Limiting fragmentation in heavy-ion collisions and percolation of strings

P. Brogueira*
Departamento de Física, IST, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

J. Dias de Deus†
CENTRA, Departamento de Física, IST, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

C. Pajares‡
IGFAE and Departamento de Fisica de Particulas, Univ. of Santigo de Compostela, E-15706, Santiago de Compostela, Spain
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The observed limiting fragmentation of charged particle distributions in heavy ion collisions is difficult to explain as it does not apply to the proton spectrum itself. On the other hand, string percolation provides a mechanism to regenerate fast particles, eventually compensating the rapidity shift (energy loss) of the nucleons. However a delicate energy-momentum compensation is required, and in our framework we see no reason for limiting fragmentation to be exact. A prediction, based on percolation arguments, is given for the charged particle density in the full rapidity interval at LHC energy ($\sqrt{s} = 5500$ GeV).

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I. INTRODUCTION

Recently, the phenomenon of limiting fragmentation, [1], or of extended longitudinal scaling, was rediscovered in the framework of high-energy heavy ion collisions [2]. In general the inclusive particle distribution, dn/dy, is a function of the central rapidity y and of the center of mass energy $\sqrt{s}$, or of $\Delta \equiv y - y_b$ and y_b, where y_b is the beam rapidity, with $\sqrt{s} = m_b e^{y_b}$. Limiting fragmentation essentially means that as Δ becomes larger than some y_b dependent threshold Δ_0, $\Delta \geqslant \Delta_0(y_b)$, dn/dy becomes a function only of Δ,

$$\frac{dn}{dy}(\Delta, y_b) \underset{\Delta > \Delta_0(y_b)}{\longrightarrow} f(\Delta), \tag{1}$$

independent of y_b. As $\Delta_0(y_b)$ decreases with y_b the region in Δ of limiting fragmentation increases with the energy.

It has been argued that limiting fragmentation reflects the fast parton distribution in the beam [3]. However, as a sizable fraction of the fast particles building up the limiting fragmentation behavior are nucleons (protons) [4], one requires specific parton correlations to generate the observed nucleons. This is done, for instance, in the dual parton model, [5], by introducing valence diquarks which, directly or indirectly, [6], produce baryons, thus preserving the flow of baryon number.

It is important to remark that the proton spectrum at high rapidity does not scale in the sense of limiting fragmentation, [4], presenting a shift $\langle \Delta \rangle_B \equiv \langle y \rangle_B - y_b$ increasing in absolute value with energy, at least for intermediate energies [7]. Theoretically, in QCD evolution models, a transfer of energy and momentum from fast partons to the sea is also expected, [8].

In such circumstances how is it possible to obtain overall limiting fragmentation? A possible solution is given by string percolation models as percolation implies not only a summation in color [9,10], but, as well, a summation in momentum [11,12]. In a sense, percolation is a mechanism for reacceleration of particles, with production of fast mesons (pions) from sea strings [13].

II. STRING PERCOLATION MODELS

In string percolation models strings are produced along the collision axis. In the impact parameter plane if the area of interaction is πR^2, the projected discs from the strings have an area πr^2 and there are $\bar{N}_s$ strings, the transverse density parameter η,

$$\eta \equiv \left(\frac{r}{R}\right)^2 \bar{N}_s, \tag{2}$$

is the relevant parameter in percolation. If $\eta \ll 1$ the strings are independent and do not overlap, if $\eta > 1$ the strings fuse and percolate.

When strings overlap, due to random color summation, the particle rapidity density dn/dy is not the sum of the particle density $\bar{n}_1$ of each string, and the average transverse momentum $\langle p_T \rangle$ is not the single string average transverse momentum $\bar{p}_1$. In general we have

$$dn/dy = F(\eta)\bar{N}_s\bar{n}_1, \tag{3}$$

and

$$\langle p_T \rangle = \bar{p}_1/\sqrt{F(\eta)}, \tag{4}$$

where $F(\eta)$ is the color summation reduction factor:

$$F(\eta) \equiv \sqrt{\frac{1 - e^{-\eta}}{\eta}}. \tag{5}$$

Note that $\eta \to 0$, $F(\eta) \to 1$, and that as $\eta \to \infty$,

$$F(\eta) \to 1/\eta^{1/2}. \tag{6}$$

*Electronic address: pedro@fisica.ist.utl.pt
†Electronic address: jdd@fisica.ist.utl.pt
‡Electronic address: pajares@fpaxp1.usc.es

Regarding the momentum summation, if we have a cluster of N strings, each string made up of partons with Feynman x-variable $-x_1$ and $+x_1$, respectively, such that $y_1 = y_b + \ln x_1$ [12], we have for the end (forward rapidity of the N-cluster string)

$$y_N = y_1 + \ln N. \tag{7}$$

With percolation just one cluster is formed and N becomes the number $\bar{N}_s$ of strings:

$$y_{\bar{N}_s} = y_1 + \ln \bar{N}_s. \tag{8}$$

A simple model can be imagined where, at relatively low energy, there is a transfer of momentum from the valence string (leading proton) with creation of identical sea strings, followed, at very higher energy, by the mechanism of percolation with regeneration of fast strings [12]. One obtains a flat distribution in rapidity ending at $y = y_{\bar{N}_s}$ [see Eq. (7)].

The consequences of percolation become very clear: a decrease of particle density at midrapidity, see Eq. (3), and an extension of the length of the forward sea rapidity distribution from $\sim y_1$ to $\sim \ln \bar{N}_s$, see Eq. (7).

If we impose in the model energy conservation in AA collision when there are $N_{\rm part}$ participating nucleons,

$$\int_0^{y_{\rm MAX}} E(y)\frac{dn}{dy}dy = \frac{N_{\rm part}}{2}\frac{\sqrt{s}}{2}, \tag{9}$$

where $E(y) = \langle m_T\rangle \cosh y$ and $\sqrt{s} = m_b e^{y_b}$ we obtain, as $y_b \to \infty$,

$$\int_0^{e^{\Delta_0}} \frac{\langle m_T\rangle}{m_b}\frac{2}{N_{\rm part}}\frac{dn}{d\Delta}d(e^{+\Delta}) = 1, \tag{10}$$

where $\Delta \equiv y - y_b$ and

$$\Delta_0 = \ln \bar{N}_s - y_b. \tag{11}$$

If we write for the asymptotic behavior of the number of strings

$$\bar{N}_s \sim s^{\lambda} \sim e^{2\lambda y_b}. \tag{12}$$

we obtain

$$\Delta_0 = -\alpha y_b, \tag{13}$$

with

$$\alpha = 1 - 2\lambda. \tag{14}$$

One should notice that $1+\lambda$ is, in this approach, the intercept of the bare Pomeron [12]. As const. $\leqslant \bar{N}_s \leqslant \sqrt{s}$ one also sees that $0 \leqslant \alpha \leqslant 1$.

In order to make practical use of Eq. (9), we need to estimate the asymptotical behavior of the ratio $\langle m_T\rangle/m_b$. We take two simple models: Model I—One assumes that the relation (4) for $\langle p_T\rangle$ applies as well to $\langle m_T\rangle$ [12]. This means that large p_T physics is dominating everywhere. Model II—One assumes that the ratio $\langle m_T\rangle/m_b$ is energy independent. Such approximation is probably more realistic, as one is mostly considering the forward rapidity contributions where the p_T behavior is expected to be almost energy independent [14].

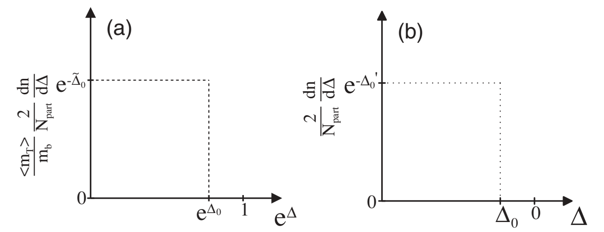

FIG. 1. (a) The integrand of Eq. (9) as a function of e^{Δ}. (b) The normalized $dn/d\Delta$ distribution as a function of Δ. In both cases $\Delta_0 = -\alpha y_b$.

Note that in $\langle m_T\rangle$ it is also implicit an averaging over rapidity. For a discussion of this problem, see [16].

We can now estimate the asymptotic behavior of the integrand of Eq. (9),

$$\frac{\langle m_T\rangle}{m_b}F(\eta)\bar{N}_s\bar{n}_1 \underset{y_b\to\infty}{\longrightarrow} e^{-\tilde{\Delta}_0}, \tag{15}$$

where $\tilde{\Delta}_0 = -\tilde{\alpha} y_b$ with, for central collisions, i.e., $F(\eta) \to 1/\eta^{1/2}$,

$$\tilde{\alpha} = 3/2\lambda \qquad \text{(Model I)}, \tag{16}$$

and

$$\tilde{\alpha} = \lambda \qquad \text{(Model II)}. \tag{17}$$

It is clear, see Fig. 1(a), that conservation of energy requires

$$-\tilde{\Delta}_0 + \Delta_0 = 0 \tag{18}$$

or $\tilde{\alpha} = \alpha$. One further obtains

$$\lambda = 2/7 \qquad \text{(Model I)}, \tag{19}$$

$$\lambda = 1/3 \qquad \text{(Model II)}. \tag{20}$$

In order to arrive at the asymptotic behavior of dn/dy we simply have to divide the integrand of Eq. (9) by $\langle m_T\rangle/m_b$ (see Fig. 1(b)) to obtain

$$dn/dy \sim e^{-\Delta_0'} \sim e^{\alpha' y_b}, \tag{21}$$

with, because of Eqs. (6) and (11),

$$\alpha' = \lambda = 2/7 \qquad \text{(Model I)}, \tag{22}$$

and

$$\alpha' = \lambda = 1/3 \qquad \text{(Model II)}. \tag{23}$$

The model (with two variations) that we have considered corresponds to a step function distribution: $dn/d\Delta \equiv e^{-\Delta_0'}$, $\Delta \leqslant \Delta_0$; $dn/d\Delta = 0$, $\Delta > \Delta_0$ (see Fig. 1(b)). It trivially satisfies limiting fragmentation, Eq. (1), with $f(\Delta) \equiv 0$.

III. GENERALIZING THE MODEL: THE FERMI DISTRIBUTION

We shall now generalize the model by introducing the Fermi distribution as a smoothing function:

$$2/N_{\rm part}dn/d\Delta = \frac{e^{-\Delta_0'}}{e^{\frac{\Delta-\Delta_0}{\delta}} + 1}, \tag{24}$$

TABLE I. The parameters λ, α, and δ of Eq. (23), in the case of Models I and II, and as obtained from PHOBOS data [2].

	λ	α	δ
Model I	2/7	3/7	0
Model II	1/3	1/3	0
RHIC data [2]			
$\sqrt{s} = 19.6$ GeV	0.26 ± 0.002	0.27 ± 0.03	0.65 ± 0.05
$\sqrt{s} = 62.4$ GeV	0.230 ± 0.008	0.23 ± 0.02	0.62 ± 0.07
$\sqrt{s} = 130$ GeV	0.249 ± 0.006	0.28 ± 0.02	0.67 ± 0.07
$\sqrt{s} = 200$ GeV	0.251 ± 0.005	0.29 ± 0.01	0.70 ± 0.06
Overall fit to RHIC data	0.247 ± 0.003	0.269 ± 0.007	0.67 ± 0.03

with $\Delta_0' = -\lambda y_b$, $\Delta_0 = -\alpha y_b$ and δ being a parameter. In the limit $\delta \to 0$ we, of course, recover the step function. A more physical argument for the Fermi distribution is based on an evolution equation for the fast valence string in the fragmentation region, with emissions of sea strings in the midrapidity region [15].

In what follows we shall not distinguish between pseudorapidity and rapidity. This ambiguity produces a difference in midrapidity plots. However, as the mass of produced particles is small (mostly pions) and as $\langle p_T \rangle$ in nuclear central collisions is large, the error introduced is small.

In Table I we present the values of λ, α, and δ in the case of Model I and Model II. We show as well the results for λ, α and δ when fitting RHIC data for $\frac{2}{N_{\text{part}}} dn/dy$, with Eq. (23). Our results are very similar to the fits of [17]. We note that the experimental values for λ and α are not very different, as in Model II. This means that the energy dependence of $\langle m_T \rangle / m_b$ is weak.

In the table we have also included the values of λ, α, and δ resulting from an overall fit to RHIC/PHOBOS data. Note that our fit is reasonable. However, we have a flat behavior as our curves approach midrapidity, while data show a tendency to decrease, as typical of pseudorapidity distributions.

In Fig. 2 we show our curves from the overall fit, in comparison with data, and our prediction for LHC, 5500 GeV data. At midrapidity we expect dn/dy to be about 1500.

IV. THE QUESTION OF LIMITING FRAGMENTATION AND CONCLUSIONS

We now come back to the question of limiting fragmentation. It is clear, from Fig. 2, that we do not have strict limiting fragmentation in the $\Delta \gtrsim 0$ region.

From Eq. (23) one sees that, for $\Delta \gg \Delta_0$, limiting fragmentation simply means

$$\frac{2}{N_{\text{part}}} \frac{dn}{d\Delta} \sim e^{-\Delta/\delta}, \tag{25}$$

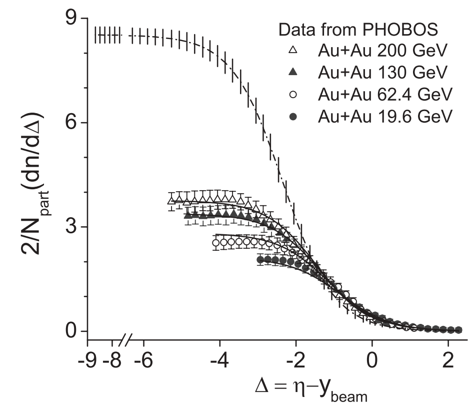

FIG. 2. Overall fits to PHOBOS/RHIC central (0–6%) data using Eq. (23)—see the table. The dotted-dashed line is a prediction for LHC, $\sqrt{s} = 5500$ GeV, with error bars estimated from the errors of the overall fit to the RHIC data.

which requires, for $\delta > 0$, the limiting fragmentation condition [see Eq. (23)]

$$-\Delta_0' + \frac{\Delta_0}{\delta} = 0, \tag{26}$$

or

$$\delta = \Delta_0/\Delta_0' = \alpha/\lambda. \tag{27}$$

The fact that we do not have limiting fragmentation is not our choice. The RHIC/PHOBOS data, fitted with the parametrization (23), clearly shows that relation (26) is not obeyed: $\alpha/\lambda \gtrsim 1$ and $\delta \simeq 2/3 < 1$ (see the table). In these circumstances, the limiting fragmentation behavior, as the energy increases, tends to disappear (as seen in Fig. 2) and the overall curve approaches the step function of Models I and II.

In our estimates, we have assumed that at present energies and for large number of participating nucleons, $F(\eta) \to 1/\eta^{1/2}$, Eq. (6), η being large enough and increasing with energy and N_{part}, [10]. So we do expect changes in our parameters in Eq. (23) when moving from central (Au-Au, 0–6%) to peripherical (Au-Au, 35–45%) collisions. Our parametrization for peripherical collisions gives $\lambda = 0.228 \pm 0.002$, $\alpha = 0.235 \pm 0.008$, and $\delta = 0.90 \pm 0.03$, to be compared with the values in the table, for central collisions. At much higher energy we expect λ, α, and δ to approach the central collision values and to become the same for all centralities, as well as for *pp* collisions.

The parameter $\Delta_0 = -\alpha y_b$ plays, in our model, the important role of controlling the separation (sharply in Models I and II) of the midrapidity dense central region—where percolation dominates—from the fragmentation region where, in general, the participating nucleons retain their individuality. The central rapidity region $\Delta_{central} = -\alpha y_b - (-y_b) = (1-\alpha) y_b$ grows with the energy. But the fragmentation region, $\Delta_{fragm.} = 0 - (-\alpha y_b) = \alpha y_b$ also increases with the energy. This is not the case of an extended plateau (Feynmann-Wilson plateau), followed by a fixed rapidity length fragmentation region.

We finally note that the experimental value found for λ, not very different from the values of Model I and Model II, 0.25, is consistent with values found for the intercept of the Pomeron in color glass saturation models extended to *AA* scattering via geometrical scaling [18].

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