Analysis of elastic α-nucleus scattering data at 240 MeV

M. A. Alvi,* J. H. Madani, and A. M. Hakmi
Department of Physics, Science Faculty, KAAU, Jeddah, Saudi Arabia
(Received 20 February 2007; published 14 June 2007)

Working within the framework of the Coulomb modified Glauber model we fit the elastic differential scattering cross section of 240 MeV α particle on ^{58}Ni using the effective N-α amplitude with one adjustable parameter. It is found that once the effective amplitude is calibrated on ^{58}Ni by varying the adjustable parameter, it very nicely reproduces the available elastic α scattering data on other nuclei at the same energy.

DOI: 10.1103/PhysRevC.75.064609 PACS number(s): 24.10.Ht, 25.55.Ci, 25.70.Bc

I. INTRODUCTION

During the past several years, the acceleration of light ions at low and intermediate energies has been the subject of much interest in the field of nucleus-nucleus collision which is complementary to nucleon-nucleus scattering. The α particle, due to its zero spin and i-spin and of relatively large binding energy per nucleon, has naturally appeared prominently in the list of light ion nuclear scattering experiments. As a result, an adequate amount of α-nucleus elastic and inelastic scattering data at several energies are available in literature [1–4].

Generally, two theoretical approaches have been used for analyzing α-nucleus elastic scattering data. One is at low and medium energies by using the phenomenological optical model potential. At these energies the behavior of α-nucleus scattering cross sections is dominated by strong absorption in the nuclear surface region. More explicitly, the cross section depends mainly on a small number of phase shifts (δ_l), i.e., to those where l-values correspond to impact parameters in the region of nuclear surface. Since a different interior wave function can generate the same phase shifts, the optical model parametrization of the interaction leads to ambiguous information regarding the optical potential. This has been found to give the existence of discrete and continuous ambiguities as well as to the uncertainty in the general shapes of the real and imaginary potentials [2,5,6].

The other approach which has frequently been used is the Glauber model [7] using optical limit approximation or the rigid projectile model (RPM) [1,8–12]. It is found that the Glauber model, though based on the high energy approximation, works reasonably well even at some lower energies provided, it is modified to account for the deviation of projectile trajectory due to Coulomb field [13–16].

The fact that the evaluation of the full Glauber amplitude for a realistic description of nuclei is a computationally difficult task, Ahmad and Alvi [17] proposed a simple semiphenomenological method of analysis for α-nucleus elastic scattering data at medium and high energies. The method consists of using an effective N-α amplitude with one adjustable parameter instead of the generally used N-α elastic scattering amplitude in the usual RPM. The small momentum transfer (q) part of the effective amplitude is fixed from N-α scattering experiments while the large q part which is assumed to simulate the nuclear medium effect as well as the use of RPM approximation, is treated phenomenologically. Using this amplitude and the realistic ground state target densities, they found excellent agreement with elastic α-nucleus scattering data at 1.37 GeV.

In the recent past, the differential cross section for elastic scattering of the α particle is measured [3] at the incident energy of 60 MeV/nucleon for ^{58}Ni, ^{116}Sn, and ^{197}Au. An optical model analysis has also been performed that shows energy dependence as well as ambiguities in parameters of the optical potential.

In this paper, motivated from the success and simplicity, we use the method of the effective N-α amplitude [17] to analyze the α-nucleus elastic scattering data [3] at 60 MeV/nucleon within the framework of Coulomb modified Glauber model. As we will see shortly, a very satisfactory description of α-nucleus elastic scattering data is achieved.

II. FORMULATION

In the rigid projectile model assuming that the effect of correlations in the target nucleus is small [10], the S-matrix element $S_N(b)$ for the elastic α-nucleus scattering may approximately be written as

$$S_N(b) \approx [1 - \Gamma_\alpha(b)]^A \qquad (1)$$

with

$$\Gamma_\alpha(b) = \frac{1}{ik}\int q\,dq\,J_0(qb)\,f_{N\alpha}(q)\,F_A(q), \qquad (2)$$

where k is the nucleon momentum corresponding to the α particle kinetic energy per nucleon, $f_{N\alpha}(q)$ is the N-α scattering amplitude, and $F_A(q)$ is the target form factor. In all our calculations, the nuclear form factors are parametrized as a sum of Gaussians:

$$F_A(q) = \sum_{j=1} a_j\, e^{-b_j q^2}, \qquad (3)$$

where a_j and b_j are the parameters. The advantage of using this parametrization is that the phase-shift function for the Coulomb potential due to the finite charge distribution of the colliding nuclei can be evaluated analytically. The parameter values of the nuclear form factors for ^{58}Ni, ^{116}Sn, and ^{197}Au are

*alveema@hotmail.com

TABLE I. Parameter values of the sum of the Gaussian parametrization of the nuclear form factor.

Nucleus	a_j	b_j (fm^2)
^{58}Ni	−9.69	0.785
	0.744	0.2519
	−4.58	0.3877
	−76.48	0.469
	5.438	1.23
	85.573	0.475
^{116}Sn	−16.15	1.37
	4.417	2.31
	−4.653	3.158
	−67.6	1.708
	14.924	2.44
	70.06	1.558
^{197}Au	−8.02	1.6477
	4.852	1.94
	35.025	1.9866
	−65.316	2.373
	−15.94	3.54
	50.399	2.982

given in Table I. They have been obtained by fitting the form factors as given by the realistic ground state charge densities [18] of ^{58}Ni, ^{116}Sn, and ^{197}Au after correcting for the finite proton charge density. We also assume that the proton and neutron densities are the same.

We take the N-α scattering amplitude to be of the form [17]

$$f_{N\alpha}(q) = \frac{ik\sigma_\alpha(1 - i\rho_\alpha)}{4\pi} e^{-\beta_\alpha^2 q^2/2}\,[1 + \lambda q^4], \quad (4)$$

where σ_α is the N-α total cross section, ρ_α the ratio of the real to the imaginary parts of the forward angle scattering amplitude, β_α^2 is the slope parameter, and λ is a free parameter. The values of the parameters of $f_{N\alpha}(q)$, namely, σ_α, ρ_α, and β_α^2 which determine the small q behavior of the N-α amplitude should be the same as for the free N-α scattering at one-fourth of the kinetic energy of the incident α particle. In the present case we need their values at 60 MeV (incident nucleon kinetic energy). The values of σ_α are obtained using the parametrization of the N-α total cross section [19] which is well represented by the formula

$$\sigma_{N\alpha} = \frac{15.84}{T_L + 0.74} + 0.133\,\frac{e^{\frac{T_L-390}{159}}}{\left[1 + e^{\frac{T_L-390}{159}}\right]}, \quad (5)$$

where $T_L(=E_{\text{lab}})$ is in MeV/nucleon. The required value of ρ_α is taken from the graph/table given in the paper of Schwaller *et al.* [20] at the energy of our interest. The values are

$$\sigma_\alpha = 275\,\text{mb}; \quad \rho_\alpha = 1.35. \quad (6)$$

Regarding the parameter β_α^2, it may be determined with a fair degree of certainty at high energies where the scattering is mostly diffractive and peaked in the forward direction. At lower energies where the scattering is nondiffractive, a cursory survey of the literature shows that it has a large uncertainty [19–21]. We, therefore, treat it as an adjustable parameter.

The elastic scattering amplitude for the scattering of a charged nuclear particle from target nucleus of mass number A and charge number Z may be written as

$$F_{el}(\theta) = F_c(\theta) + \frac{i}{2K}\sum_{l=0}^{\infty}(2l+1)\,e^{2i\sigma_l}[1 - S_l]P_l(\cos\theta), \quad (7)$$

where $F_c(\theta)$ is the point Coulomb scattering amplitude, σ_l is the point Coulomb phase shift, K is the c.m. momentum of the system, $P_l(\cos\theta)$ is the Legendre polynomial, and S_l is the elastic S-matrix element. Following [22–24], we evaluate S_l approximately from the relation

$$S_l \approx S_N(b)\,e^{i\chi_c(b)}|_{Kb=l+1/2}, \quad (8)$$

where b is the impact parameter, $S_N(b)$ is the nuclear part of the S-matrix element in the impact parameter space. The quantity $\chi_c(b)$ is the difference between the phase-shift functions of the Coulomb potential due to the extended charge distributions of the interacting nuclei and the corresponding point charges. Both $S_N(b)$ and $\chi_c(b)$ are obtained by invoking the Glauber high energy approximation. Here, it may be mentioned that the $\chi_c(b)$ has been neglected in most of the elastic scattering studies.

One of the basic assumptions of the Glauber model is that the projectile follows a straight line trajectory during a collision with the target nucleus. This is not a good approximation at lower energies. However, for charged particle scattering some improvements over this approximation can be affected following the method Faldt and Pilkuhn [25] proposed in connection with the charged pion-nucleus scattering and latter applied by others [14–16,23]. The essential point of the method [25] is to replace the straight line trajectory of the Glauber model by the Rutherford trajectory and evaluate the Glauber S-matrix at a distance of closest approach b' instead of b. The quantity b' is related to b as

$$b' = \frac{\eta}{K} + \sqrt{\left(\frac{\eta}{K}\right)^2 + b^2}; \quad \eta = \frac{2Z_A e^2}{\hbar v}.$$

Finally, the elastic differential scattering cross sections for the α-nucleus are calculated using the expression

$$\frac{d\sigma}{d\Omega} = |F_{el}(\theta)|^2. \quad (9)$$

III. ANALYSIS

A. Elastic α-^{58}Ni scattering data

Using the values of parameters σ_α and ρ_α of $f_{N\alpha}(q)$ as given by Eq. (6) and the Gaussian form factor for the ^{58}Ni from Table I we fit the α-^{58}Ni elastic differential scattering cross-section data [3] by varying the parameters β_α^2 and λ. The result of the two-parameters fit is shown by the solid curve in the Fig. 1. The corresponding parameters values are

$$\text{Set A.} \quad \sigma_\alpha = 275\,\text{mb}; \quad \rho_\alpha = 1.35$$
$$\beta_\alpha^2 = 44\,\text{GeV}^{-2}; \quad \lambda = 100\,\text{GeV}^{-4}.$$

It is seen that the calculation agrees very nicely with the experiment over the whole momentum transfer covered by the data. It is important to note that a calculation by Ahmad [26]

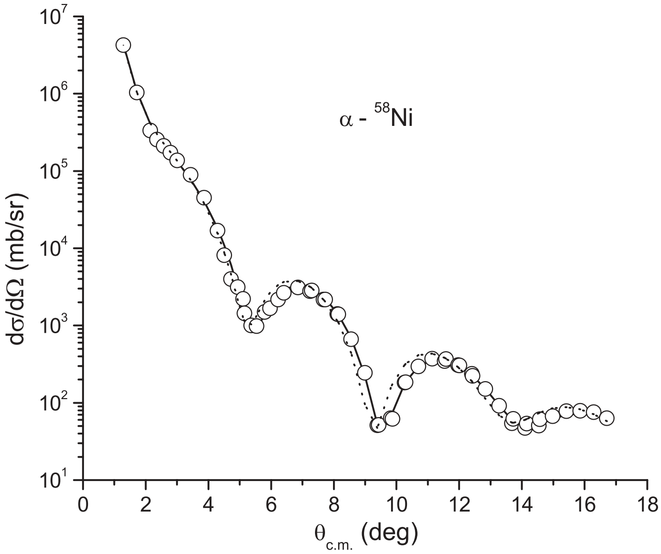

FIG. 1. Elastic differential scattering cross section for α-^{58}Ni scattering at 240 MeV. The solid and dashed curves show the results with and without trajectory deviation due to the Coulomb field using Set A, respectively. The open circles represent the experimental data [3].

using N-α phase-shifts shows $\beta_\alpha^2 = 38.75\,\text{GeV}^{-2}$ at about 20 MeV neutron lab energy. This value of the slope parameter is very close to that achieved above in our calculation. It has already been stated that at lower energies very many different values for β_α^2 have been used in the literature. Therefore, it is quite justified to say that the solid curve for ^{58}Ni in Fig. 1 represents a single parameter fit to the experimental data.

The dashed curve in Fig. 1 shows the predictions of the calculation without considering the deviation of the projectile trajectory due to the Coulomb field between the α particle and the target ^{58}Ni nucleus with the same parameter values as in Set A. There is a qualitative difference, though small between the two calculations starting from momentum transfer q is as low as 0.4 fm^{-1}. This suggests the importance of accounting the modification of the trajectory due to the Coulomb field at low energy.

B. Elastic scattering of α-^{116}Sn and ^{197}Au

The fact that at the high energy effective N-α amplitude once determined by fitting α-nucleus elastic scattering data [17] reproduces very nicely the elastic differential cross sections on neighboring nuclei at the same energy, it is tempting to see if a similar situation exists at this energy too.

In Figs. 2 and 3, we compare the results of parameter free calculations (solid curve) for the elastic α-^{116}Sn and α-^{197}Au scattering at 240 MeV with the experimental data. The calculations have been made with the parameter Set A of the effective N-α amplitude and the form factors as given in Table I. It is seen that calculations agree fairly well with the experimental data. In particular the positions of cross-sections maxima and minima are well reproduced for both nuclei. However, there is some quantitative difference between theory

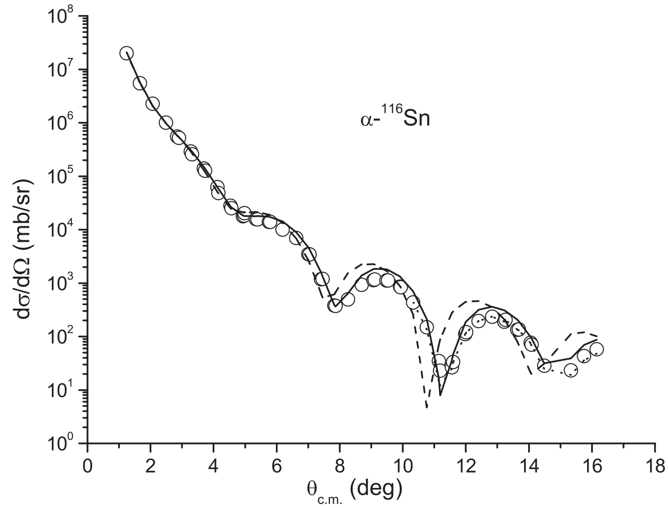

FIG. 2. Elastic differential scattering cross section for α-^{116}Sn scattering at 240 MeV. The descriptions of solid and dashed curves are the same as in Fig. 1. The dotted curve shows the optical model fitting by Clark *et al*. [3].

and experiment at higher q specially for the ^{197}Au nucleus. In the figures we also show by the dashed curve the predictions of our calculations without considering the deviation of the projectile trajectory due to the Coulomb field between the α particle and respective nuclei with the same parameter values of the effective N-α amplitude as above. As expected there is a large qualitative difference between the calculations with and without considering the trajectory deviation due to the Coulomb field.

With regards to some disagreement at higher q, we, just for comparison, in Figs. 2 and 3 show by dotted curves the six parameters optical model fitting by Clark *et al*. [3]. It is seen that even their optical model fitting shows disagreement at higher q for the α-^{197}Au elastic scattering data. It is also useful to point out that Ahmad and Alvi [17] using the effective

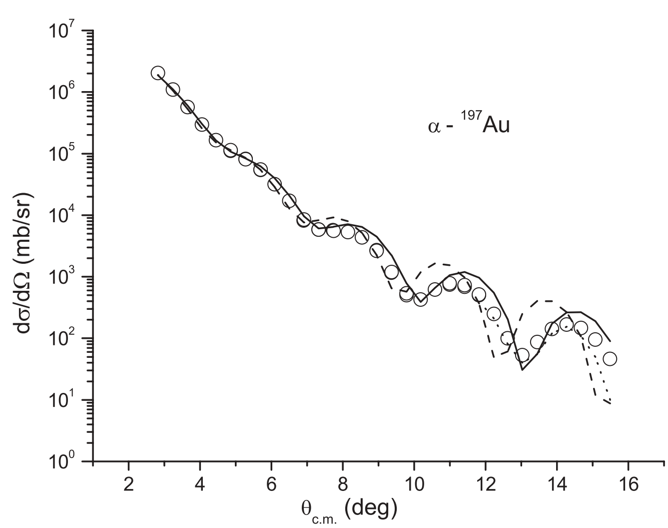

FIG. 3. Elastic differential scattering cross section for α-^{197}Au scattering at 240 MeV. The descriptions of the curves are the same as in Fig. 2.

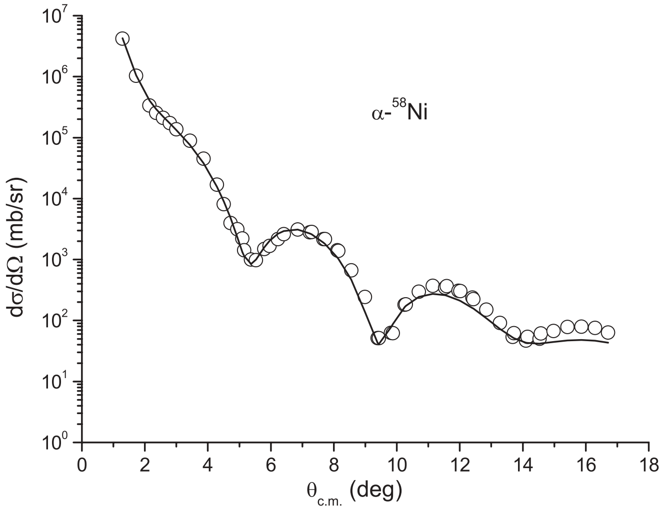

FIG. 4. Elastic differential scattering cross section for α-^{58}Ni scattering at 240 MeV. The solid curve represents the calculation using parameter Set B.

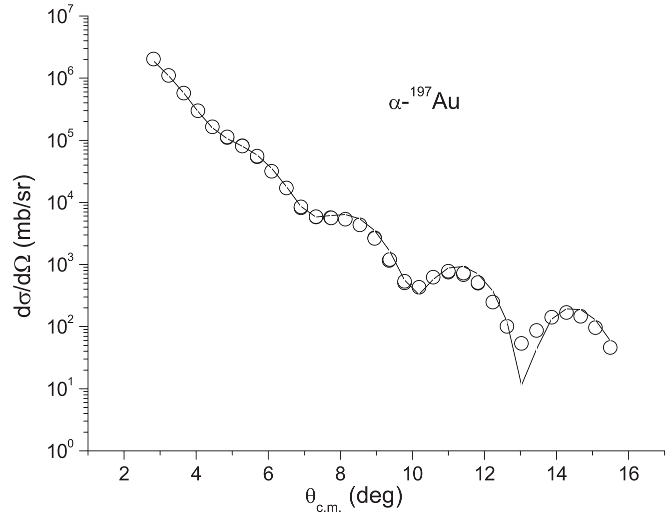

FIG. 6. Elastic differential scattering cross section for α-^{197}Au scattering at 240 MeV. The description of the curve is the same as in Fig. 4.

N-α amplitude got impressive success in reproducing α-nucleus elastic scattering data only to very neighboring mass nuclei while far mass nucleus they too have discrepancies between the calculated values and the experimental data.

In view of the large mass difference between the calibrated ^{58}Ni nucleus and the nuclei ^{116}Sn and ^{197}Au, we also show in Figs. 4–6 the results of our calculation with average values of the parameters β_α^2 and λ. For this, we first fit nicely the individual α-nucleus elastic scattering data by varying β_α^2 and λ of $f_{N\alpha}(q)$ and then taking the average values of these parameters. The corresponding values are

$$\text{Set B.}\quad \sigma_\alpha = 275\,\text{mb}; \qquad \rho_\alpha = 1.35$$
$$\beta_\alpha^2 = 49.28\,\text{GeV}^{-2}; \quad \lambda = 210\,\text{GeV}^{-4}.$$

It is seen that the data are very nicely reproduced especially for ^{116}Sn and ^{197}Au over the whole angular range. During the course of fitting we found that it is β_α^2 which demands the larger value than the free one for higher mass nuclei. At present we do not have any explanation for the above behavior except that more studies are needed before we come to any definite conclusion.

Finally, in view of the above remarks, the reasonably good account of experimental data suggests that the effective N-α amplitude method works well even at this low energy. On the whole it does not seem unjustified to conclude that once the effective N-α amplitude is calibrated on a target nucleus of known structure, it is fairly stable over a broad range of target mass nuclei at a given energy (this statement needs more study both theoretically and experimentally).

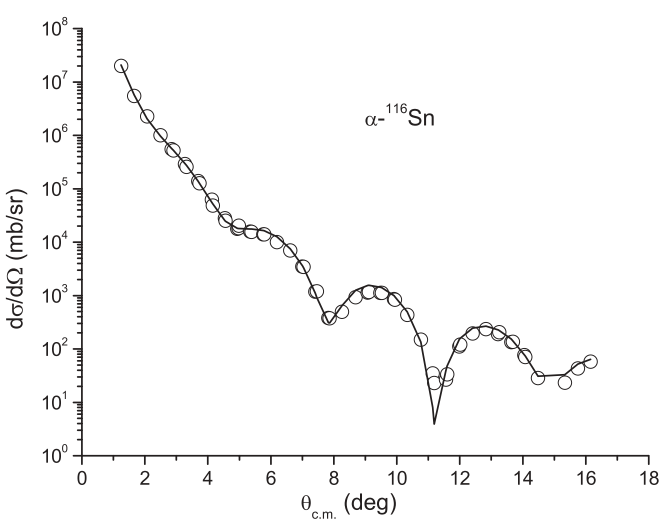

FIG. 5. Elastic differential scattering cross section for α-^{116}Sn scattering at 240 MeV. The description of the curve is the same as in Fig. 4.

IV. SUMMARY AND CONCLUSION

In this work we have presented a theoretical study of α-^{58}Ni, α-^{116}Sn, and α-^{197}Au elastic scattering at 240 MeV using the Coulomb modified Glauber multiple scattering theory. The main feature of the present study is the use of an effective N-α amplitude with effectively one adjustable parameter instead of the generally used N-α elastic scattering amplitude in the usual rigid projectile model (RPM) for the evaluation of the Glauber amplitude.

To appreciate the usefulness of our work it should be recalled that the optical model analysis though successful in many respects in accounting the α-nucleus elastic scattering data, but more important is the fact that it suffers from the existence of discrete and continuous ambiguities as well as uncertainty in the general shape of the real and imaginary potentials. Whereas, the Glauber theory which is essentially based on the high energy approximation is found to be successful even at low energies without such ambiguities provided that it is suitably modified to account for the deviation of the projectile trajectory due to the Coulomb field [14–16].

Needless to say the form of effective N-α amplitude is such that its small q part is the same as the measured N-α amplitude at the corresponding incident nucleon kinetic energy. It is the large q part which is assumed to simulate the nuclear medium effect, treated phenomenologically. By using the parameters of the effective N-α amplitude at energy one-fourth of the energy of α particle from N-α scattering experiments we obtained a very good fit to the elastic α-^{58}Ni scattering data at 240 MeV by varying effectively one adjustable parameter λ. It is also found that the same amplitude reproduces α-^{116}Sn and α-^{197}Au elastic scattering data at 240 MeV quite well. There are some small quantitative difference between our parameter free calculation and the experiment at higher angles especially for the ^{197}Au nucleus. Whatever may be the reason, the same discrepancy is also visible in the six parameters optical model analysis by Clark *et al*. [3]. As expected, we have found a large qualitative difference between our results with and without considering the trajectory deviation due to the Coulomb field.

Coming back to the small discrepancies for parameter free calculations at higher q values particularly for the ^{197}Au nucleus, we suspect it may be due to some weak mass dependence in λ of the effective N-α amplitude. But our results with average parameter values of the N-α amplitude shows the preference of higher values of parameter β_α^2.

Finally, on the whole we found that the effective N-α amplitude method of analysis of elastic α-nucleus scattering data is quite encouraging and works reasonably well even at low energy. In addition it would not be unjustified to conclude that once the sole parameter λ of the effective N-α amplitude is fixed from the elastic scattering data on a target nucleus of known ground state density, it is fairly stable over a wide range of target nuclei at a given energy (in order to confirm this, one needs elastic α-nucleus data in a large domain of incident energies and target nuclei).

ACKNOWLEDGMENTS

The authors are grateful to Dr. Faraj S. Alhazmi, chairman, physics department for providing the facilities to carry out this work. They are also thankful to Prof. I. Ahmad for fruitful discussions and suggestions.

[1] A. Chaumeaux *et al*., Nucl. Phys. **A267**, 413 (1976).
[2] B. Bonin *et al*., Nucl. Phys. **A445**, 381 (1985).
[3] H. L. Clark, Y. W. Lui, and D. H. Youngblood, Nucl. Phys. **A589**, 416 (1995).
[4] M. Avrigeanu, W. von Oertzen, A. J. M. Plompen, and V. Avrigeanu, Nucl. Phys. **A723**, 104 (2003).
[5] K. A. G. Rao *et al*., Phys. Rev. C **62**, 014607 (2000).
[6] L. Trache *et al*., Phys. Rev. C **61**, 024612 (2000).
[7] R. J. Glauber, in *Lectures in Theoretical Physics: High-Energy Collision Theory*, Boulder, edited by W. E. Brittin (Interscience Publishers, New York, 1959), Vol. 1, p. 315.
[8] G. D. Alkhazov *et al*., Nucl. Phys. **A280**, 365 (1977).
[9] V. Franco and G. K. Varma, Phys. Rev. C **18**, 349 (1978).
[10] I. Ahmad, J. Phys. G: Nucl. Phys. **4**, 1695 (1978).
[11] I. Ahmad, J. Phys. G: Nucl. Phys. **6**, 947 (1980).
[12] G. Faldt and I. Hulthage, Nucl. Phys. **A316**, 253 (1979).
[13] J. Chauvin, D. Lebrun, A. Lounis, and M. Buenerd, Phys. Rev. C **28**, 1970 (1983).
[14] A. Vitturi and F. Zardi, Phys. Rev. C **36**, 1404 (1987).
[15] S. M. Lenzi *et al*., Nucl. Phys. **A536**, 118 (1992).
[16] S. K. Charagi and S. K. Gupta, Phys. Rev. C **41**, 1610 (1990).
[17] I. Ahmad and M. A. Alvi, Phys. Rev. C **28**, 2543 (1983).
[18] H. De Varies *et al*., At. Data and Nucl. Data Tables **36**, 495 (1987).
[19] I. Ahmad, M. A. Abdulmomen, and L. A. Al-Khattabi, Int. J. Mod. Phys. E **8**, 229 (1999).
[20] P. Schwaller *et al*., Nucl. Phys. **A316**, 317 (1979).
[21] I. Ahmad, M. A. Abdulmomen, and M. A. Alvi, Int. J. Mod. Phys. E **11**, 519 (2002).
[22] Z. Kirzon and A. Dar, Nucl. Phys. **A237**, 319 (1975).
[23] M. A. Alvi and I. Ahmad, J. Phys. G: Nucl. Phys. **8**, 1209 (1982).
[24] I. Ahmad and M. A. Alvi, Int. J. Mod. Phys. E **13**, 1225 (2004).
[25] G. Faldt and H. Pilkuhn, Phys. Lett. **B46**, 337 (1973).
[26] I. Ahmad (private communication, 1998).