quad (50)$$

We may note also that Eq. (49) may then be written as

$$\begin{aligned} B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f') &= \frac{1}{2} \left[B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_0 \right. \\ &\left. + \sum_{i=1}^3 s_i B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_i \right], \end{aligned} \quad (51)$$

where s_1, s_2, s_3 denote the Stokes parameters characterizing the state of polarization of the beam and

$$B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_i = \sum_{\mu, \mu' = -1, 1} B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'} (\sigma_i)_{\mu\mu'}. \quad (52)$$

It may be noted that s_0 representing the intensity of the beam is chosen as 1. It is worth noting that the bilinears $B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{\mu\mu'}$ are known individually, if the $B_{m_\Lambda}^\Lambda(\lambda s_f; \lambda' s_f')_{i=0,1,2,3}$ are determined empirically.

C. Target analyzing powers

The differential cross section Eq. (39) for a polarized target may readily be expressed in the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} [1 + \mathbf{A} \cdot \mathbf{P}], \quad (53)$$

where the target analyzing power \mathbf{A} is given in terms of its spherical components $A_0^1 = A_z$; $A_{\pm 1}^1 = \mp \frac{1}{\sqrt{2}} [A_x \pm i A_y]$ by

$$\begin{aligned} \frac{d\sigma_0}{d\Omega} A_v^1 &= \frac{1}{\sqrt{2}} \frac{q}{k} \sum_{s_f} [s_f]^2 \sum_{\lambda, \lambda'} (-1)^\lambda [\lambda] [\lambda'] \\ &\times W \left(\frac{1}{2} 1 s_f \lambda; \frac{1}{2} \lambda' \right) B_v^1(\lambda s_f; \lambda' s_f). \end{aligned} \quad (54)$$

D. Polarization in the final state

Starting with Eq. (46) and effecting a change of basis from the channel spin states to the individual spin states of the meson and the nucleon in the final state through

$$\begin{aligned} \rho_{m_s m_N; m'_s m'_N}^f &= \sum_{s_f, s'_f} C \left(s \frac{1}{2} s_f; m_s m_N m_f \right) \rho_{s_f m_f; s'_f m'_f}^f \\ &\times C \left(s \frac{1}{2} s'_f; m'_s m'_N m'_f \right), \end{aligned} \quad (55)$$

we may express ρ^f in the form

$$\rho^f = \frac{1}{2[s]^2} \sum_{\lambda_s, \lambda_N, \lambda_f} \left(\left(S^{\lambda_s}(s, s) \otimes S^{\lambda_N} \left(\frac{1}{2}, \frac{1}{2} \right) \right)^{\lambda_f} \cdot E^{\lambda_f} \right) \quad (56)$$

where

$$\begin{aligned} E_{\mu_f}^{\lambda_f} &= \sqrt{2} [s] \sum_{s_f, s'_f} (-1)^{s'_f - s_f} (-1)^{\lambda_s + \lambda_N - \lambda_f} \\ &\times [s'_f] [s_f]^2 [\lambda_s] [\lambda_N] \begin{Bmatrix} s & \frac{1}{2} & s'_f \\ s & \frac{1}{2} & s_f \\ \lambda_s & \lambda_N & \lambda_f \end{Bmatrix} T_{\mu_f}^{\lambda_f}, \end{aligned} \quad (57)$$

and $\{\}$ denotes the Wigner 9j symbol [47]. The irreducible tensors $E_{\mu_f}^{\lambda_f}$ describe clearly that the meson and nucleon spins are entangled in the final state through Eq. (57). It is clear from Eqs. (48) to (50) that the entanglement persists even if the target and beam are unpolarized. If no observations are made on the spin state of the recoil nucleon in the final state, the state of polarization of the emitted meson is characterized by the density matrix ρ^s , whose elements are given by

$$\rho_{m_s m'_s}^s = \sum_{m_N = -\frac{1}{2}}^{\frac{1}{2}} \rho_{m_s m_N; m'_s m'_N}^f. \quad (58)$$

Likewise, if no observations are made on the meson spin state, the recoil nucleon polarization is specified by the density matrix ρ^r whose elements are given by

$$\rho_{m_N m'_N}^r = \sum_{m_s = -\frac{1}{2}}^{\frac{1}{2}} \rho_{m_s m_N; m'_s m'_N}^f. \quad (59)$$

E. Recoil nucleon polarization and nucleon spin transfer

Expressing ρ^r given by Eq. (59) in the form

$$\rho^r = \frac{\text{Tr} \rho^r}{2} [1 + \boldsymbol{\sigma} \cdot \mathbf{R}], \quad (60)$$

the recoil nucleon polarization \mathbf{R} is given in terms of its spherical components $R_0^1 = R_z$; $R_{\pm 1}^1 = \mp \frac{1}{\sqrt{2}} [R_x \pm i R_y]$ by

$$\frac{d\sigma_0}{d\Omega} R_{\mu_f}^1 = \frac{q}{k} \sum_{s_f, s'_f} G_r T_{\mu_f}^1 \quad (61)$$

where the geometrical factor G_r is given by

$$G_r = \sqrt{2} [s'_f] [s_f]^2 W \left(s \frac{1}{2} s_f 1; s'_f \frac{1}{2} \right). \quad (62)$$

If the target is polarized, it is clear from Eqs. (43) and (47) that $R_{\mu_f}^1$ is dependent on \mathbf{P} and this can be brought out by expressing Eq. (61) in the form

$$\frac{d\sigma_0}{d\Omega} R_{\mu_f}^1 = \sum_{\mu_i} \mathcal{R}_{\mu_f \mu_i}'' P_{\mu_i}^1 \quad (63)$$

in terms of the nucleon spin transfers $\mathcal{R}_{\mu_f \mu_i}''$ which are readily given by

$$\begin{aligned} \mathcal{R}_{\mu_f \mu_i}'' &= \frac{q}{k} \sum_{\Lambda} C(1 \Lambda 1; \mu_i m_{\Lambda} \mu_f) \sum_{s_f, s'_f} G_r \\ &\times \sum_{\lambda, \lambda'} G B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f), \end{aligned} \quad (64)$$

where G_r , G and $B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f)$ are given respectively by Eqs. (62), (45), and (49). It may be noted that the recoil nucleon is polarized even in the absence of target polarization.

F. Meson polarization and target nucleon to meson spin transfer

Using $\tau_{\mu_s}^{\lambda_s} = S_{\mu_s}^{\lambda_s}(s, s)$, we may express ρ^s given by Eq. (58) in the form

$$\rho^s = \frac{\text{Tr} \rho^s}{2s + 1} \sum_{\lambda_3=0}^{2s} (\tau^{\lambda_s} \cdot \mathbf{t}^{\lambda_s}), \quad (65)$$

in terms of Fano statistical tensors $t_{\mu_s}^{\lambda_s}$ of rank λ_s which are given by

$$\frac{d\sigma_0}{d\Omega} t_{\mu_s}^{\lambda_s} = \frac{q}{k} \sum_{s_f, s'_f} G_s T_{\mu_s}^{\lambda_s}, \quad (66)$$

where the geometrical factor G_s is given by

$$G_s = (-1)^{s_f - s'_f} [s_f]^2 [s'_f] W \left(\frac{1}{2} s s_f \lambda_f; s'_f s \right). \quad (67)$$

Noting from Eqs. (43) and (47) that $T_{\mu_s}^{\lambda_s}$ are dependent on \mathbf{P} , when the target is polarized, we may define the target nucleon to meson spin transfer $\mathcal{T}_{\mu_s \mu_i}^{\lambda_s 1}$ through

$$\frac{d\sigma_0}{d\Omega} t_{\mu_s}^{\lambda_s} = \sum_{\mu_i} \mathcal{T}_{\mu_s \mu_i}^{\lambda_s 1} P_{\mu_i}^1 \quad (68)$$

so that

$$\begin{aligned} T_{\mu_s \mu_i}^{\lambda_s 1} &= \frac{q}{k} \sum_{\Lambda} C(1\Lambda \lambda_s; \mu_i m_{\Lambda} \mu_s) \\ &\times \sum_{s_f, s'_f} G_s \sum_{\lambda, \lambda'} G B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f). \end{aligned} \quad (69)$$

The meson is polarized even if the target is unpolarized. In such a case, the Fano statistical tensors $T_{\mu_s}^{\lambda_s}$ characterizing the meson polarization are given by Eq. (66), where use is made of Eq. (48) for $T_{\mu_s}^{\lambda_s}$ on the right-hand side of Eq. (66).

IV. SUMMARY AND OUTLOOK

Photo production of mesons with arbitrary spin-parity- s^{π} is described in terms of a set of $4(2s+1)$ independent irreducible tensor amplitudes, which are expressible in terms of a single compact formula given by Eq. (9) in terms of the ‘‘magnetic’’ and ‘‘electric’’ multipole amplitudes $\mathcal{M}_{l s_f; L}^j$ and $\mathcal{E}_{l s_f; L}^j$, respectively, introduced through Eq. (6). All hadron spin observables viz., the target analyzing power, meson and recoil nucleon polarizations, target to meson and target to recoil nucleon spin transfers and spin correlations and beam analyzing powers have been expressed in terms of the bilinear tensors $B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f)_{\mu \mu'}$ of rank $\Lambda = 0, \dots, 2(s+1)$ defined by Eq. (44). The unpolarized differential cross section itself is clearly proportional to $B_0^0(\lambda s_f; \lambda s_f)_0$, whereas hadron spin observables in experiments with an unpolarized beam are given by $B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f)_0$ in accordance with Eq. (50). It is also clear from Eqs. (51) and (52) that each element $B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f)_{\mu \mu'}$ for $\mu, \mu' = \pm 1$ is known if the four $B_{m_{\Lambda}}^{\Lambda}(\lambda s_f; \lambda' s'_f)_i$ for $i = 0, 1, 2, 3$ with unpolarized and appropriate partially polarized beams are measured for a given Λ and m_{Λ} . Thus our approach employing irreducible tensor amplitudes leads to a systematic and elegant procedure to analyze experimental data on all spin and polarization observables including beam analyzing powers and the differential cross section for photo production of mesons with arbitrary spin-parity s^{π} . Further work is in progress.

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APPENDIX A: CONNECTION BETWEEN IRREDUCIBLE TENSOR AMPLITUDES AND CGLN AMPLITUDES FOR PION PRODUCTION

For the particular case of pseudoscalar meson photoproduction, the final-state channel spin can take only one value, namely $s_f = \frac{1}{2}$, because $s = 0$. It is clear that $\mathcal{F}_{m_{\Lambda}}^{\lambda}(\frac{1}{2}, \mu)$

for $\lambda = 0, 1$ represent respectively the nucleon spin independent and spin dependent amplitudes. The CGLN amplitudes \mathcal{F}_i , $i = 1, \dots, 4$ in terms of which the reaction amplitude \mathcal{F} is expressed by Eq. (7.2) of [1] are well known. Rewriting Eq. (7.2) of Ref. [1] conveniently as

$$\mathcal{F} = L + i \boldsymbol{\sigma} \cdot \mathbf{K}, \quad (A1)$$

in terms of the nucleon spin-independent and spin-dependent amplitudes L and \mathbf{K} , respectively, we may note

$$L = \hat{q} \cdot (\hat{k} \times \hat{\epsilon}) \mathcal{F}_2 \quad (A2)$$

$$\begin{aligned} \mathbf{K} &= \hat{\epsilon} (\mathcal{F}_1 - \mathcal{F}_2 \cos \theta) \\ &+ (\hat{q} \cdot \hat{\epsilon}) [\hat{q} \mathcal{F}_4 + \hat{k} (\mathcal{F}_2 + \mathcal{F}_3)], \end{aligned} \quad (A3)$$

where $\hat{\epsilon}$ denotes photon polarization that is orthogonal to \hat{k} . We may define $L(\mu)$ and $K_{m_{\Lambda}}^1(\mu)$ by substituting $\hat{\epsilon} = \hat{u}_{\mu}$ in Eqs. (A2) and (A3), respectively, after expressing \mathbf{K} in terms of its spherical components $K_0^1 = K_z$; $K_{\pm 1}^1 = \mp \frac{1}{\sqrt{2}}(K_x \pm i K_y)$. We thus have

$$\mathcal{F}_0^0 \left(\frac{1}{2}, \pm 1 \right) = L(\pm 1) = \mp \frac{i}{\sqrt{2}} \mathcal{F}_2 \sin \theta \quad (A4)$$

$$\begin{aligned} \mathcal{F}_0^1 \left(\frac{1}{2}, \pm 1 \right) &= i K_0^1(\pm 1) \\ &= \frac{i}{\sqrt{2}} \sin \theta (\mathcal{F}_2 + \mathcal{F}_3 + \mathcal{F}_4 \cos \theta) \end{aligned} \quad (A5)$$

$$\begin{aligned} \mathcal{F}_{-1}^1 \left(\frac{1}{2}, \pm 1 \right) &= i K_{-1}^1(\pm 1) \\ &= \pm i \left(\mathcal{F}_1 - \mathcal{F}_2 \cos \theta + \mathcal{F}_4 \frac{\sin^2 \theta}{2} \right) \end{aligned} \quad (A6)$$

$$\mathcal{F}_1^1 \left(\frac{1}{2}, \pm 1 \right) = i K_1^1(\pm 1) = \mp i \mathcal{F}_4 \frac{\sin^2 \theta}{2}, \quad (A7)$$

where the irreducible tensor amplitudes are explicitly given, on using Eq. (9), with $\varphi = 0$ in Madison frame by

$$\begin{aligned} \mathcal{F}_0^0 \left(\frac{1}{2}, \pm 1 \right) &= \mp \frac{4\pi}{\sqrt{2}} \sin \theta \sum_l P_l' \frac{1}{\sqrt{l(l+1)}} \\ &\times [(l+1) \mathcal{M}_{l,l}^{l+\frac{1}{2}} + l \mathcal{M}_{l,l}^{l-\frac{1}{2}}] \end{aligned} \quad (A8)$$

$$\begin{aligned} \mathcal{F}_0^1 \left(\frac{1}{2}, \pm 1 \right) &= \frac{4\pi}{\sqrt{2}} \sin \theta \sum_l P_l' \left[-\frac{\mathcal{M}_{l,l}^{l+\frac{1}{2}}}{\sqrt{l(l+1)}} + \frac{\mathcal{M}_{l,l}^{l-\frac{1}{2}}}{\sqrt{l(l+1)}} \right. \\ &\left. - \sqrt{\frac{(l+2)}{(l+1)}} \mathcal{E}_{l,l+1}^{l+\frac{1}{2}} + \sqrt{\frac{(l-1)}{l}} \mathcal{E}_{l,l-1}^{l-\frac{1}{2}} \right] \end{aligned} \quad (A9)$$

$$\begin{aligned} \mathcal{F}_{-1}^1 \left(\frac{1}{2}, \pm 1 \right) &= \pm 2\pi \sum_l P_l \left[\sqrt{l(l+1)} (\mathcal{M}_{l,l}^{l+\frac{1}{2}} - \mathcal{M}_{l,l}^{l-\frac{1}{2}}) \right. \\ &\left. - \sqrt{(l+2)(l+1)} \mathcal{E}_{l,l+1}^{l+\frac{1}{2}} - \sqrt{(l-1)} \mathcal{E}_{l,l-1}^{l-\frac{1}{2}} \right] \end{aligned} \quad (A10)$$

$$\mathcal{F}_1^{\pm}(\frac{1}{2}, \pm 1) = \pm 2\pi \sin^2 \theta \sum_l P_l'' \left[-\frac{\mathcal{M}_{l,l}^{l+\frac{1}{2}}}{\sqrt{l(l+1)}} + \frac{\mathcal{M}_{l,l}^{l-\frac{1}{2}}}{\sqrt{l(l+1)}} - \frac{\mathcal{E}_{l,l+1}^{l+\frac{1}{2}}}{\sqrt{(l+2)(l+1)}} - \frac{\mathcal{E}_{l,l-1}^{l-\frac{1}{2}}}{\sqrt{l(l-1)}} \right]. \quad (\text{A11})$$

Comparing (A4) to (A7) with (A8) to (A11) and using Eqs. (7.3) to (7.6) of CGLN [1], for \mathcal{F}_i , $i = 1, \dots, 4$, we may identify

$$\mathcal{M}_{l,L}^{l\pm\frac{1}{2}} = i \frac{\sqrt{L(L+1)}}{4\pi} M_{l\pm} \quad (\text{A12})$$

$$\mathcal{E}_{l,L}^{l\pm\frac{1}{2}} = -i \frac{\sqrt{L(L+1)}}{4\pi} E_{l\pm} \quad (\text{A13})$$

in terms of the multipole amplitudes, $M_{l\pm}$ and $E_{l\pm}$ of CGLN [1].

The photoproduction amplitudes for specific reactions are readily obtained using Eq. (10)

$$\langle n\pi^+ | \mathcal{F} | \gamma p \rangle = -\frac{\sqrt{2}}{3} (\mathcal{F}^{1\frac{1}{2}} - \mathcal{F}^{1\frac{3}{2}} + \sqrt{3}\mathcal{F}^{0\frac{1}{2}}) \quad (\text{A14})$$

$$\langle p\pi^0 | \mathcal{F} | \gamma p \rangle = \frac{1}{3} (2\mathcal{F}^{1\frac{3}{2}} + \mathcal{F}^{1\frac{1}{2}} + \sqrt{3}\mathcal{F}^{0\frac{1}{2}}) \quad (\text{A15})$$

$$\langle p\pi^- | \mathcal{F} | \gamma n \rangle = \frac{\sqrt{2}}{3} (\mathcal{F}^{1\frac{3}{2}} - \mathcal{F}^{1\frac{1}{2}} + \sqrt{3}\mathcal{F}^{0\frac{1}{2}}) \quad (\text{A16})$$

$$\langle n\pi^0 | \mathcal{F} | \gamma n \rangle = \frac{1}{3} (2\mathcal{F}^{1\frac{3}{2}} + \mathcal{F}^{1\frac{1}{2}} - \sqrt{3}\mathcal{F}^{0\frac{1}{2}}). \quad (\text{A17})$$

It may be noted that if the different charge states of the pion are defined in the real three dimensional isospin space through

$$\pi^0 = \pi_z; \quad \pi^\pm = \frac{1}{\sqrt{2}}(\pi_x \pm i \pi_y), \quad (\text{A18})$$

an extra overall minus sign has to be attached to (A14), because the three charged state of the pion are identified through $|1\nu_m\rangle$, $\nu_m = 0, \pm 1$ in deriving (A14) to (A17). We may then compare the above with

$$\langle n\pi^+ | \mathcal{F} | \gamma p \rangle = \sqrt{2}(\mathcal{F}^{(-)} + \mathcal{F}^{(0)}) \quad (\text{A19})$$

$$\langle p\pi^0 | \mathcal{F} | \gamma p \rangle = \mathcal{F}^{(+)} + \mathcal{F}^{(0)} \quad (\text{A20})$$

$$\langle p\pi^- | \mathcal{F} | \gamma n \rangle = -\sqrt{2}(\mathcal{F}^{(-)} - \mathcal{F}^{(0)}) \quad (\text{A21})$$

$$\langle n\pi^0 | \mathcal{F} | \gamma n \rangle = \mathcal{F}^{(+)} - \mathcal{F}^{(0)} \quad (\text{A22})$$

in terms of the traditional isospin amplitudes of CGLN and hence identify

$$\mathcal{F}^{(0)} = \frac{1}{\sqrt{3}}\mathcal{F}^{0\frac{1}{2}} \quad (\text{A23})$$

$$\mathcal{F}^{(+)} = \frac{1}{3}(\mathcal{F}^{1\frac{1}{2}} + 2\mathcal{F}^{1\frac{3}{2}}) \quad (\text{A24})$$

$$\mathcal{F}^{(-)} = \frac{1}{3}(\mathcal{F}^{1\frac{1}{2}} - \mathcal{F}^{1\frac{3}{2}}). \quad (\text{A25})$$

APPENDIX B: HELICITY AMPLITUDES

The helicity amplitudes may be defined through

$$H_{\mu_s, \mu_f; \mu_i, \mu} \equiv \langle s\mu_s; \frac{1}{2}\mu_f | \mathcal{F} | \frac{1}{2}\mu_i; 1\mu \rangle \quad (\text{B1})$$

where μ and μ_s denote the photon and meson spin projections along \mathbf{k} and \mathbf{q} , respectively, whereas $|\frac{1}{2}\mu_i\rangle$ and $|\frac{1}{2}\mu_f\rangle$ denote initial and final helicity states with conventional phases following [5] and having spin projections μ_i and μ_f along $-\mathbf{k}$ and $-\mathbf{q}$, respectively. We may note that the helicity eigenstate of the photon is given by

$$|1\mu\rangle \equiv \hat{\boldsymbol{\epsilon}}_\mu = -\mu \hat{\mathbf{u}}_\mu; \quad \mu = \pm 1, \quad (\text{B2})$$

following Eq. (1). We also observe that

$$\left| s\mu_s; \frac{1}{2}\mu_f \right\rangle = \sum_{s_f} C \left(s \frac{1}{2} s_f; \mu_s - \mu_f \mu'_f \right) \left| \left(s \frac{1}{2} \right) s_f \mu'_f \right\rangle \quad (\text{B3})$$

in terms of channel spin states $|(s\frac{1}{2})_{s_f}\mu'_f\rangle$ that are expressed as

$$\left| \left(s \frac{1}{2} \right) s_f \mu'_f \right\rangle = \sum_{m_f} d_{m_f \mu'_f}^{s_f}(\theta) \left| \left(s \frac{1}{2} \right) s_f m_f \right\rangle \quad (\text{B4})$$

with respect to \mathbf{q} as the quantization axis. The polar coordinates of \mathbf{q} are $(q, \theta, 0)$ in the Madison frame [44] where the plane containing \mathbf{q} and \mathbf{k} is chosen as the reaction plane with the z axis along \mathbf{k} . Thus, we have

$$H_{\mu_s, \mu_f; \mu_i, \mu} = -\mu \sum_{s_f m_f} C \left(s \frac{1}{2} s_f; \mu_s - \mu_f \mu'_f \right) d_{m_f \mu'_f}^{s_f}(\theta) \times \sum_{m_i} \left\langle \left(s \frac{1}{2} \right) s_f m_f \left| \mathcal{F}(\mu) \right| \frac{1}{2} m_i \right\rangle \delta_{m_i, -\mu_i}, \quad (\text{B5})$$

where the matrix elements are expressible as

$$\begin{aligned} & \left\langle \left(s \frac{1}{2} \right) s_f m_f \left| \mathcal{F}(\mu) \right| \frac{1}{2} m_i \right\rangle \\ &= \sum_{\lambda} C \left(\frac{1}{2} \lambda s_f; m_i - m_\lambda m_f \right) (-1)^{m_\lambda} [\lambda] \mathcal{F}_{m_\lambda}^\lambda(s_f, \mu) \end{aligned} \quad (\text{B6})$$

in terms of the irreducible tensor amplitudes given by Eq. (9) with $\phi = 0$. Using Eq. (B5) and Eq. (11), we obtain the symmetry relation,

$$H_{-\mu_s, -\mu_f; -\mu_i, -\mu} = -\pi (-1)^{s+\mu_s+\mu_i-\mu_f} H_{\mu_s, \mu_f; \mu_i, \mu}. \quad (\text{B7})$$

In the particular case of pseudoscalar meson photoproduction with $s = \mu_s = 0$, $s_f = \frac{1}{2}$ and $\pi = -1$, the helicity amplitudes are explicitly given by

$$H_{-\frac{1}{2}; -\frac{1}{2}1} = \sqrt{2} \sin \frac{\theta}{2} \mathcal{F}_1^1(\frac{1}{2}, 1) - \cos \frac{\theta}{2} [\mathcal{F}_0^0(\frac{1}{2}, 1) + \mathcal{F}_0^1(\frac{1}{2}, 1)] \quad (\text{B8})$$

$$H_{-\frac{1}{2}; \frac{1}{2}1} = -\sqrt{2} \cos \frac{\theta}{2} \mathcal{F}_{-1}^1(\frac{1}{2}, 1) - \sin \frac{\theta}{2} [\mathcal{F}_0^0(\frac{1}{2}, 1) - \mathcal{F}_0^1(\frac{1}{2}, 1)] \quad (\text{B9})$$

$$H_{\frac{1}{2};-\frac{1}{2}1} = \sqrt{2} \cos \frac{\theta}{2} \mathcal{F}_1^1\left(\frac{1}{2}, 1\right) + \sin \frac{\theta}{2} \left[\mathcal{F}_0^0\left(\frac{1}{2}, 1\right) + \mathcal{F}_0^1\left(\frac{1}{2}, 1\right) \right] \quad (\text{B10})$$

$$H_{\frac{1}{2};\frac{1}{2}1} = \sqrt{2} \sin \frac{\theta}{2} \mathcal{F}_{-1}^1\left(\frac{1}{2}, 1\right) - \cos \frac{\theta}{2} \left[\mathcal{F}_0^0\left(\frac{1}{2}, 1\right) - \mathcal{F}_0^1\left(\frac{1}{2}, 1\right) \right], \quad (\text{B11})$$

satisfying

$$H_{-\mu_f;-\mu_i-\mu} = (-1)^{\mu_i-\mu_f} H_{\mu_f;\mu_i\mu}, \quad (\text{B12})$$

in exact agreement with Walker [16], where the notation $H_1, H_2, H_3,$ and H_4 is used, respectively, for the four independent helicity amplitudes (B8),..., (B11). Using (A8) to (A11) in (B6), we readily obtain the helicity amplitudes in terms of the CGLN amplitudes as

$$H_1 = -\frac{1}{\sqrt{2}} \sin \theta \cos \frac{\theta}{2} [\mathcal{F}_3 + \mathcal{F}_4] \quad (\text{B13})$$

$$H_2 = \sqrt{2} \cos \frac{\theta}{2} [(\mathcal{F}_2 - \mathcal{F}_1) + \frac{1}{2}(1 - \cos \theta)(\mathcal{F}_3 - \mathcal{F}_4)] \quad (\text{B14})$$

$$H_3 = \frac{1}{\sqrt{2}} \sin \theta \sin \frac{\theta}{2} (\mathcal{F}_3 - \mathcal{F}_4) \quad (\text{B15})$$

$$H_4 = \sqrt{2} \sin \frac{\theta}{2} [(\mathcal{F}_1 + \mathcal{F}_2) + \frac{1}{2}(1 + \cos \theta)(\mathcal{F}_3 + \mathcal{F}_4)], \quad (\text{B16})$$

which are in exact agreement with Walker [16].

In the case of vector meson photoproduction with $s = 1$, we may once again use Eqs. (B6) in (B5) to obtain explicitly the 12 independent helicity amplitudes in terms of the irreducible tensor amplitudes as

$$\begin{aligned} H_{-\frac{1}{2};-\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ -\sqrt{\frac{3}{2}}(1 - \cos \theta) \left[\mathcal{F}_0^1\left(\frac{3}{2}, 1\right) + \mathcal{F}_0^2\left(\frac{3}{2}, 1\right) \right] \right. \\ &\quad \left. - (1 + \cos \theta) \mathcal{F}_2^2\left(\frac{3}{2}, 1\right) \right\} + \sin \frac{\theta}{2} \left\{ -\frac{\sqrt{3}}{2}(1 - \cos \theta) \right. \\ &\quad \times \left[\mathcal{F}_{-1}^1\left(\frac{3}{2}, 1\right) + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2\left(\frac{3}{2}, 1\right) \right] - \frac{\sqrt{3}}{2}(1 + \cos \theta) \\ &\quad \times \left. \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) + \sqrt{3} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \right\} \quad (\text{B17}) \end{aligned}$$

$$\begin{aligned} H_{\frac{1}{2};-\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ \sqrt{\frac{2}{3}} \mathcal{F}_1^1\left(\frac{1}{2}, 1\right) + \sqrt{\frac{3}{2}}(1 - \cos \theta) \left[\mathcal{F}_{-1}^1\left(\frac{3}{2}, 1\right) \right. \right. \\ &\quad \left. \left. + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2\left(\frac{3}{2}, 1\right) \right] + \frac{1}{\sqrt{6}}(3 \cos \theta - 1) \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) \right. \right. \\ &\quad \left. \left. + \sqrt{3} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \right\} + \sin \frac{\theta}{2} \left\{ \frac{1}{\sqrt{3}} \left[\mathcal{F}_0^0\left(\frac{1}{2}, 1\right) \right. \right. \\ &\quad \left. \left. + \mathcal{F}_0^1\left(\frac{1}{2}, 1\right) \right] + \frac{1}{\sqrt{3}}(3 \cos \theta + 1) \left[\mathcal{F}_0^1\left(\frac{3}{2}, 1\right) \right. \right. \\ &\quad \left. \left. + \mathcal{F}_0^2\left(\frac{3}{2}, 1\right) \right] - \sqrt{2}(1 + \cos \theta) \mathcal{F}_2^2\left(\frac{3}{2}, 1\right) \right\} \quad (\text{B18}) \end{aligned}$$

$$\begin{aligned} H_{\frac{1}{2};\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ -\sqrt{\frac{2}{3}} \left[\mathcal{F}_0^0\left(\frac{1}{2}, 1\right) + \mathcal{F}_0^1\left(\frac{1}{2}, 1\right) \right] \right. \\ &\quad \left. - \frac{1}{\sqrt{6}}(3 \cos \theta - 1) \left[\mathcal{F}_0^1\left(\frac{3}{2}, 1\right) + \mathcal{F}_0^2\left(\frac{3}{2}, 1\right) \right] \right. \\ &\quad \left. - (1 - \cos \theta) \mathcal{F}_2^2\left(\frac{3}{2}, 1\right) \right\} + \sin \frac{\theta}{2} \left\{ \frac{2}{\sqrt{3}} \mathcal{F}_1^1\left(\frac{1}{2}, 1\right) \right. \\ &\quad \left. - \frac{\sqrt{3}}{2}(1 + \cos \theta) \left[\mathcal{F}_{-1}^1\left(\frac{3}{2}, 1\right) + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2\left(\frac{3}{2}, 1\right) \right] \right. \\ &\quad \left. + \frac{1}{2\sqrt{3}}(3 \cos \theta + 1) \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) + \sqrt{3} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \right\} \quad (\text{B19}) \end{aligned}$$

$$\begin{aligned} H_{-\frac{1}{2};\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ \frac{\sqrt{3}}{2}(1 - \cos \theta) \left[\mathcal{F}_{-1}^1\left(\frac{3}{2}, 1\right) \right. \right. \\ &\quad \left. \left. - \sqrt{3} \mathcal{F}_{-1}^2\left(\frac{3}{2}, 1\right) \right] + \frac{\sqrt{3}}{2}(1 + \cos \theta) \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) \right. \right. \\ &\quad \left. \left. - \frac{1}{\sqrt{3}} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \right\} + \sin \frac{\theta}{2} \left\{ \sqrt{\frac{3}{2}}(1 + \cos \theta) \right. \\ &\quad \times \left[\mathcal{F}_0^1\left(\frac{3}{2}, 1\right) - \mathcal{F}_0^2\left(\frac{3}{2}, 1\right) \right] - (1 - \cos \theta) \\ &\quad \times \left. \mathcal{F}_{-2}^2\left(\frac{3}{2}, 1\right) \right\} \quad (\text{B20}) \end{aligned}$$

$$\begin{aligned} H_{\frac{1}{2};\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ -\frac{1}{\sqrt{3}} \left[\mathcal{F}_0^0\left(\frac{1}{2}, 1\right) - \mathcal{F}_0^1\left(\frac{1}{2}, 1\right) \right] \right. \\ &\quad \left. - \frac{1}{\sqrt{3}}(3 \cos \theta - 1) \left[\mathcal{F}_0^1\left(\frac{3}{2}, 1\right) - \mathcal{F}_0^2\left(\frac{3}{2}, 1\right) \right] \right. \\ &\quad \left. + \sqrt{2}(1 - \cos \theta) \mathcal{F}_{-2}^2\left(\frac{3}{2}, 1\right) \right\} \\ &\quad + \sin \frac{\theta}{2} \left\{ \sqrt{\frac{2}{3}} \mathcal{F}_{-1}^1\left(\frac{1}{2}, 1\right) + \sqrt{\frac{3}{2}}(1 + \cos \theta) \right. \\ &\quad \times \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) - \frac{1}{\sqrt{3}} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \\ &\quad \left. - \frac{1}{\sqrt{6}}(3 \cos \theta + 1) \left[\mathcal{F}_{-1}^1\left(\frac{3}{2}, 1\right) - \sqrt{3} \mathcal{F}_{-1}^2\left(\frac{3}{2}, 1\right) \right] \right\} \quad (\text{B21}) \end{aligned}$$

$$\begin{aligned} H_{\frac{1}{2};-\frac{1}{2}1} &= \cos \frac{\theta}{2} \left\{ -\frac{2}{\sqrt{3}} \mathcal{F}_{-1}^1\left(\frac{1}{2}, 1\right) + \frac{\sqrt{3}}{2}(1 - \cos \theta) \right. \\ &\quad \times \left. \left[\mathcal{F}_1^1\left(\frac{3}{2}, 1\right) - \frac{1}{\sqrt{3}} \mathcal{F}_1^2\left(\frac{3}{2}, 1\right) \right] \right\} \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{2\sqrt{3}}(3 \cos \theta - 1) \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) \right. \\
 & \left. - \sqrt{3} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] \Big\} + \sin \frac{\theta}{2} \left\{ -\sqrt{\frac{2}{3}} \left[\mathcal{F}_0^0 \left(\frac{1}{2}, 1 \right) \right. \right. \\
 & \left. \left. - \mathcal{F}_0^1 \left(\frac{1}{2}, 1 \right) \right] - \frac{1}{\sqrt{6}}(3 \cos \theta + 1) \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. - \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] - (1 + \cos \theta) \mathcal{F}_{-2}^2 \left(\frac{3}{2}, 1 \right) \right\} \quad (\text{B22})
 \end{aligned}$$

$$\begin{aligned}
 H_{-1-\frac{1}{2};-\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ -\frac{2}{\sqrt{3}} \mathcal{F}_1^1 \left(\frac{1}{2}, 1 \right) + \frac{\sqrt{3}}{2}(1 - \cos \theta) \right. \\
 & \times \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] \\
 & + \frac{1}{2\sqrt{3}}(3 \cos \theta - 1) \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) \right. \\
 & \left. + \sqrt{3} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] \Big\} + \sin \frac{\theta}{2} \left\{ -\sqrt{\frac{2}{3}} \left[\mathcal{F}_0^0 \left(\frac{1}{2}, 1 \right) \right. \right. \\
 & \left. \left. + \mathcal{F}_0^1 \left(\frac{1}{2}, 1 \right) \right] + \frac{1}{\sqrt{6}}(3 \cos \theta + 1) \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. + \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] - (1 + \cos \theta) \mathcal{F}_2^2 \left(\frac{3}{2}, 1 \right) \right\} \quad (\text{B23})
 \end{aligned}$$

$$\begin{aligned}
 H_{0-\frac{1}{2};-\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ \frac{1}{\sqrt{3}} \left[\mathcal{F}_0^0 \left(\frac{1}{2}, 1 \right) + \mathcal{F}_0^1 \left(\frac{1}{2}, 1 \right) \right] \right. \\
 & \left. - \frac{1}{\sqrt{3}}(3 \cos \theta - 1) \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) + \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] \right. \\
 & \left. - \sqrt{2}(1 - \cos \theta) \mathcal{F}_2^2 \left(\frac{3}{2}, 1 \right) \right\} \\
 & + \sin \frac{\theta}{2} \left\{ -\sqrt{\frac{2}{3}} \mathcal{F}_1^1 \left(\frac{1}{2}, 1 \right) - \sqrt{\frac{3}{2}}(1 + \cos \theta) \right. \\
 & \times \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] \\
 & \left. + \frac{1}{\sqrt{6}}(3 \cos \theta + 1) \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) + \sqrt{3} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] \right\} \quad (\text{B24})
 \end{aligned}$$

$$\begin{aligned}
 H_{1-\frac{1}{2};-\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ \frac{\sqrt{3}}{2}(1 + \cos \theta) \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. + \frac{1}{\sqrt{3}} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] + \frac{\sqrt{3}}{2}(1 - \cos \theta) \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. + \sqrt{3} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] \right\} + \sin \frac{\theta}{2} \left\{ -\sqrt{\frac{3}{2}}(1 + \cos \theta) \right.
 \end{aligned}$$

$$\begin{aligned}
 & \times \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) + \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] - (1 - \cos \theta) \\
 & \times \mathcal{F}_2^2 \left(\frac{3}{2}, 1 \right) \Big\} \quad (\text{B25})
 \end{aligned}$$

$$\begin{aligned}
 H_{-1-\frac{1}{2};\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ \sqrt{\frac{2}{3}} \left[\mathcal{F}_0^0 \left(\frac{1}{2}, 1 \right) - \mathcal{F}_0^1 \left(\frac{1}{2}, 1 \right) \right] \right. \\
 & \left. - \frac{1}{\sqrt{6}}(3 \cos \theta - 1) \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) - \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] \right. \\
 & \left. + (1 - \cos \theta) \mathcal{F}_{-2}^2 \left(\frac{3}{2}, 1 \right) \right\} \\
 & + \sin \frac{\theta}{2} \left\{ -\frac{2}{\sqrt{3}} \mathcal{F}_{-1}^1 \left(\frac{1}{2}, 1 \right) + \frac{\sqrt{3}}{2}(1 + \cos \theta) \right. \\
 & \times \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) - \frac{1}{\sqrt{3}} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] \\
 & \left. - \frac{1}{2\sqrt{3}}(3 \cos \theta + 1) \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. - \sqrt{3} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] \right\} \quad (\text{B26})
 \end{aligned}$$

$$\begin{aligned}
 H_{0-\frac{1}{2};\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ \sqrt{\frac{2}{3}} \mathcal{F}_{-1}^1 \left(\frac{1}{2}, 1 \right) + \sqrt{\frac{3}{2}}(1 - \cos \theta) \right. \\
 & \times \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) - \frac{1}{\sqrt{3}} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] + \frac{1}{\sqrt{6}}(3 \cos \theta - 1) \\
 & \times \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) - \sqrt{3} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] \Big\} + \sin \frac{\theta}{2} \left\{ \frac{1}{\sqrt{3}} \right. \\
 & \times \left[\mathcal{F}_0^0 \left(\frac{1}{2}, 1 \right) - \mathcal{F}_0^1 \left(\frac{1}{2}, 1 \right) \right] - \frac{1}{\sqrt{3}}(3 \cos \theta + 1) \\
 & \times \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) - \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] - \left[\sqrt{2}(1 + \cos \theta) \right. \\
 & \left. \times \mathcal{F}_{-2}^2 \left(\frac{3}{2}, 1 \right) \right] \Big\} \quad (\text{B27})
 \end{aligned}$$

$$\begin{aligned}
 H_{1-\frac{1}{2};\frac{1}{2}1} & = \cos \frac{\theta}{2} \left\{ -\sqrt{\frac{3}{2}}(1 - \cos \theta) \left[\mathcal{F}_0^1 \left(\frac{3}{2}, 1 \right) \right. \right. \\
 & \left. \left. - \mathcal{F}_0^2 \left(\frac{3}{2}, 1 \right) \right] \right. \\
 & \left. + (1 + \cos \theta) \mathcal{F}_{-2}^2 \left(\frac{3}{2}, 1 \right) \right\} + \sin \frac{\theta}{2} \left\{ \frac{\sqrt{3}}{2}(1 + \cos \theta) \right. \\
 & \times \left[\mathcal{F}_{-1}^1 \left(\frac{3}{2}, 1 \right) - \sqrt{3} \mathcal{F}_{-1}^2 \left(\frac{3}{2}, 1 \right) \right] + \frac{\sqrt{3}}{2}(1 - \cos \theta) \\
 & \times \left[\mathcal{F}_1^1 \left(\frac{3}{2}, 1 \right) - \frac{1}{\sqrt{3}} \mathcal{F}_1^2 \left(\frac{3}{2}, 1 \right) \right] \Big\}, \quad (\text{B28})
 \end{aligned}$$

which are identifiable, respectively, with H_{i,μ_s} , $i = 1, \dots, 4$ of Ref. [32]. It may be noted that the above formulas are not restricted to threshold production but are applicable at higher energies as well. Even at low energies, the higher-order partial waves could be expected to play a role in determining the spin observables.

Likewise, in anticipation of future experimental developments, explicit expressions for the helicity amplitudes for

photoproduction of higher spin mesons may also be obtained, not only at threshold but also at all energies using Eq. (B6) in Eq. (B5). Our approach thus provides incidentally an elegant methodology to obtain the helicity amplitudes in terms of the partial wave multipole amplitudes for all orders not only in the case of vector meson photoproduction but also for photoproduction of mesons with arbitrary spin-parity s^π , even if s is greater than 1.

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