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Evolution of nuclear shells with the Skyrme density dependent interaction

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We present the evolution of the shell structure of nuclei in Hartree-Fock calculations using Skyrme's density-dependent effective nucleon-nucleon interaction. The role of the tensor part of the Skyrme interaction to the Hartree-Fock spin-orbit splitting in spherical spin unsaturated nuclei is reanalyzed. The contribution of a finite range tensor force to the spin-orbit splitting in closed shell nuclei is calculated. It is found that the exact matrix elements of a Gaussian and of a one-pion exchange tensor potential could be written as a product Skyrme's short range expression times a suppression factor which is almost constant for closed shell nuclei with mass number $A \geqslant 48$. The suppression factor is ~ 0.15 for the one-pion exchange potential.

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I. INTRODUCTION

The shell structure is a distinctive feature of nuclei and is characterized by the existence of magic numbers that are a consequence of the spin-orbit interaction [1,2]. The spin-orbit interaction can be understood in a mean-field approach that leads to a one-body potential containing a central part and a spin-orbit part. In spin-saturated nuclei the spin-orbit part stems from the spin-orbit nucleon-nucleon interaction. In spin unsaturated nuclei there are additional contributions coming either from the exchange part of the central two-body force or from the tensor force [3–5]. In view of the recent progress related to the discovery of exotic nuclei (neutron or proton rich) a major problem is to understand how the shell structure evolves from stable to unstable nuclei. Presently there is much concern about the role of the tensor force in the shell evolution and the structure of exotic nuclei [6–11].

In a previous work [3] we estimated the contribution of the tensor part of the Skyrme interaction to the Hartree-Fock spin-orbit splitting in several magic nuclei and adjusted the strength of the tensor force such as to obtain a good global fit. In the present article we extend the previous study to exotic nuclei, most of which were unknown at that time. This extension sheds a new light on the previous results.

The tensor term in the Skyrme interaction is written as a δ function in the internucleon separation multiplied by momentum-dependent terms (Sec. IV). The momentum dependence takes the finite range of the interaction into account. Contrary to the view that it plays a minor role because of its δ-type structure [6], this interaction has the same effect as a finite size interaction due to its momentum dependence. We will show that the Skyrme interaction provides a good mechanism for describing the evolution of the shell structure in exotic nuclei.

Otsuka *et al.* [6] have pointed out that the nuclear tensor force has a rather long range and that the use of the energy density (1) may not be justified. The goal of the present article is to show that expressions (1) together with Eqs. (16) and (17), given in Sec. II of this article, can still be used to study the contribution of finite range tensor forces, even if the range is that of the one-pion exchange force. Shell gaps are mainly determined by the spin-orbit splitting of the states with highest l in any shell and our study is restricted to these states. The spin-orbit splitting is less important in states with lower l because it is hidden by pairing effects and other forms of configuration mixing.

In Sec. II we derive an expression for the leading contribution of the tensor force to the Skyrme energy density functional using some results from the article of Negele and Vautherin [12] on the density matrix expansion method. Section III presents some numerical calculations that show that the main effect of a longer-range interaction is to introduce a suppression factor that is almost constant for all nuclei with mass number greater than $A \sim 28$. Section IV recalls the expression of the tensor for a short-range tensor interaction. In Sec. V we present results for single-particle levels of Sn isotopes, $N = 82$ isotones and Ca isotopes, where the tensor force considerably improves the agreement with the experiment when its parameters are properly chosen.

The conclusion is that the Skyrme energy functional with the tensor force is adequate to describe the evolution of shell effects.

II. CONTRIBUTION OF A SHORT-RANGE TENSOR FORCE

The Skyrme parametrization of a short-range tensor force leads to a contribution to the energy density

$$\Delta H(\mathbf{r}) = \tfrac{1}{2}\alpha\left[J_n^2(\mathbf{r}) + J_p^2(\mathbf{r})\right] + \beta J_n(\mathbf{r})J_p(\mathbf{r}), \tag{1}$$

where the $J_q(\mathbf{r})$ $(q = n, p)$ are spin-orbit densities and α and β are parameters defined in Sec. V. They represent the combined effect of the tensor plus central exchange interactions. If the radial wave functions depend only on the orbital angular momentum l and not on j, then the spin-orbit densities are zero if both components of a spin-orbit doublet are filled. Then the energy density (1) brings contributions only to spin unsaturated

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nuclei. Thus $\Delta H(\mathbf{r})$ would be almost zero in ^{40}Ca, which is a double closed-shell nucleus. It would be large for ^{208}Pb, which has spin unsaturated shells for both neutrons and protons. The energy functional (1) leads to a simple modification of the single-particle spin-orbit potential for both protons and neutrons (see Sec. V).

The purpose of the present section is to derive the form of $\Delta H(\mathbf{r})$ for a short-range tensor interaction. We focus on the contribution of the neutron-proton interaction. The starting point is a two-body tensor potential

$$V_T(r) = v_T(r)\vec{\tau}_1 \cdot \vec{\tau}_2 \left[\frac{1}{3r^2}(\vec{\sigma}_1 \cdot \vec{\mathrm{r}})(\vec{\sigma}_2 \cdot \vec{\mathrm{r}}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2\right] \tag{2}$$

like the one arising from one-pion exchange. The effect of the isospin dependence in Eq. (2) is to make $\beta \approx 2\alpha$. An interaction with no isospin dependence would have $\beta = 0$.

According to Negele and Vautherin [12] the expectation value of the tensor interaction with Hartree-Fock wave functions is

$$\langle V_T^{np}\rangle = -\int d^3\mathbf{r}_1 d^3\mathbf{r}_2 v_T(\mathbf{r}_1 - \mathbf{r}_2)|\vec{\rho}_n(\mathbf{r}_1, \mathbf{r}_2) \cdot \vec{\rho}_p(\mathbf{r}_1, \mathbf{r}_2)|. \tag{3}$$

The expressions for the nn and pp contributions are similar but are each multiplied by a factor $1/2$ from the isospin dependence of Eq. (2). Negele and Vautherin give a factorization of the spin-density matrices for spherical nuclei in which subshells are either completely full or completely empty. It is

$$\vec{\rho}(\mathbf{r}_1, \mathbf{r}_2) = i(\mathbf{r}_1 \times \mathbf{r}_2)\rho_1(\mathbf{r}_1, \mathbf{r}_2), \tag{4}$$

where

$$\rho_1(\mathbf{r}_1, \mathbf{r}_2) = \pm\sum_{njl} \frac{1}{2\pi r_1^2 r_2^2} R_{njl}(r_1) R_{njl}(r_2) P_l'(\cos\theta). \tag{5}$$

P_l' is the derivative of the Legendre polynomial P_l, θ is the angle between the directions of $\mathbf{r}_1$ and $\mathbf{r}_2$ and the $\pm$ sign in Eq. (5) stands for $j = l \pm \frac{1}{2}$. For a short-range interaction $\theta \approx 0$ in Eq. (5) and $P_l'(\cos\theta) \approx l(l+1)/2$. This is the origin of the spin-orbit splitting factor in Eq. (6) of Ref. [3]

$$(2j+1)[j(j+1) - l(l+1) - 3/4] = \pm 2l(l+1) \quad \text{if} \quad \mathrm{j} = \mathrm{l} \pm 1/2. \tag{6}$$

If the radial wave functions $R_{njl}(r)$ are the same for $j = l \pm 1/2$ then the contribution of a particular l-level to $\rho_1(\mathbf{r}_1, \mathbf{r}_2)$ vanishes if both j components are either completely occupied or completely empty.

When the interaction $V_T(r)$ has a sufficiently short range the $\langle V_T^{np}\rangle$ simplifies to

$$\langle V_T^{np}\rangle = \frac{\pi}{6}\int d^3\mathbf{r} J_n(r) J_p(r) \int v(s) s^4 ds, \tag{7}$$

where

$$J_q(r) = 2r\rho_{1q}(\mathbf{r}, \mathbf{r}), \tag{8}$$

which is the spin-orbit density in Eq. (6) of Ref. [3]. Equation (6) shows that $J_q(r) > 0$ when the lower component of a spin-orbit doublet is being filled and goes to zero when both components are filled.

III. A FINITE RANGE SUPPRESSION FACTOR

The analysis in the present section shows that for a tensor interaction with a range of the order of the one pion exchange potential and for single-particle states with the largest l for a given A, the effect of the finite range interaction is to multiply Eq. (7) by a simple suppression factor that is almost the same for any nuclei with mass number greater than $A = 28$. As a consequence one should be able to parametrize the contribution of a tensor force to the energy density by the simple form in Eq. (1) with values of α and β that are constant for all nuclei. This means that the original short range Skyrme density-dependent form used in Ref. [3] remains entirely valid and that finite range effects can be incorporated by using suitable values of α and β (see Sec. V).

We start with Eq. (3) and make a change of variables in the expression for $\langle V_T^{np}\rangle$, which becomes

$$\langle V_T^{np}\rangle = 8\pi^2 \int_0^\infty F(r) dr, \tag{9}$$

where

$$F(r) = \int_0^{2r} ds_r \int_0^\pi d\theta \sin^3\theta\, v_T(\mathbf{s})(r_1 r_2)^4 \rho_{1n}(\mathbf{r}_1, \mathbf{r}_2) \times \rho_{1p}(\mathbf{r}_1, \mathbf{r}_2), \tag{10}$$

with $r_1 = r + s_r/2, r_2 = r - s_r/2$ and θ the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$. There are three other angles that have been integrated out to give a factor $8\pi^2$. The formula contains the factor $|\mathbf{r}_1 \times \mathbf{r}_2|^2 = (r^2 - s_r^2/4)^2 \sin^2\theta$. The spin densities ρ_{1q} are defined in Eq. (5). The squared distance $|\mathbf{s}|$ between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is

$$|\mathbf{s}|^2 = |\mathbf{r}_1 - \mathbf{r}_2|^2 = s_r^2 + 4\left(r^2 - s_r^2/4\right)\sin^2\frac{\theta}{2}, \tag{11}$$

For states with maximum l in any shell the function $F(r)$ has a single peak at r_m near the maximum of the radial wave functions. The short-range approximation to $F(r)$, denoted by $F_0(r)$, holds when the range of the interaction is much less than r_m. The important values of s_r are much less than r_m and the angle integral has contributions only from small values of θ. Then $F(r)$ is replaced by

$$F_0(r) = \int_0^\infty ds_r \int_0^\infty d\theta\theta^3 v_T(\mathbf{s}) r^8 \rho_{1n}(\mathbf{r}, \mathbf{r})\rho_{1p}(\mathbf{r}, r), \tag{12}$$

with $|\mathbf{s}|^2 = {s_r}^2 + r^2\theta^2$. The short range approximation (7) to $\langle V_T^{np}\rangle$ can be obtained from (12) by using the relation (5).

The numbers in Table I are calculated with oscillator radial wave function

$$R_l(r) = N_l r^{l+1} \exp\left(-\frac{r^2}{2b^2}\right).$$

TABLE I. Values of the suppression factors $I(l, r_m/a)$, $S^G(r_m)$, S^G, $S^Y(r_m)$, and S^Y for the Gaussian and one-pion exchange potentials for various values of A and l.

A	l	r_m(fm)	$I(L, r_m/a)$	$S^G(r_m)$	S^G	$S^Y(r_m)$	S^Y
28	2	3.08	0.550	0.579	0.459	0.197	0.159
48	3	3.89	0.515	0.521	0.436	0.171	0.147
90	4	4.83	0.511	0.507	0.440	0.166	0.146
132	4,5	5.43	0.520	0.511	0.449	0.166	0.147
208	5,6	6.34	0.516	0.504	0.452	0.164	0.145

They have a maximum at $r_m = b\sqrt{l+1}$. The radial suppression factor $S(r)$ and the total suppression factor S are defined by

$$S(r) = \frac{F(r)}{F_0(r)}, \quad S = \frac{\int F(r)dr}{\int F_0(r)dr}. \tag{13}$$

There is a simple approximation $I(l, r/a)$ for the suppression factor $S^G(r)$ for a Gaussian interaction $V(r) = V_0 \exp(-r^2/a^2)$, which is given by

$$I(l, r/a) = \left(\frac{4r^2/a^2}{l(l+1) + 4r^2/a^2}\right)^2 \left[\frac{1}{1 + a^2/b^2}\right]^{1/2} \tag{14}$$

with $l = (l_n + l_p)/2$. The form of Eq. (14) arises from the replacement of $r^2 - s_r^2/4$ and of $r_1 r_2$ by r^2 in Eqs. (10) and (11), an approximation which is valid when $s_r \sim a \ll r$. Then the integral (10) separates into a product of an angle integral and an integral over s_r for a Gaussian interaction. The first factor is independent of the radial wave functions.

Values of $I(l, r_m/a)$ for several closed-shell nuclei are given in Table I. The range $a = 1.2$ fm is taken from Ref. [9]. There is quite a strong dependence on r for fixed l but $I(l, r/a)$ is almost constant at $r = r_m$ because $l(l+1)a^2/r_m^2$ does not change much for all the nuclei considered. The table also gives values of $S^G(r_m)$ and the total suppression factor S^G calculated by numerical integration. The approximate formula (14) for $S^G(r_m)$ is accurate to within 5%. The results indicate that, for a Gaussian potential and for states with maximum l, one can use the short-range approximation (7) with a reduced interaction strength.

The last two columns of the table give values of $S^Y(r_m)$ and S^Y for the tensor part of a Yukawa (one-pion exchange) potential that has a longer range and a form factor with radial dependence $v_T(x) = V_0 \exp(-x)(1/x + 3/x^2 + 3/x^3)$, where $x = \mu s$ with $\mu = 0.70$ fm^{-1}. It has a $1/s^3$ singularity at $\mathbf{s} = 0$ but this is canceled by the $\sin^3\theta$ factor in the integral for the matrix element. The suppression factors $S^Y(r_m)$ and S^Y, calculated by numerical integration, are almost constant. This shows that it is reasonable to use the Skyrme parametrization (1) to study the contribution of the tensor force to spin-orbit splittings for states with maximum l even for the one-pion exchange potential.

Our calculations show that the suppression factor $S(r)$ is an increasing function of r that goes to zero as $r \to 0$ and to 1 for larger r. The numbers in the table show that the total suppression factor S is less than the suppression factor evaluated at r_m for both the Gaussian and one-pion exchange potentials.

Early calculations showed that the Yukawa one-pion exchange potential play an important role in describing the ^{208}Pb levels [13]. Note that the results given above for the Gaussian potential shape are important in view of the fact that such interactions are used in shell-model calculations (cf. Otsuka *et al.* [9]). The main difference between the values for $S^G(r_m)$ and $S^Y(r_m)$ in Table I is due to the range of the Gaussian interaction. The values become very similar for all the nuclei in the table if the range of the Gaussian interaction is increased from 1.2 to 2.1 fm.

IV. THE TENSOR PART OF THE SKYRME INTERACTION

The parameters of the Skyrme interaction were originally determined in Hartree-Fock calculations to reproduce the total binding energies and charge radii of closed-shell nuclei [4]. Further extensive calculations were made later [5]. Several improved parameter sets were found. They differ mainly through the single-particle spectra. In the present article as in our previous work, we shall use the parameter set SIII, which gives good overall single-particle spectra. In Ref. [3] a tensor force was added and a range of its strength was found such as to maintain a good quality of the single-particle spectra of ^{48}Ca, ^{56}Ni, ^{90}Zr, and ^{208}Pb.

As in Ref. [3], in the configuration space the tensor interaction has the following form

$$\begin{aligned} V_T &= \tfrac{1}{2}T\big\{\big[(\vec{\sigma}_1\cdot\vec{k}')(\vec{\sigma}_2\cdot\vec{k}') - \tfrac{1}{3}k'^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)\big]\delta(\vec{r}_1-\vec{r}_2) \\ &\quad + \delta(\vec{r}_1-\vec{r}_2)\big[(\vec{\sigma}_1\cdot\vec{k})(\vec{\sigma}_2\cdot\vec{k}) - \tfrac{1}{3}k^2(\vec{\sigma}_1\cdot\vec{\sigma}_2)\big] \\ &\quad \times \delta(\vec{r}_1-\vec{r}_1)\big\} + U\big\{(\vec{\sigma}_1\cdot\vec{k}')\delta(\vec{r}_1-\vec{r}_2)(\vec{\sigma}_1\cdot\vec{k}) \\ &\quad - \tfrac{1}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)[\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}]\big\}. \end{aligned} \tag{15}$$

The parameters T and U measure the strength of the tensor force in even and odd states of relative motion.

V. RESULTS

The analysis in Secs. II and III show that the simple form (1) is a good approximation to the contribution of the tensor forces to the energy density. Values of α and β can be taken to be constant for states with maximum l in nuclei with $A \geqslant 48$ even for forces with a range of the one pion exchange potential.

Both the central exchange and the tensor interactions give contributions to the spin-orbit single particle potential to be added to the spin-orbit interaction. The additional contribution are [3]

$$\Delta W_n = (\alpha J_n + \beta J_p)\vec{\ell}\cdot\vec{s} \tag{16}$$

$$\Delta W_p = (\alpha J_p + \beta J_n)\vec{\ell}\cdot\vec{s} \tag{17}$$

with $\alpha = \alpha_T + \alpha_c$ and $\beta = \beta_T + \beta_c$. For the Skyrme SIII interaction the parameters of the central exchange part are [5]

$$\alpha_c = \tfrac{1}{8}(t_1 - t_2) = 61.25\,\mathrm{MeV fm}^5, \quad \beta_c = 0, \tag{18}$$

where t_1 and t_2 are two of the Skyrme interaction parameters. In terms of the tensor parameters T and U one has

$$\alpha_T = \tfrac{5}{12}U, \quad \beta_T = \tfrac{5}{24}(T+U). \tag{19}$$

Equations (16) and (17) imply that the mechanism invoked by Otsuka *et al.* is intrinsic to the Skyrme energy density formalism. These equations show that the filling of proton (neutron) levels influences the spin-orbit splitting of neutron (proton) levels whenever $\beta \neq 0$. The normal spin-orbit single particle potential is

$$V_{so} = W_0 \frac{1}{r}\left(\frac{d\rho}{dr} + \frac{d\rho_q}{dr}\right)\vec{\ell}\cdot\vec{s} \quad \text{with} \quad \frac{d\rho}{dr} < 0. \tag{20}$$

When β is positive the neutron (proton) spin-orbit splitting is reduced as protons (neutrons) fill a $j = l + 1/2$ level because $J_{p(n)} > 0$. This effect is clearly seen in Fig. 4 of Otsuka *et al.* [7].

In Ref. [3] we searched for sets of parameters α and β that simultaneously fit absolute values of single-particle levels in the closed-shell nuclei ^{48}Ca, ^{56}Ni, ^{48}Zr, and ^{208}Pb. We found that the common optimal values were located in a right angled triangle with sides $\alpha = -80$ MeV fm^5, $\beta = 80$ MeV fm^5, and hypotenuse $\alpha + \beta = 0$. Here we relax these constraints and try to analyze single-particle energies some nuclei far from the stability line. The experimental data did not exist in 1977 when we discussed the global fit for closed-shell nuclei [3]. Our present choice of parameters is guided by the recent results of Ref. [10] on the $Z = 50$ isotopes and $N = 82$ isotones which were analyzed in a HF + BCS approach based on the Skyrme interaction SLy5 [14] with refitted values of T and U plus a pairing force. To see whether one can obtain the correct trend in the evolution of single-particle levels we look at energy differences between them. These differences can give a clear indication of the formation of closed shells from the size of the gaps. Absolute values of single-particle energies depend not only on the tensor but also on other parts of the Skyrme interaction. Here we are not concerned with making the best fits to absolute energies.

In the present article we still use the SIII version of the Skyrme interaction [5] for comparison with the previous work. We maintain the conditions $\alpha < 0$ and $\beta > 0$ but take values outside the triangle found before that are not inconsistent with the previous findings [3]. We show that values $\alpha_T = -180$ MeV fm^5 and $\beta_T = 120$ MeV fm^5, or equivalently $\alpha = -118.75$ MeV fm^5 and $\beta = 120$ MeV fm^5, give a reasonably good fit to $Z = 50$ isotopes and $N = 82$ isotones. These values are similar to the ones fitted by Brown *et al.* [8] in a recent article. For a more general orientation we also discuss the role of this parametrization on Ca isotopes.

We conclude this section with some remarks on ^{208}Pb and ^{90}Zr. The proton $h_{11/2}$ and neutron $i_{13/2}$ in ^{208}Pb are filled and J_p and J_n are both positive with comparable magnitudes. Because $\alpha \approx -\beta$ we have $\Delta W_n \approx \Delta W_p \approx 0$ and the tensor forces hardly change the spin-orbit splitting. The situation is different for ^{90}Zr. There $J_p = 0$ and the effect of the tensor forces is to increase the spin-orbit splitting for neutrons and reduce it for protons. The shell gaps for protons and neutrons are both increased significantly and the stability of the double closed shell at ^{90}Zr is enhanced.

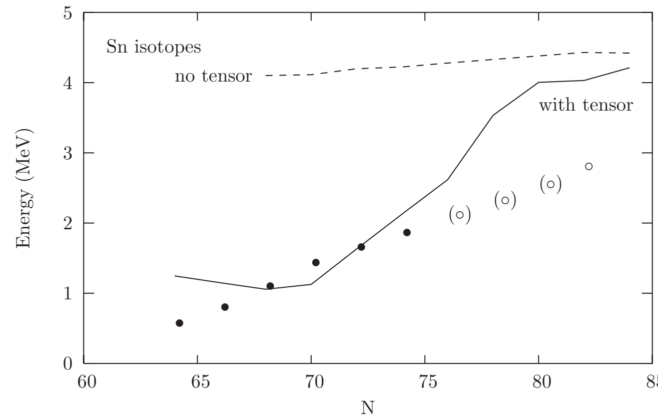

FIG. 1. The proton single particle energy difference between $1h_{11/2}$ and $1g_{7/2}$ in Sn isotopes ($Z = 50$, $N = 64$–82) calculated without and with tensor force $\alpha = -118.75$ MeV fm^5, $\beta = 120$ MeV fm^5. Data points are from Ref. [15]. Solid dots give information from transfer reactions. Open circles are obtained from methods less sensitive to the single-particle nature. The parentheses indicate less certain or indirect assignments.

A. Sn isotopes

Figure 1 shows the HF results for the proton single particle energy difference between $1h_{11/2}$ and $1g_{7/2}$ in Sn isotopes ($Z = 50$, $N = 64$–82) with and without tensor force. One can see that the effect of the tensor force is indeed important. The experimental pattern is satisfactorily reproduced with this simple approach. In the more sophisticated HF+BCS calculations of Ref. [10] the theoretical results beyond ^{126}Sn are better. However, in that region the experimental situation is less certain because the corresponding values have been

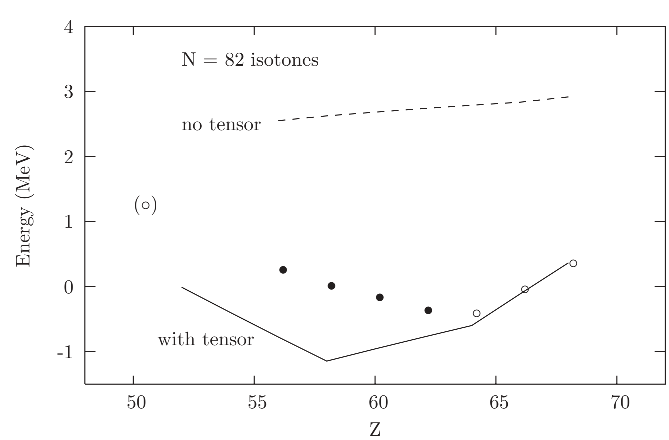

FIG. 2. The neutron single-particle energy difference between $1i_{13/2}$ and $1h_{9/2}$ in $N = 82$ isotones calculated with and without tensor force. Data points are from Ref. [15]. Solid dots give information from transfer reactions. Open circles are obtained from methods less sensitive to the single-particle nature. The parentheses indicate less certain or indirect assignments.

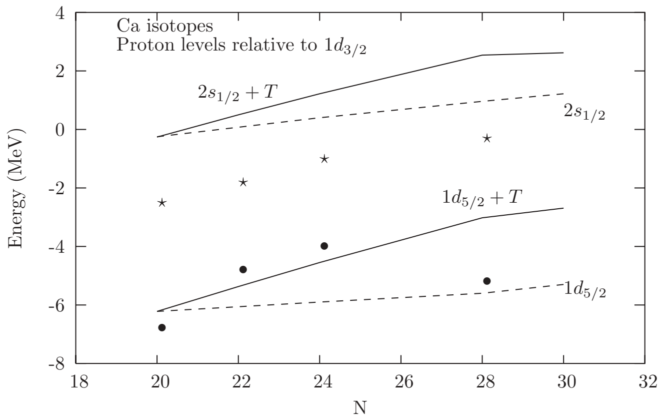

FIG. 3. The proton single-particle energies in Ca isotopes relative to $1d3/2$ level, calculated with tensor force (nlj + T): $\alpha = -118.75$ MeV fm^5, $\beta = 120$ MeV fm^5 and without tensor force (*nlj*): $\alpha = 61.25$ MeV fm^5, $\beta = 0$. Data points are from Ref. [16]: solid dots for $1d5/2$ and stars for $2s1/2$.

assigned using methods that are less sensitive to the single-particle nature of the levels [15]. For the double magic nucleus ^{132}Sn the effect of the tensor force and of the central exchange part cancels out because $J_p \approx J_n$. For isotopes with $Z = 56$–62 the comparison with the experiment is not possible because the $1h_{11/2}$ level becomes unbound in these calculations.

B. $N = 82$ isotones

In Fig. 2 we present neutron single-particle energy differences between $1i_{13/2}$ and $1h_{9/2}$ in $N = 82$ isotones calculated with and without tensor force and compare them with data from Ref. [15]. Again the role of the tensor force is considerable, bringing down the energy difference $e(1i_{13/2}) - e(1h_{9/2})$ close to the best known experimental values. For $Z \leqslant 50$ and for $Z \geqslant 70$ the $1h_{9/2}$ level becomes unbound both with and without tensor force.

C. Ca isotopes

As the Skyrme interaction SIII was fitted to closed shell nuclei it has the peculiarity that the central exchange interaction produces some undesirable effects in the middle of a shell. The predicted single-particle levels have the wrong order when compared with the experimental levels and wrong levels are occupied [5]. This happens in the absence of the tensor interaction, but when a tensor interaction with adequate parameters is added the problem is solved. In particular the parameters $\alpha = -118.75$ MeV fm^5 and $\beta = 120$ MeV fm^5 remove this anomaly in ^{50}Ca. The reason is a considerable increase of the spin-orbit in the $1f$ shell that shifts the $1f_{7/2}$ above the $2p$ levels. However, the anomaly persists for $\alpha = 0$ and $\beta = 80$ MeV fm^5 located at the edge of the above mentioned triangle.

In addition, from Fig. 3 one can see that the effect of the tensor interaction is important and improves the spin-orbit splitting in the $1d$ shell. In the $2s_{1/2}$ shell the trend is correct but the theoretical results are above the experimental points, with or without tensor. The pattern is quite similar to that obtained in Ref. [6] in a shell-model approach.

VI. CONCLUSIONS

The short range approximation for the contribution of a tensor force to the spin-orbit splitting in nuclei was studied in Secs. II and III for both a Gaussian and a Yukawa (one-pion exchange) interactions for states with maximum l in any shell for several nuclei with mass number between $A = 28$ and $A = 208$. It was shown that the exact matrix elements of the one-pion exchange tensor potential could be expressed as a product of the short-range expression (7) and a suppression factor $S^Y \approx 0.147$ that is almost constant for nuclei with mass number $A \geqslant 48$. It is only slightly larger, i.e., $S^Y \approx 0.16$ for nuclei near ^{28}Si. Thus the short-range formulas (1), (16), and (17) with constant α and β should give qualitatively good results for a Yukawa one-pion exchange potential.

We have made a new fit to the parameters α and β in the parametrization (16) and (17) of the tensor contribution to the spin-orbit coupling using data on $Z = 82$ isotopes and $N = 82$ isotones. The tensor force makes a dramatic difference to the single-particle energy difference between the $h_{11/2}$ and $g_{7/2}$ single-particle levels. A similar situation holds for the energy difference between the $i_{13/2}$ and $h_{9/2}$ single-particle levels in $N = 82$ isotones. In both cases the calculation with the addition of the tensor force give a good description of the experimental data. The case with Ca isotopes is similar to ^{90}Zr. The tensor force reduces the spin-orbit splitting for protons and increases it for neutrons. This brings the order of single particle into a better agreement with experiment.

The mechanism observed by Otsuka and collaborators [6,7,9] that the filling of neutron levels influences the proton spin-orbit splitting and vice versa is intrinsic to the Skyrme energy density approach and is very simple in that theory. In addition, with the Skyrme density formalism, one can easily study the combined contribution of the central exchange and tensor *NN* interactions to the spin-orbit potential.

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