Azimuthal angle dependence of Coulomb and nuclear interactions between two deformed nuclei

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(Received 22 March 2006; revised manuscript received 7 December 2006; published 18 June 2007)

The azimuthal angle (ϕ) variation of the Coulomb and nuclear heavy ion (HI) potentials is studied in the framework of the double folding model, which is derived from realistic nuclear density distributions and a nucleon-nucleon (NN) interaction. The present calculation shows that the variation of HI potentials with the azimuthal angle depends strongly on the range of the NN forces. For the long-range Coulomb force, the maximum variation with ϕ is about 0.9%, and for HI potential derived from zero-range NN interaction the ϕ-variation can reach up to 90.0%. Our calculations are compared with the recent ϕ-dependence of the HI potential derived from proximity method. The present realistic ϕ-dependence calculations of the HI potential is completely different from the results of the proximity calculations.

DOI: 10.1103/PhysRevC.75.064610 PACS number(s): 24.10.−i

The calculations of the nucleus-nucleus potential between two deformed, oriented nuclei have been of much interest [1–9]. The double folding model [2,4,10] plays a fundamental role in deriving the heavy ion (HI) potential. For two deformed density distributions, the calculation of the nucleus-nucleus potential is a hard task due to the numerical computation of a six-dimensional integral, which is very time consuming. This problem is quite relevant because most nuclei have permanent and/or vibrational deformations. Recently, many authors have considered this problem to study the synthesis of new and superheavy elements [4,5,7,8], since the collision of deformed nuclei is a path to the far side of the proposed island of superheavy nuclei [11]. For the collision between either two spherical [12], or spherical-deformed nuclei [13], the HI potential (including the Coulomb part) had been derived microscopically by different methods. The double folding model [2,4,10] is one of the successful methods to derive the HI potential starting from a finite range NN force. Moreover, this model is the only one used to treat correctly the Coulomb potential between two heavy ions.

For two deformed nuclei, the double folding model can be simplified when their symmetry axes are coplanar [1,2]. For arbitrary orientations of the nuclei, the six-dimensional integral of the double folding model is difficult to calculate without making approximations. Due to this difficulty, many authors considered different approximate methods to derive both the nuclear and Coulomb HI potentials [5–7]. For example, in Refs. [6,7] the pocket formula of the proximity potential was used to study orientation dependence of the HI potential. In another study [14], the zero-range NN force was used to reduce the six-dimensional integration to three dimensions. The Coulomb potential, in these recent studies, was calculated from a simplified equation derived by Wong [15], or by assuming the nucleus to be of uniform charge distribution with sharp cutoff edge [5]. The two methods for calculating the Coulomb potential produce a large error in both internal and surface regions [16].

Moreover, the azimuthal angle dependence of the HI potential between the two deformed nuclei is neglected in most cases (coplanar symmetry axes) or treated by approximate methods [7]. This shows the need to study the heavy ion potential for arbitrary orientation of the symmetry axes of the two deformed nuclei using a realistic NN force. The aim of the present work is to show the importance of the ϕ dependence of the HI potential for arbitrary orientation of the symmetry axes of two interacting deformed nuclei. The method described in Ref. [4] is implemented, and it is based on the multipole expansion of the deformed density distribution,

$$\rho(\bar{r}) = \sum_{l,m} \rho_l(r) Y_{lm}(\theta, \phi). \tag{1}$$

For an axially symmetric nucleus, having reflection symmetry across a plane normal to its axis of symmetry and passing its center, the values of l in Eq. (1) are restricted to even values, and the value of m is zero.

The multipole expansion has the advantage of reducing the six-dimensional integral in the folding model, to the sum of the products of three single dimensional integrals. Moreover, the series converges rapidly and the contributions of higher multipole terms become negligible. Assuming two unequal deformed nuclei with their symmetry axes are noncoplanar, the folding integral for the finite range NN force, $V_{NN}(s)$, is given by

$$V(R, \hat{\Omega}_P, \hat{\Omega}_T) = \int \rho_P(r_1, \hat{\Omega}_P) V_{NN}(s) \rho_T(r_2, \hat{\Omega}_T) d\boldsymbol{r}_1 d\boldsymbol{r}_2 , \tag{2}$$

where $\bar{R}$ is the separation vector joining the two center of masses of the interacting nuclei, $\hat{\Omega}_P$, ($\hat{\Omega}_T$) is the direction of symmetry axis of projectile (target), and $\rho_P(\rho_T)$ denotes the density distribution of the projectile (target), and the other symbols are indicated in Fig. 1. Following Greiner *et al.* [2,4], ρ_P and ρ_T are first expanded, using Eq. (1), then the Fourier

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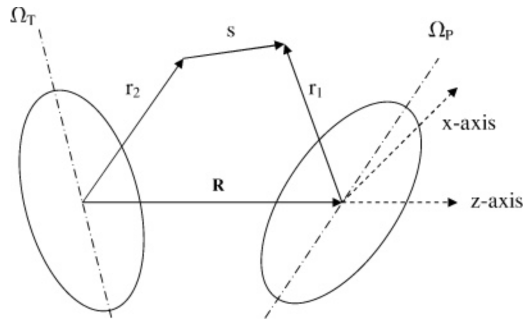

FIG. 1. The coordinate system for the interacting deformed-deformed nuclei.

transform of the finite range NN interaction $V_{NN}(s = R + r_1 - r_2)$ is taken to separate the coupled coordinators we can get

$$V(R,\hat{\Omega}_P,\hat{\Omega}_T) = (4\pi)^3 \sum_{\lambda_1\lambda_2\lambda} i^{\lambda_1+\lambda-\lambda_2}(2\lambda+1)\begin{pmatrix}\lambda & \lambda_1 & \lambda_2\\ 0 & 0 & 0\end{pmatrix} \times \int dk k^2 j_\lambda(kR)\tilde{V}_{NN}(k)A^T_{\lambda_2}(k)A^P_{\lambda_1}(k) \times \sum_{m=-\lambda_1}^{+\lambda_1}(-1)^m \begin{pmatrix}\lambda & \lambda_1 & \lambda_2\\ 0 & m & -m\end{pmatrix} \times Y^*_{\lambda_1 m}(\hat{\Omega}_P)Y_{\lambda_2 m}(\hat{\Omega}_T), \quad (3)$$

where $\tilde{V}_{NN}(k)$ is the Fourier transform of the NN force given by,

$$\tilde{V}_{NN}(k) = \frac{1}{(2\pi)^3}\int d\boldsymbol{x} e^{-ik.x} V_{NN}(\boldsymbol{x}) \quad (4)$$

and the quantities $A_\lambda(k)$ are given by

$$A_\lambda(k) = \int dr r^2 \rho_\lambda(r) j_\lambda(kr). \quad (5)$$

Equation (3) is reduced to Eq. (17) in Ref. [2] for the case of coplanar $\hat{\Omega}_1 and \hat{\Omega}_2$ by replacing $Y_{\lambda m}(\beta,\phi)$ by

$$Y_{\lambda m}(\beta,0) = \sqrt{\frac{2\lambda+1}{4\pi}} d^\lambda_{m0}(\beta).$$

In the present calculations, we use the well-known M3Y effective NN force [2,17] in its form

$$V(s) = \left[7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s}\right] - 276.0\left(1 - 0.005\frac{E_L}{A_P}\right)\delta(s) \quad (6)$$

the first and the second major terms are the direct and exchange contributions to the NN force . We choose the nuclear pair $^{238}U + ^{238}U$ as an example to study the azimuthal angle dependence of both Coulomb and nuclear HI potentials for the deformed-deformed the interacting pair. We study also, the ϕ-dependence of the interaction barrier and its position for $^{150}Nd + ^{150}Nd$ collision considered recently in Ref. [7]. It is assumed that this collision leads to the production of superheavy nucleus with the atomic number $z = 120$ [18].

In the present calculations, the matter or charge density distributions for the ^{238}U nucleus are represented by the Fermi shape

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + e^{\frac{r-R(\theta)}{a}}}, \quad (7)$$

where $\cos\theta = \hat{r}\cdot\hat{\Omega}$ and the radius $R(\theta)$ is given by

$$R(\theta) = R_0[1 + \delta_2 Y_{20}(\theta) + \delta_4 Y_{40}(\theta)]. \quad (8)$$

The values of the parameters R_0 and a are taken from Ref. [19], δ_2 and δ_4 are the quadrupole and hexadecapole deformation parameters, respectively.

In the present work, we use the values $\delta_2 = 0.331$ and $\delta_4 = 0$ or 0.087, the Coulomb potential $V_c(R,\hat{\Omega}_p,\hat{\Omega}_T)$, the direct $V_D(R,\hat{\Omega}_P,\hat{\Omega}_T)$, and zero-range exchange $V_{ex}(R,\hat{\Omega}_P,\hat{\Omega}_T)$ contributions of the $^{238}U + ^{238}U$ potential are calculated for four orientations (β_P,β_T) of the symmetry axes of the deformed nuclei. These orientations are $(30^\circ,30^\circ)$, $(60^\circ,60^\circ)$, $(90^\circ,90^\circ)$, and $(45^\circ,135^\circ)$. Define $\hat{\Omega}_\mathbf{i} = (\beta_\mathbf{i},\phi_\mathbf{i})$ then, for each (β_P,β_T) value, we consider the azimuthal angle variation from $\phi_T = 0^0$ to $\phi_T = 180^0$, and $\phi_P = 0^0$ also ϕ is set to represent ϕ_T.

Figures 2(a)–2(d) show the azimuthal angle variation of the direct and exchange parts of the $^{238}U + ^{238}U$ nuclear HI potential calculated at the two separation distances $R = 12.0$ and 15.0 fm. The figures show the potentials for four different values of relative orientation angles (β_P,β_T) and $\delta_4 = 0$.

Figures 2(a) and 2(b) show the direct term, while Figs. 2(c) and 2(d) show the exchange term of the HI potential. Figures 3(a)–3(d) are the same as Figs. 2(a)–2(d) except they are calculated with the value of hexadecapole deformation parameter $\delta_4 = 0.087$.

The present study shows that the ϕ-variation of the nucleus-nucleus Coulomb interaction is small and has a maximum percentage of 0.9%, around the barrier region, for the orientation angles $(\beta_P,\beta_T) \equiv (30^\circ,30^\circ)$. For the same orientation angles the change of ϕ from 0° to 180° (ϕ-variation range) increases the Coulomb potential by 7.0 and 4.6 MeV at $R = 12.0$ and 14.0 fm, respectively. For $(60^\circ,60^\circ)$, V_C increases by 6.6 MeV and 4.0 MeV for the same ϕ-range, and the same values of separation distance R. At $R = 12.0$ fm the ϕ variation decreases the value of V_C for the two orientations $(90^\circ,90^\circ)$ and $(45^\circ,135^\circ)$ by 2.5 MeV and 9.0 MeV, respectively, and by 1.3 MeV and 6.1 MeV at $R = 14.0$ fm.

In contrast to the long-range Coulomb interaction, the nuclear part of the HI potential calculated assuming zero range exchange NN force has the strongest ϕ-variation at the orientation angles $(\beta_P = \beta_T = 60^0)$ as shown in Figs. 2(c) and 2(d). The ϕ-variation is stronger compared to the direct part of the HI potential calculated using finite range NN force shown in Figs. 2(a) and 2(b). As the separation distance R increases, the percentage variation of $V_{ex}(R,\beta_P,\beta_T,\phi)$ increases as ϕ changes in its range. For example, $V_{ex}(R,30^\circ,30^\circ,\phi)$ becomes more attractive by 15%, 25%, and 36.8% at a separation distance set of values $R = 12.0$, 14.0, and 17.0 fm, respectively. For the direct HI potential these percentages become 11.7%, 15.9%, and 27.7% for the same set of R values.

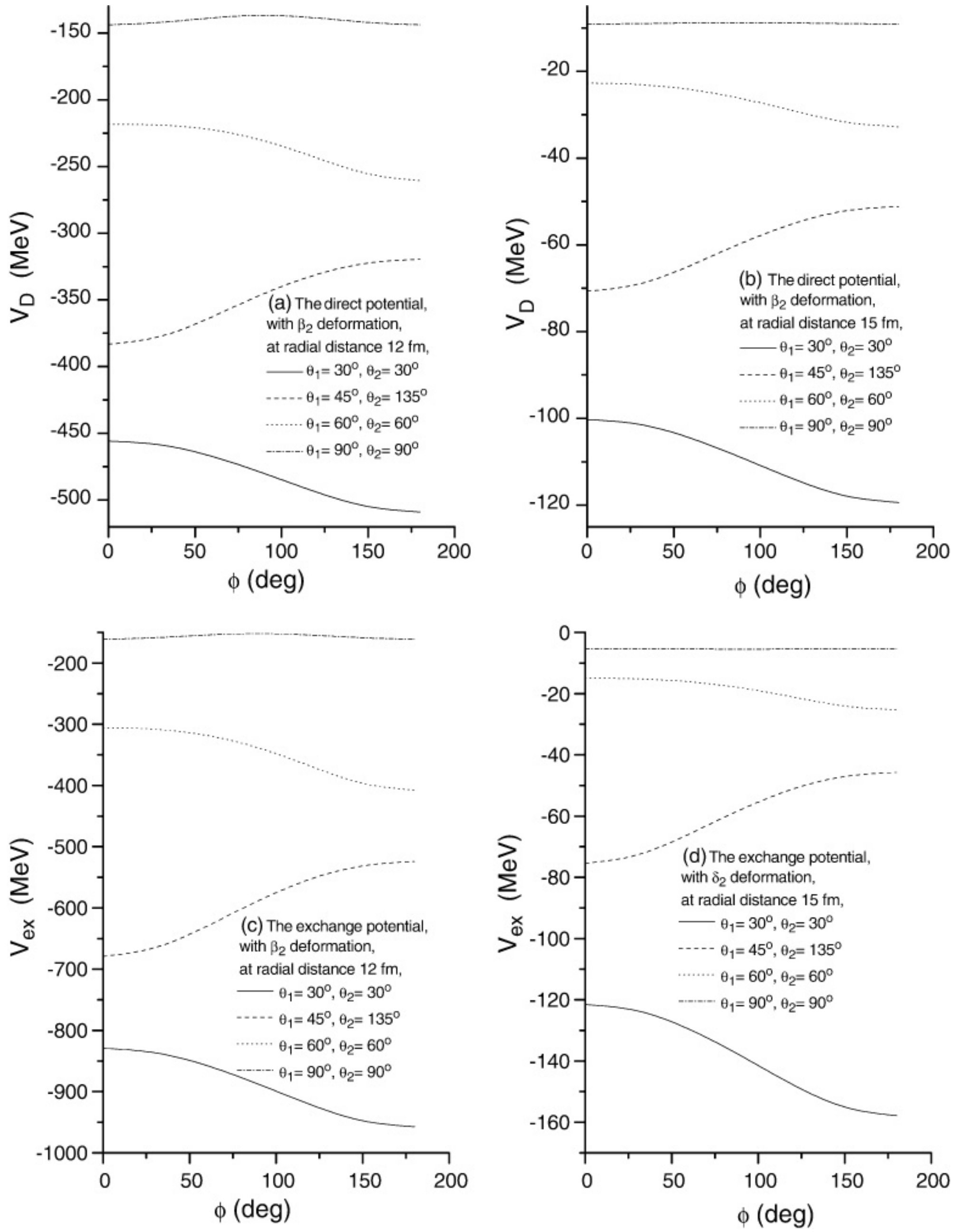

FIG. 2. The azimuthal angle variation of the nuclear HI potential for the $^{238}U + ^{238}U$ pair considering the quadrupole deformation only. (a) The direct part calculated at separation distance value $R = 12.0$ fm. (b) The direct part calculated at $R = 15.0$ fm. (c) The exchange part at $R = 12.0$ fm. (d) The exchange part at $R = 15.0$ fm.

As R increases the percentage change of $V_{ex}(R, 60^\circ, 60^\circ, \phi)$ becomes stronger and it reaches up to about 90% at $R = 17.0$ fm.

Figures 3(a)–3(d) show the same quantities as Figs. 2(a)–2(d), but the calculation is based on the value of the hexadecapole deformation parameter $\delta_4 = 0.087$ instead of $\delta_4 = 0.0$ used in Figs. 2(a)–2(d). Comparing the corresponding figures, one concludes that the presence of δ_4 in both deformed nuclei does not affect the behavior of direct and exchange parts of the HI potential with the variation of azimuthal angle ϕ. The presence of δ_4 deformation enhances strongly the ϕ dependence of the potential at large values of the separation distance R. Table I presents the values of the Coulomb, direct, and exchange contributions to the HI potential calculated at four-different orientations and for two values of the separation distance R for the case $\delta_4 = 0.087$. The first and second lines for each orientation represent the value of the potential at $\phi = 0^\circ$ and $\phi = 180^\circ$ or 90°.

Figures 2, 3, and Table I show that, for separation distance $R \geqslant 12.0$fm the HI potentials are enhanced for the orientations $(30^\circ, 30^\circ)$ and $(60^\circ, 60^\circ)$ as the azimuthal angle ϕ increases, while for the orientation $(45^\circ, 135^\circ)$ it is the other way around,

TABLE I. The ϕ-variation of the Coulomb, direct, and exchange parts in the HI potential (MeV) between the $^{238}U + ^{238}U$ interacting pair at four different sets of orientation angles.

β_1 (deg)	β_2 (deg)	ϕ (deg)	$R = 14$ fm			$R = 17$ fm		
			V_C	V_D	V_{Ex}	V_C	V_D	V_{Ex}
30.0	30.0	0.0	905.4	−180.9	−266.7	738.4	−20.6	−15.5
		180.0	910.0	−209.7	−332.0	740.0	−26.3	−21.2
60.0	60.0	0.0	860.2	−56.4	−48.9	711.7	−2.3	−0.96
		180.0	864.2	−75.1	−77.2	713.3	−3.63	−1.8
90.0	90.0	0.0	842.6	−26.6	−18.5	700.9	−0.81	−0.39
		90.0	841.3	−25.5	−18.2	700.4	−0.82	−0.40
45.0	135.0	0.0	887.0	−137.8	−185.0	726.6	−11.76	−7.75
		180.0	880.9	−105.9	−122.8	724.0	−7.34	−4.04

the change in the HI interaction between the nuclear interacting pair is depressed. This can be understood by purely geometrical considerations concerning the overlap region between the two nuclei for these orientations. For $R \geqslant 12.0$ fm, the value of the overlap integral,

$$\int d\boldsymbol{r}\rho_P(\boldsymbol{r}), \hat{\Omega}_P)\rho_T(\boldsymbol{r} + \boldsymbol{R}, \hat{\Omega}_T),$$

increases as ϕ increases for the orientations $(\beta_P, \beta_T) = (30^\circ, 30^\circ)$ and $(60^\circ, 60^\circ)$, while it decreases for $(45^\circ, 135^\circ)$.

The variation of the Coulomb and nuclear potentials near the position of the Coulomb barrier is of great importance in calculating the fusion [20] and fission cross sections [21]. Since the fusion cross section (σ_f) varies almost as the logarithmic function containing the difference between the total energy and the height of the Coulomb barrier [22], we expect that any small variation of the values of the Coulomb barrier parameters affects significantly σ_f. Recent calculations of σ_f for two nuclei having permanent and/or vibrational deformations do not include ϕ-dependence in the HI potential. Moreover, these calculations include the effect of deformation of one or both nuclei using too simplified methods [22]. This shows the importance of studying the ϕ-dependence of the HI potential using realistic calculations, which is the aim of the present work.

In the field of producing superheavy elements, no fusion experiments are made with both reaction partners as deformed nuclei. Some authors considered this type of reaction to study the azimuthal angle dependence of the Coulomb barrier parameters for two deformed nuclei using simplified models. For example, the authors in Ref. [7] considered the ϕ-variation for the Coulomb barrier of the reaction $^{150}Nd + ^{150}Nd$ and $^{180}Hf + ^{180}Hf$. They used the proximity method to derive the nuclear part of the HI potential. The Coulomb part was calculated using the approximate Wong's formulaes [15]. They found that the ϕ-dependence of identical and nonidentical target-projectile combinations plays an important role in determining the interacting barrier height and position for the orientation $(90^\circ, 90^\circ)$ of the two nuclei.

We compare the calculated results for the ϕ-dependence of the nuclear part of the $^{238}U + ^{238}U$ potential with the same quantity calculated for $^{150}Nd + ^{150}Nd$ using the proximity method [7]. For $\beta_P = \beta_T = 90^\circ$ the orientation considered in Ref. [7], the HI potentials are symmetric around $\phi = 90^\circ$. The nuclear potentials for the $^{238}U + ^{238}U$ pair vary by less than 3.5% around the Coulomb barrier, when ϕ is changed from $0^\circ - 90^\circ$, while the proximity potentials in Ref. [7] vary by more than 18.0% at their minimum. The presence of δ_4 deformation with positive value affects the values of both the proximity and the folding potentials in the surface region of the HI potential by more than 24%. The hexadecapole deformation lowers the folding potentials, while it raises the potentials in the proximity calculations presented in Ref. [7].

The effect of the azimuthal angle variation on the parameters of the physically significant Coulomb (fusion) barrier for the $^{238}U + ^{238}U$ interacting pair is given in Table II. The table shows the maximum percentage of the ϕ-variation of the barrier height V_B and its radius R_B for different orientation

TABLE II. The maximum percentage variation of the Coulomb barrier height and its position with the azimuthal angle at different orientations of the symmetry axes of the $^{238}U + ^{238}U$ interacting nuclei.

β_P (deg)	β_T (deg)	ϕ range (deg)	$\delta_2 = 0.331, \delta_4 = 0.0$		$\delta_2 = 0.331, \delta_4 = 0.087$	
			R_B(%)	V_B(%)	R_B(%)	V_B(%)
30	30	180	1.913	−1.450	3.927	−3.119
60	60	180	3.094	−2.414	2.623	−2.062
90	90	90	−0.369	0.065	0.289	−0.379
45	135	0–180	−3.126	2.683	−4.760	4.943

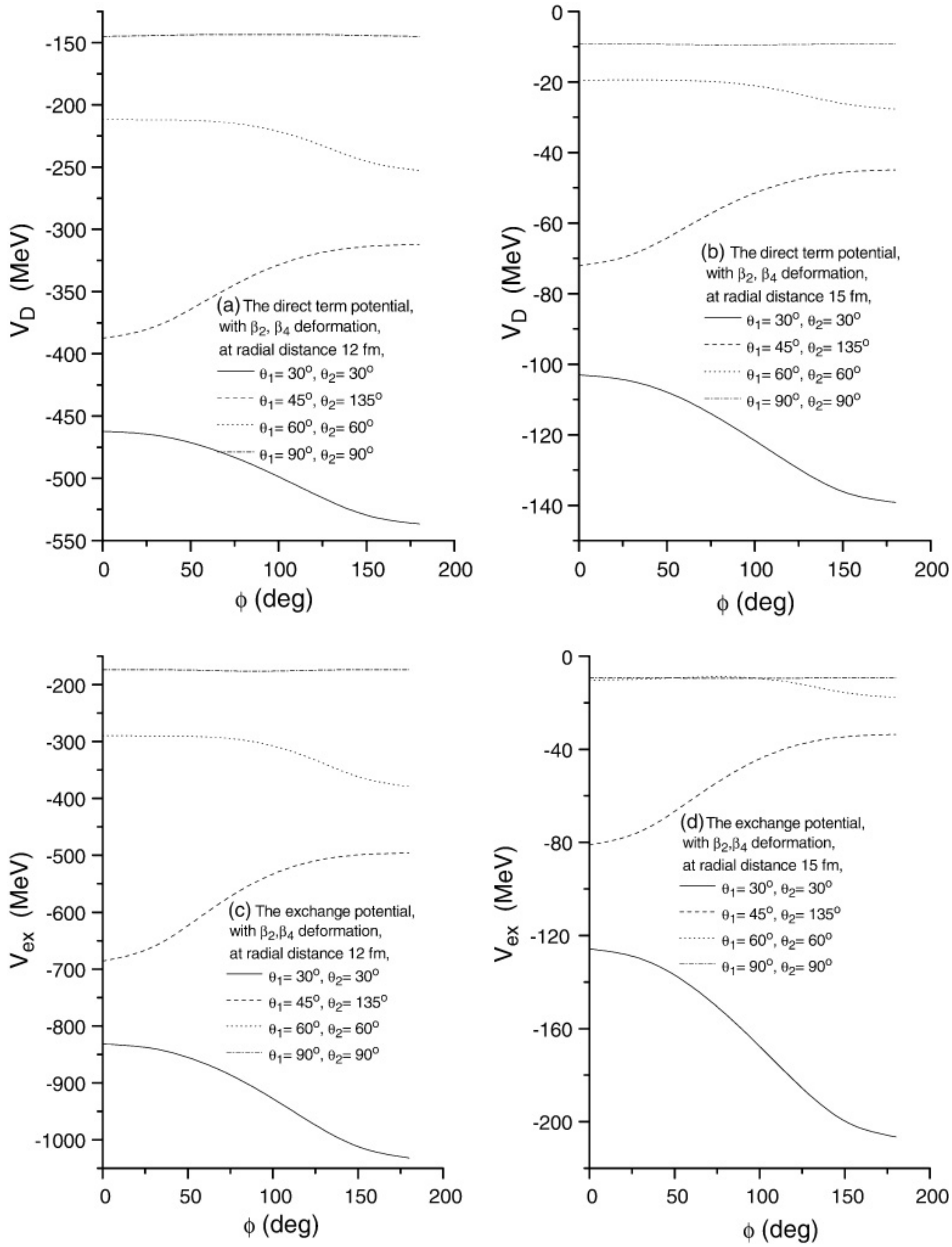

FIG. 3. The azimuthal angle variation of the nuclear HI potential for the $^{238}U + ^{238}U$ pair considering the quadrupole and the hexadecapole deformation with $\delta_4 = 0.087$. (a) The direct part calculated at separation distance value $R = 12.0$ fm. (b) The direct part calculated at $R = 15.0$ fm. (c) The exchange part at $R = 12.0$ fm. (d) The exchange part at $R = 15.0$ fm.

angles of the symmetry axes of the two interacting nuclei. The two cases $\delta_4 = 0.0$ and $\delta_4 = 0.087$ are presented in Table II. The percentage changes in both V_B and R_B show that the ϕ-change enhances strongly the fusion cross section for the orientations (30°, 30°) and (60°, 60°), while it produced opposite effect on fusion cross section for (45°, 135°). For the orientation (90°, 90°) Table II shows a small variation of R_B, V_B with the azimuthal angle, since this result is completely different from that found in Ref. [7], we extend our calculations of R_B and V_B derived from realistic M3Y-NN force to one of the reactions considered in Ref. [7]. We used for the ^{150}Nd nucleus the same values of density and deformation parameters considered in Ref. [7]. We calculated the azimuthal angle variation of R_B and V_B at the orientation (90°, 90°) for the three values of the hexadecapole deformation parameters $\delta_4 = 0.0$, 0.107, and -0.107. Since the Nd-nucleus has prolate quadrupole deformation, the three cases were denoted by pp, p^+p^+, and p^-p^- collisions, respectively. For the orientation (90°, 90°) the HI potential for the pp collision calculated using M3Y-NN force predicts the value of R_B which gradually decreases by an amount 0.23% , while V_B gradually increases by 0.03% when ϕ increases from 0° to 90°

TABLE III. The values of the Coulomb barrier height and its position for the values of the azimuthal angle 0° and 90° for the ^{150}Nd + ^{150}Nd interacting nuclei.

β_P (deg)	β_T (deg)	$\delta_2 = 0.243, \delta_4 = 0.0$			$\delta_2 = 0.243, \delta_4 = 0.107$			$\delta_2 = 0.243, \delta_4 = -0.107$		
		ϕ (deg)	R_B (fm)	V_B (MeV)	ϕ (deg)	R_B (fm)	V_B (MeV)	ϕ (deg)	R_B (fm)	V_B (MeV)
90	90	0	13.00	367.60	0	13.19	359.78	0	12.91	371.67
90	90	90	12.97	367.70	90	13.20	359.44	90	12.77	375.09

as shown in table III. These smaller percentages are enhanced to 1.09% and 0.9% for the $p^- p^-$ collision.

For the $p^+ p^+$ collision R_B increases by 0.08% while V_B decreases by 0.1% for the ϕ-variation in the range 0–90°. These results are in correspondence with that obtained for the ^{238}U + ^{238}U pair (the value of δ_2 for ^{238}U is greater by 36% compared with δ_2 of ^{150}Nd). The results of the proximity method show a gradual variation of R_B and V_B with increasing ϕ for the pp collision only. When δ_4 is added the variation of R_B and V_B with ϕ becomes oscillating.

Table III shows the values of V_B and R_B at the orientation (90°, 90°) for pp, $p^+ p^+$ and $p^- p^-$ collisions.

We can summarize the results of our study on the variation of the HI potential with the azimuthal angle in the following points.

(i) The percentage variation of the HI potential when ϕ changes in the range 0°–180° depends on the values of the deformation parameters and range of the NN force used to derive the HI potential. Large and vanishing ranges of the NN force produce a weak and strong azimuthal angle variation of the HI potential. As the values of deformation parameters increase, the ϕ variation of the HI potential increases. It is larger in case of the ^{238}U + ^{238}U interaction compared with the Nd-Nd potential because the quadrupole deformation parameter has the values $\delta_2 = 0.331$ and 0.243 for U and Nd nuclei, respectively.

(ii) The relative orientation of the two interacting nuclei plays an important role in producing the ϕ variation of the HI potential. This variation vanishes if β_P and/or β_T have/has zero value. It is too small if $(\beta_P, \beta_T) =$ (90°, 90°) and becomes stronger for the orientations (30°, 30°), (60°, 60°).

(iii) ϕ-variation in the range of 0–180° enhances the HI potentials for the orientations (30°, 30°) and (60°, 60°), while it reduces the interactions for the orientations (45°, 135°) and (90°, 90°).

(iv) The HI-potential derived from the folding model with M3Y-NN force has different ϕ variation than that derived from the the proximity approach [7].

(v) ϕ-variation affects appreciably the parameters of the Coulomb barrier, which is a physically important quantity in the field of heavy ion collision. It determines the fusion cross section and is used in the superheavy nuclei production field.

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