^{12}C-^{12}C elastic scattering at 1.016, 1.449, and 2.4 GeV and the *NN* amplitude

Deeksha Chauhan and Z. A. Khan*
Department of Physics, Aligarh Muslim University, Aligarh-202002, India
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Working within the framework of the Coulomb modified Glauber model, we analyze the elastic angular distribution and reaction cross section for the ^{12}C-^{12}C system at 1.016, 1.449, and 2.4 GeV. The elastic *S* matrix is evaluated using the effective profile function approach, and a correlation expansion for the Glauber amplitude is obtained. We emphasize the parametrization of the basic (input) *NN* amplitude, which may be used for a wide range of angles. Retaining the first two terms of the correlation expansion and using the realistic densities for the colliding nuclei, we find that (i) the consideration of higher momentum transfer components, and hence the nondiffractive behavior, of the *NN* amplitude provides a more satisfactory account of the data than does the conventional (one-term) Gaussian parametrization for the *NN* amplitude, (ii) the in-medium effects seem to reduce the (free) *NN* total cross section and influence the other parameters of the *NN* amplitude as well, (iii) the phase of the *NN* amplitude does not help in improving the theoretical situation, and (iv) the c.m. correlations play an important role at the energies considered. We also discuss the suitability of the effective profile function approach in the present context.

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I. INTRODUCTION

Over the past about three decades, the Glauber multiple scattering theory (GMST) [1,2] has been the most successful tool for providing microscopic description of hadron-nucleus collisions at intermediate energies [3–6]. The results of this description highlight the importance of the successive higher order scatterings (correlations) at high momentum transfers and provide useful information about the matter density distributions.

Encouraged by this, several authors have extended GMST to study nucleus-nucleus collisions [7–11] at intermediate energies. But because the analytic evaluation of the full Glauber amplitude for the realistic description of nuclei is a computationally difficult task [12,13], most of the analyses have been made by invoking the so-called optical-limit approximation (OLA) [12,13], which considers only the leading term in an expansion of the nucleus-nucleus phase shift function. This leading term depends upon the one-body densities of the colliding nuclei, while the neglected terms depend upon the two-body and higher order densities. These analyses show that the OLA works reasonably well provided that the conventional Glauber amplitude is suitably modified to account for the deviation of the projectile trajectory due to the Coulomb field [10,11]. It is also observed that the predictions of the OLA become less satisfactory especially at large momentum transfers as the projectile energy increases [10,11]. This feature of the OLA may be understood by noting that at lower energies, the scattering is sensitive mainly to the surface regions of the colliding nuclei in which the contribution of the neglected higher order terms in the expansion of the nucleus-nucleus phase shift function [13] may be negligibly small. However, with an increase in projectile energy, the scattering now becomes more sensitive to the interior regions of the nuclei where suppressing the higher order terms in the phase shift function expansion may not be justified. Thus, one expects that the inclusion of higher order terms in the OLA may improve the theoretical situation at relatively higher energies.

In an attempt to improve the OLA results, El-Gogary *et al.* [14] performed full Glauber series calculations employing the techniques developed by Yin *et al.* [15] and Huang [16]. Using the double-Gaussian single-particle densities for the colliding nuclei and the conventional (one-term) Gaussian parametrization for the input *NN* amplitude (GNN), the authors [14] calculated the elastic angular distribution for the ^{12}C-^{12}C system at 1.016, 1.449, and 2.4 GeV. The center-of-mass (c.m.) correlation is accounted for by applying the commonly used global c.m. correlation correction factor for all orders of scattering, which strictly holds for single Gaussian (harmonic oscillator) densities. The results of this calculation, though better than the OLA results, show noticeable disagreement at 200 MeV/nucleon. In a study using a similar approach, El-Gogary *et al.* [17] also reported a somewhat improved calculation in which the c.m. correlation is treated in a consistent manner. It is found that the consistent treatment of the c.m. correlation does not significantly improve the theoretical situation.

In another approach, Abu-Ibrahim and Suzuki [18] performed a Glauber model analysis of ^{12}C-^{12}C and ^{12}C-^{208}Pb elastic scattering data at 200 MeV/nucleon by considering the nucleon-target (NT) interaction as "elementary." The profile function Γ_{NT} for the NT amplitude has either been calculated in terms of the profile function for the *NN* scattering amplitude or obtained by assuming a suitable parametrization for the NT profile function. The application of NT formalism shows substantial improvement over the earlier results at small momentum transfers. However, in this case also, noticeable disagreement between theory and experiment occurs at large momentum transfers.

Later, Ahmad *et al.* [19] analyzed ^{12}C-^{12}C elastic scattering data at 1.016, 1.449, and 2.4 GeV within the framework of

*Electronic address: zak_atif@yahoo.com

the Coulomb modified Glauber model [10,11]. Retaining the first two terms of the nuclear phase expansion series [13] and using the realistic densities for the colliding nuclei, a good description of the experimental data at 1.016 and 1.449 GeV was achieved by taking into account the phase of the *NN* amplitude. At 2.4 GeV, the second-order phase term provides some improvement over the OLA results; however, the theoretical situation remains unsatisfactory.

Keeping in mind the unsatisfactory results of the Glauber model analyses [14,17–19] of ^{12}C-^{12}C elastic scattering data at 200 MeV/nucleon, Ahmad *et al*. [20,21] proposed phenomenological methods for analyzing the heavy-ion elastic scattering data within the framework of the Coulomb modified Glauber model. In these methods, instead of using the GNN, the authors evaluated it in terms of either the (i) phenomenological effective *NN* potential or (ii) effective *NN* phase shift function, the parameters of which are varied up to the extent of getting the best possible fits to the experimental data. Application of these methods to some ^{12}C-nucleus and ^{16}O-nucleus systems shows that a very satisfactory description of the elastic scattering data at several energies can be obtained in this way. In particular, the ^{12}C-^{12}C elastic scattering data at 200 MeV/nucleon is nicely reproduced. Moreover, these results show that the values of the effective *NN* total cross sections $\sigma_{NN}^{\rm eff}$ are smaller than the corresponding free values σ_{NN}^{f} [22]. This feature of $\sigma_{NN}^{\rm eff}$ agrees qualitatively with the results of the microscopic studies that show that in-medium *NN* total cross sections are smaller than the corresponding free values mainly because of Pauli blocking [23,24]. Furthermore, it has been observed that for a given pair of colliding nuclei, the deviation of $\sigma_{NN}^{\rm eff}$ from the free value is quite large (∼40%) at 200 MeV/nucleon; the enhanced transparency in the nucleus-nucleus collisions due to much smaller values of σ_{NN}^{f} at this energy makes the collision more sensitive to the behavior of the *NN* interaction in the interior region where the *NN* interaction may be reduced considerably due to medium effects.

Coming to the phenomenological approach for heavy-ion elastic scattering within the framework of OLA of the Glauber model (PAGM) [20,21], it may be emphasized that the success of PAGM seems to lie in the choice of the input *NN* amplitude, as the results of the previous OLA calculations with the GNN [10,19] are not found to be as satisfactory as the phenomenological one. The reason why PAGM works so well may be understood from the following argument.

As discussed in Ref. [20], the GNN may be well suited at high energies where the small angle *NN* scattering is mostly diffractive and peaked in the forward direction, but the same may not be very appropriate for describing the *NN* scattering at lower energies, as the scattering in this case is nondiffractive. Therefore, one conjectures that the *NN* amplitude, as obtained in Refs. [20,21], might be closer to the realistic description of *NN* scattering at lower energies, and hence the success of PAGM may be connected with the better choice of the *NN* amplitude. Moreover, the above discussion gets further support if we note that in the Glauber model calculations for nucleus-nucleus collisions at intermediate energies [10,14,17,19], the required *NN* scattering parameter values, except for σ_{NN}^{f} [22], are found to be very different in different studies. Apart from this, there are two more factors missing in the Glauber model calculations with the GNN. One is the large q behavior of the *NN* amplitude, which might be of some significance for collisions between lighter nuclei whose form factors fall rather smoothly, and the other factor concerns the nuclear medium effects on *NN* scattering. To include the first factor, we, in this work, parametrize the *NN* amplitude in the same form as in Refs. [25,26], whereas the nuclear medium effects may be incorporated, rather indirectly, through reasonable variation in the parameters of the *NN* amplitude. One hopes that our study may not only tell about the in-medium *NN* total cross section, as reported in earlier studies [20,21,23,24], but also give some information about the medium effects on other parameters.

In this work, we propose to analyze the elastic angular distribution and reaction cross section for the ^{12}C-^{12}C system at 1.016, 1.449, and 2.4 GeV. The analysis is based upon the Coulomb modified [10,11] correlation expansion for the Glauber amplitude [27], the first term of which corresponds to the well-known optical-limit result, and the others depend successively upon the two-, three-, and many-body densities of the colliding nuclei. In the following, we content ourselves with considering up to the two-body density term, which may be considered as the leading correction term [19] to the optical-limit result. Section II consists of a brief review of the correlation expansion for the Glauber amplitude. The numerical results are presented, discussed, and summarized in Sec. III.

II. CORRELATION EXPANSION FOR THE GLAUBER AMPLITUDE

According to the Glauber model, the scattering amplitude describing the elastic scattering of a projectile nucleus with ground state wave function ψ_B on a target nucleus with ground state wave function ψ_A may be written as (see, for example, Refs. [12,13])

$$F(\vec{q}) = \frac{iK}{2\pi}\int d^2b\,\exp(i\vec{q}\cdot\vec{b})[1 - S_{\rm el}(\vec{b})], \tag{1}$$

$$S_{\rm el}(\vec{b}) = \left(\psi_A\psi_B\left|\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\Gamma_{NN}(\vec{b}-\vec{s}_i+\vec{s}'_j)]\right|\psi_B\psi_A\right), \tag{2}$$

where A and B are the mass numbers of the target and projectile nuclei, respectively, $\vec{s}_i(\vec{s}'_j)$ are the projections of the target (projectile) nucleon coordinates on the plane perpendicular to the incident momentum $\vec{K}$, and the *NN* profile function Γ_{NN} is related to the *NN* amplitude f_{NN} as

$$\Gamma_{NN}(\vec{b}) = \frac{1}{2\pi ik}\int d^2q\,\exp(-i\vec{q}\cdot\vec{b})f_{NN}(\vec{q}), \tag{3}$$

where k is the incident nucleon momentum corresponding to the projectile kinetic energy per nucleon.

Next, to obtain the required expansion for the elastic scattering amplitude, we follow Ahmad [27] and write the

$\hat{S}$ matrix element $S_{\rm el}$ in terms of an effective profile γ_{ij} as

$$S_{\rm el}(\vec{b}) = \left(\psi_A\psi_B\left|\prod_{i=1}^{A}\prod_{j=1}^{B}[(1-\Gamma_{00})+\gamma_{ij}]\right|\psi_B\psi_A\right), \quad (4)$$

where

$$\gamma_{ij} = \Gamma_{00} - \Gamma_{NN}(\vec{b}-\vec{s}_i+\vec{s}_j'), \quad (5)$$

and

$$\Gamma_{00} = \int \rho_A(\vec{r})\rho_B(\vec{r}')\Gamma_{NN}(\vec{b}-\vec{s}+\vec{s}')\,d\vec{r}\,d\vec{r}'. \quad (6)$$

In Eq. (6), ρ_A and ρ_B are the ground state densities of the target and projectile, respectively.

Now it is easy to see that the double product in Eq. (4) may be expanded as

$$S_{\rm el}(\vec{b}) = S_0(\vec{b}) + \sum_{l=2}^{AB} S_l(\vec{b}), \quad (7)$$

where

$$S_0(\vec{b}) = (1-\Gamma_{00})^{AB}, \quad (8)$$

and

$$S_l(\vec{b}) = (\psi_A\psi_B|\hat{S}_l(\vec{b})|\psi_B\psi_A), \quad (9)$$

with

$$\hat{S}_l(\vec{b}) = \frac{1}{l!}(1-\Gamma_{00})^{AB-l}\sum_{i_1,j_1}{}'\sum_{i_2,j_2}{}'\cdots\sum_{i_l,j_l}{}'\gamma_{i_1,j_1}\gamma_{i_2,j_2}\cdots\gamma_{i_l,j_l}. \quad (10)$$

The primes on the summation signs indicate the restriction that two pairs of indices cannot be equal at the same time (for example, if $i_1 = i_2$ then $j_1 \neq j_2$ and vice versa). The sum in Eq. (7) starts from $l = 2$, since the $l = 1$ term does not contribute to the elastic scattering.

Substituting the expansion (7) in Eq. (1), one obtains the following (correlation) expansion for the elastic scattering amplitude:

$$F(\vec{q}) = F_0(\vec{q}) + \sum_{l=2}^{AB} F_l(\vec{q}), \quad (11)$$

where

$$F_0(\vec{q}) = \frac{iK}{2\pi}\int \exp(i\vec{q}\cdot\vec{b})[1-S_0(\vec{b})]d^2b, \quad (12)$$

and

$$F_l(\vec{q}) = -\frac{iK}{2\pi}\int \exp(i\vec{q}\cdot\vec{b})(\psi_A\psi_B|\hat{S}_l|\psi_B\psi_A)d^2b. \quad (13)$$

The first term F_0 in Eq. (11), which depends upon the intrinsic ground state densities of the colliding nuclei, corresponds to the optical-limit result of Czyz and Maximon [12]. The other terms $F_l(l \geqslant 2)$ involve the lth-body density of both the target and projectile nuclei and may be regarded as providing corrections to the optical-limit calculation.

As mentioned in Sec. I, we restrict ourselves up to F_2 in the expression (11) for $F(\vec{q})$, because it is expected to provide a leading correction to the optical-limit term F_0 [19]. More explicitly,

$$F_2(\vec{q}) = -\frac{iK}{2\pi}\frac{1}{(2\pi ik)^2}\frac{AB}{2}\int d^2be^{i\vec{q}.\vec{b}}(1-\Gamma_{00})^{AB-2} \times[(A-1)(B-1)(G_{22}-G_{00})+(B-1) \times(G_{21}-G_{00})+(A-1)(G_{12}-G_{00})], \quad (14)$$

where

$$G_{22}(\vec{b}) = \int d^2q_1\,d^2q_2e^{-i(\vec{q}_1+\vec{q}_2)\cdot\vec{b}}F_A^{(2)}(\vec{q}_1,\vec{q}_2)F_B^{(2)} \times(-\vec{q}_1,-\vec{q}_2)f_{NN}(\vec{q}_1)f_{NN}(\vec{q}_2), \quad (15)$$

$$G_{21}(\vec{b}) = \int d^2q_1\,d^2q_2e^{-i(\vec{q}_1+\vec{q}_2)\cdot\vec{b}}F_A(\vec{q}_1+\vec{q}_2)F_B^{(2)} \times(-\vec{q}_1,-\vec{q}_2)f_{NN}(\vec{q}_1)f_{NN}(\vec{q}_2), \quad (16)$$

$$G_{12}(\vec{b}) = \int d^2q_1\,d^2q_2e^{-i(\vec{q}_1+\vec{q}_2)\cdot\vec{b}}F_A^{(2)}(\vec{q}_1,\vec{q}_2)F_B \times(-\vec{q}_1-\vec{q}_2)f_{NN}(\vec{q}_1)f_{NN}(\vec{q}_2), \quad (17)$$

and

$$G_{00}(\vec{b}) = \left(\int d^2qe^{-i\vec{q}\cdot\vec{b}}F_A(\vec{q})F_B(\vec{q})f_{NN}(\vec{q})\right)^2. \quad (18)$$

The quantities $F_{A(B)}(\vec{q})$ and $F_{A(B)}^{(2)}(\vec{q}_1,\vec{q}_2)$ in the above expressions are the one- and two-body (intrinsic) form factors of the target (projectile) nucleus, respectively,

$$F_{A(B)}(\vec{q}) = \int \rho_{A(B)}(\vec{r})e^{i\vec{q}\cdot\vec{r}}d\vec{r} \quad (19)$$

$$F_{A(B)}^{(2)}(\vec{q}_1,\vec{q}_2) = \int \rho_{A(B)}^{(2)}(\vec{r}_1,\vec{r}_2)e^{i(\vec{q}_1\cdot\vec{r}_1+\vec{q}_2\cdot\vec{r}_2)}\,d\vec{r}_1\,d\vec{r}_2, \quad (20)$$

where $\rho_{A(B)}^{(2)}(\vec{r}_1,\vec{r}_2)$ is the two-body (intrinsic) density of the target (projectile) nucleus. For the intrinsic two-body form factor, we use the following expression as obtained in Ref. [27]:

$$F^{(2)}(\vec{q}_1,\vec{q}_2) = \theta(\vec{q}_1+\vec{q}_2)\left[\frac{F(\vec{q}_1)F(\vec{q}_2)}{\theta(\vec{q}_1)\theta(\vec{q}_2)} - \tilde{g}_c\left(\frac{\vec{q}_1-\vec{q}_2}{2}\right)D_M(\vec{q}_1+\vec{q}_2)\right], \quad (21)$$

where $\theta(\vec{q})$ is the c.m. correlation correction factor [27], and $\tilde{g}_c(q)$ and $D_M(q)$ are the same as defined in Ref. [27].

In the above discussion we did not consider the effects due to Coulomb scattering. This consideration is important for nucleus-nucleus collisions over a wide range of momentum transfer [28]. In this case, though, we are concerned with the extended charge Coulomb effects, but, as discussed in Ref. [28], the same can be incorporated in the same way as for the proton-nucleus collisions. Thus the expression (1), with Coulomb effects, takes the form

$$F(\vec{q}) = f_c(\vec{q}) + \frac{iK}{2\pi}\int d^2b\exp(i\vec{q}\cdot\vec{b})e^{i\chi_{pt}(\vec{b})} \times[1-e^{i\chi_c(\vec{b})}S_{\rm el}(\vec{b})], \quad (22)$$

where f_c, χ_{pt} and χ_c are, respectively, the point Coulomb scattering amplitude, the point Coulomb phase shift, and the Coulomb phase shift due to the charge distributions of the colliding nuclei [28]. Expression (22) has been further modified [10,11] to account for the deviation in the eikonal trajectory because of the Coulomb field. This deviation can be incorporated [11] by replacing b in $S_{\rm el}(b)$ by b', which is the distance of the closest approach in Rutherford orbits and is given by

$$kb' = \eta + (\eta^2 + k^2b^2)^{1/2}, \qquad (23)$$

where $\eta = Z_A Z_B e^2/\hbar v$ is the Sommerfeld parameter with $Z_A(Z_B)$ as the target (projectile) atomic number and v the projectile velocity.

With these considerations, the elastic angular distribution for the ^{12}C-^{12}C system is then calculated using the expression for two identical bosons:

$$\frac{d\sigma}{d\Omega} = |F(q \leftrightarrow \theta) + F(q \leftrightarrow \pi - \theta)|^2, \qquad (24)$$

where θ is the scattering angle, and the reaction cross section is evaluated using the expression

$$\sigma_R = \int d^2b[1 - |S_{\rm el}(b)|^2]. \qquad (25)$$

Finally, it is to be pointed out that the distinction between protons and neutrons may be incorporated in F_0 only, as it is expected to be the leading term in the scattering amplitude. The term F_2, however, involves the average values of the *NN* parameters. With this modification, Eq. (12) takes the form

$$F_0(\vec{q}) = \frac{iK}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\left[1 - \left(1 - \Gamma_{00}^{pp}\right)^{Z_AZ_B}\left(1 - \Gamma_{00}^{np}\right)^{N_BZ_A} \times \left(1 - \Gamma_{00}^{pn}\right)^{Z_BN_A}\left(1 - \Gamma_{00}^{nn}\right)^{N_AN_B}\right]d^2b, \qquad (26)$$

with

$$\Gamma_{00}^{mn} = \int \rho_A(\vec{r})\rho_B(\vec{r}')\Gamma_{NN}^{mn}(\vec{b} - \vec{s} + \vec{s}')d\vec{r}\,d\vec{r}' \qquad (27)$$

where $N_A(N_B)$ is the number of neutrons in the target (projectile) nucleus, and each of m and n stand for a proton and a neutron.

III. RESULTS AND DISCUSSION

Following the approach outlined in Sec. II, we analyze the ^{12}C-^{12}C elastic angular distribution and reaction cross section at 1.016, 1.449, and 2.4 GeV. The inputs needed in the calculation are the *NN* amplitude, the nuclear form factor, and the oscillator constant [6].

Following Golovanova and Iskra [25], the *NN* amplitude that may be used for a wide range of angles is parametrized as

$$f_{NN}(\vec{q}) = \frac{ik\sigma_{NN}}{4\pi}\sum_{n=0}^{\infty} A_{n+1}\left(\frac{\sigma_{NN}}{4\pi\beta_{NN}^2}\right)^n \times \frac{(1 - i\rho_{NN})^{n+1}}{(n+1)}\exp\left[\frac{-\beta_{NN}^2q^2}{2(n+1)}\right], \qquad (28)$$

where

$$A_{n+1} = \frac{A_1}{n(n+1)} + \frac{A_2}{(n-1)n} + \frac{A_3}{(n-2)(n-1)} + \cdots + \frac{A_n}{1.2}, \qquad (29)$$

with $A_1 = 1$.

The amplitude (28) has three adjustable parameters σ_{NN}, ρ_{NN}, and β_{NN}^2, the values of which are obtained by providing the best possible description of elastic angular distribution [19] and reaction cross section [29] for the ^{12}C-^{12}C system at energies under consideration. In addition to this, we impose two more conditions on the choice of σ_{NN}, ρ_{NN}, and β_{NN}^2. In the first, we vary them in such a way that the (free) *NN* total cross section σ_{NN}^f [22] and ratio of the real to the imaginary parts of the forward *NN* amplitude ρ_{NN}^f [30] are also reproduced, while in the second case, we ignore such condition and allow more freedom to the *NN* amplitude parameters in order to see if something could be said about the behavior of the *NN* amplitude inside the nuclear medium. Here it may be noted that for $n = 0$, the above form of f_{NN} reduces to the usually parametrized (one-term) Gaussian *NN* amplitude (GNN) [31,32].

For computational simplicity, we parametrize the required nuclear form factor as a sum of Gaussians, that is,

$$F(\vec{q}) = \sum_j a_j e^{-b_jq^2}, \qquad (30)$$

where a_j and b_j are parameters whose values are taken from Ref. [27]. These values were determined by fitting the proton form factor as obtained from the charge density [33] after correcting for the finite size of the proton. The value of the oscillator constant for ^{12}C is from Bassel and Wilkin [34].

The results of the calculations for ^{12}C-^{12}C elastic angular distribution and reaction cross section at 1.016, 1.449, and 2.4 GeV are presented in Figs. 1–6. In Figs. 1, 2, 4, and 5, one-term, two-terms, and three-terms correspond, respectively, to $n = 0$, 1, and 2 in the *NN* amplitude (28), and the theoretical values of the reaction cross sections are given for each form of the *NN* amplitude.

Figure 1 depicts the results reflecting our search for values of σ_{NN}, ρ_{NN}, and β_{NN}^2 that would provide a simultaneous description of elastic angular distribution and reaction cross section for the ^{12}C-^{12}C system and also reproduce the σ_{NN}^f [22] and ρ_{NN}^f [30] at the energies under consideration. The values of the parameters σ_{NN}, ρ_{NN}, and β_{NN}^2 obtained in this way are reported in Table I. In the same table, σ_{NN}^t and ρ_{NN}^t give, respectively, our theoretical estimates for the (free) *NN* total cross section and ratio of the real to the imaginary parts of the forward *NN* amplitude, the values of which are obtained from Eq. (28) using the above-mentioned values of σ_{NN}, ρ_{NN}, and β_{NN}^2. It is found that the consideration of two terms in the *NN* amplitude (28) provides a more satisfactory explanation of both the reaction cross section and the elastic angular distribution throughout the available range of momentum transfer than does the GNN. Furthermore, we notice that the *NN* amplitude (28) with three terms does not provide substantial improvement over the results with two

TABLE I. Values of *NN* amplitude parameters (σ_{NN}, ρ_{NN}, and β_{NN}^2), which provide simultaneous description of elastic angular distribution and reaction cross section for ^{12}C-^{12}C system, and also reproduce the (free) *NN* total cross section (σ_{NN}^f) [22] and ratio of the real to the imaginary parts of the forward *NN* amplitude (ρ_{NN}^f) [30]. Quantities σ_{NN}^t and ρ_{NN}^t represent, respectively, our theoretical estimates corresponding to σ_{NN}^f and ρ_{NN}^f, and are obtained from Eq. (28) using the values of *NN* amplitude parameters as given in this table. γ_{NN} is the optimum phase of the *NN* amplitude (see text).

Energy (GeV)	No. of terms in *NN* amplitude (28)	*NN*	σ_{NN} (fm^2)	ρ_{NN}	β_{NN}^2 (fm^2)	σ_{NN}^t (fm^2)	ρ_{NN}^t [a]	γ_{NN} (fm^2)
	one	*pp*(*nn*)	3.30	1.600	0.741 – i0.000	3.30	1.60	1.317
		pn(*np*)	8.81	1.200	0.168 – i0.000	8.81	1.20	−0.858
1.016	two	*pp*(*nn*)	3.31	1.495	2.129 – i0.725	3.30	1.60	0.053
		pn(*np*)	5.04	1.461	0.105 – i0.300	8.81	1.20	−0.043
	three	*pp*(*nn*)	3.36	1.457	1.996 – i0.515	3.30	1.60	0.004
		pn(*np*)	5.31	1.403	0.264 – i0.532	8.81	1.20	0.003
	one	*pp*(*nn*)	2.52	1.100	0.662 – i0.000	2.52	1.10	0.827
		pn(*np*)	6.04	0.700	0.316 – i0.000	6.04	0.70	−0.564
1.449	two	*pp*(*nn*)	2.48	1.048	1.424 – i0.392	2.52	1.10	0.275
		pn(*np*)	5.47	0.721	1.021 – i1.025	6.04	0.70	−0.164
	three	*pp*(*nn*)	2.51	1.019	1.266 – i0.191	2.52	1.10	−0.062
		pn(*np*)	5.49	0.725	1.236 – i1.293	6.04	0.70	0.035
	one	*pp*(*nn*)	2.16	1.000	0.750 + i0.000	2.16	1.00	0.821
		pn(*np*)	4.05	0.300	0.534 + i0.000	4.05	0.30	−1.135
2.4	two	*pp*(*nn*)	2.34	0.881	0.318 + i0.876	2.16	1.00	0.037
		pn(*np*)	3.66	0.345	0.534 – i0.538	4.05	0.30	−0.045
	three	*pp*(*nn*)	2.35	0.877	0.391 + i0.776	2.16	1.00	0.016
		pn(*np*)	3.61	0.360	0.548 – i0.576	4.05	0.30	−0.019

[a]Values of ρ_{NN}^t in this column lie within the uncertainties reported in Ref. [30].

terms. This indicates that, in the present context, the *NN* amplitude (28) with two terms may not only cover the relatively large scattering angles but also describe the nondiffractive behavior of *NN* scattering at relatively lower energies.

Figure 2 shows the effects of the phase of the *NN* amplitude, which has been taken into account [35] by multiplying Eq. (28) by the phase factor $e^{-i\gamma_{NN}q^2/2}$ and treating the phase of the *NN* amplitude γ_{NN} as a free parameter. Keeping the values of the parameters σ_{NN}, ρ_{NN}, and β_{NN}^2 the same, as given in Table I, we find that the phase of the *NN* amplitude, γ_{NN}, does not help in improving the results with two or three terms in the *NN* amplitude, obtained in Fig. 1, in which γ_{NN} was fixed to be zero for all incident energies. Regarding this, it is worth mentioning that our results agree with those obtained by Ahmad *et al.* [19] at 1.449 and 2.4 GeV. However, at 1.016 GeV, the present calculation totally disagrees with the findings of Ahmad *et al.* [19], where it was shown that consideration of γ_{NN} is important for providing a satisfactory description of the data. The values of the optimum γ_{NN} obtained in our case are reported in the last column of Table I.

As mentioned in Sec. I, the complexity in the analytic evaluation of the Glauber model *S* matrix for nucleus-nucleus collisions, in terms of realistic description of nuclei, has led many authors to adopt various approximate schemes for studying such collisions at intermediate energies. These studies involve the GNN as their basic input. It was noted that except for σ_{NN}, whose values were calculated using the parametrization for σ_{pp} and σ_{np} [22] which fits the experimental *NN* total cross section data [36] nicely over a wide energy range (∼10 MeV to 1 GeV), different authors [10,14,17,19] have explored different values of ρ_{NN} and β_{NN}^2 at a given incident energy, in order to provide a satisfactory explanation of nucleus-nucleus scattering data at intermediate energies within the frameworks of their approaches. At first, this seems to be quite a concern, as it may lead to different descriptions of *NN* scattering at a given incident energy. However, if we rely on the values of ρ_{pp} and ρ_{np} obtained from the phase shifts and Coulomb interference measurements [30], we find that the values of ρ_{NN} used in Refs. [10,14,17,19] lie well within the uncertainties in the average values of ρ_{pp} and ρ_{pn}. Thus, the only parameter that has been fixed according to the need of nucleus-nucleus scattering data, in the energy range under consideration, is the slope parameter β_{NN}^2 of the *NN* amplitude. Unfortunately, the values of β_{NN}^2 used in the literature [10,14,17,19] are quite different at a given energy. Because of this, the parameter β_{NN}^2 is not a very well-defined quantity, and hence the conventional Glauber model analyses of nucleus-nucleus collisions are not found to be completely parameter free at intermediate energies. However, we are still of the opinion that whatever approach is taken for the microscopic calculation of the Glauber model *S* matrix, the values of the *NN* scattering parameters should be consistent within reasonable variations. Otherwise, it will lead to inconsistency in the results of various approaches, and it will be difficult to assess the suitability of a particular approach to performing Glauber model calculations for similar systems.

It is in this spirit that we also performed calculations for the ^{12}C-^{12}C elastic angular distribution and reaction cross section using the approach of Franco and Varma [13], according to which the Glauber model *S* matrix has been related to the

TABLE II. Values of NN amplitude parameters (σ_{NN}, ρ_{NN}, and β^2_{NN}), which provide simultaneous descriptions of elastic angular distribution and reaction cross section for ^{12}C-^{12}C system, without imposing any condition of reproducing the (free) NN total cross section (σ^f_{NN}) [22] and ratio of the real to the imaginary parts of the forward NN amplitude (ρ^f_{NN}) [30]. Quantities $\sigma^{\rm eff}_{NN}$ and $\rho^{\rm eff}_{NN}$ represent, respectively, the (effective) or (in-medium) values of the NN total cross section and ratio of the real to the imaginary parts of the forward NN amplitude (see text), which are obtained from Eq. (28) using the values of NN amplitude parameters given in this table. γ_{NN} is the optimum phase of the NN amplitude (see text).

Energy (GeV)	No. of terms in NN amplitude (28)	NN	σ_{NN} (fm^2)	ρ_{NN}	β^2_{NN} (fm^2)	$\sigma^{\rm eff}_{NN}$ (fm^2)	$\rho^{\rm eff}_{NN}$	γ_{NN} (fm^2)
	one	$pp(nn)$	2.72	1.042	0.668 – i0.000	2.72	1.042	0.0254
		$pn(np)$	6.31	1.951	0.341 – i0.000	6.31	1.951	−0.0073
1.016	two	$pp(nn)$	3.11	1.616	1.488 – i0.383	3.02	1.813	0.0008
		$pn(np)$	5.36	1.408	0.308 – i0.204	6.49	1.849	0.0005
	three	$pp(nn)$	2.96	1.592	1.713 – i0.100	2.77	1.823	0.0006
		$pn(np)$	5.44	1.238	0.608 – i0.625	6.74	1.307	−0.0004
	one	$pp(nn)$	2.41	1.055	0.558 – i0.000	2.41	1.055	−1.206
		$pn(np)$	3.05	1.454	1.264 – i0.000	3.05	1.454	0.721
1.449	two	$pp(nn)$	2.52	1.245	1.706 – i0.125	2.49	1.333	−0.009
		$pn(np)$	5.19	0.662	0.883 – i1.237	5.69	0.623	0.007
	three	$pp(nn)$	2.52	1.242	1.737 – i0.145	2.48	1.339	−0.007
		$pn(np)$	5.20	0.662	0.848 – i1.223	5.80	0.608	0.006
	one	$pp(nn)$	1.45	0.738	1.343 – i0.000	1.45	0.738	−0.009
		$pn(np)$	3.87	0.020	0.691 – i0.000	3.87	0.020	0.151
2.4	two	$pp(nn)$	1.62	0.889	0.122 + i0.927	1.52	0.962	−0.042
		$pn(np)$	3.63	0.023	0.963 – i0.863	3.78	−0.012	0.188
	three	$pp(nn)$	1.57	0.973	0.122 + i0.684	1.44	1.065	0.001
		$pn(np)$	3.60	0.021	0.993 – i0.519	3.83	−0.012	−0.010

nucleus-nucleus phase shift function $\chi(\vec{b})$ through the relation

$$S_{\rm el}(\vec{b}) = e^{i\chi(\vec{b})}, \tag{31}$$

with

$$\chi(\vec{b}) = \sum_{i=1}^{\infty} \chi_i(\vec{b}), \tag{32}$$

where the various orders of the phase shifts ($\chi_1, \chi_2, \ldots$) have the same expressions as obtained in Refs. [13,19].

Retaining the terms up to second order in the phase expansion series (32) and taking two terms in the NN amplitude (28) with its parameters (σ_{NN}, ρ_{NN}, and β^2_{NN}) the same as reported in Table I, we present in Fig. 3 our results for ^{12}C-^{12}C elastic angular distribution and reaction cross section at 1.016, 1.449, and 2.4 GeV. For the sake of comparison, Fig. 3 also contains our results obtained (in Fig. 1) using the effective profile function approach as discussed in Sec. II. It is found that the effective profile function approach gives relatively better results than the phase expansion approach, when one uses the same form, with the same values of the parameters, of the NN amplitude. In this context, however, it has been checked that we do not find any other set of values for the parameters of NN amplitude that can improve the results obtained within the framework of the phase expansion approach. Thus, the findings of the present work suggest that the NN amplitude (28) involving two terms, with the parameter values as reported in Table I, is a possible choice for describing the behavior of the (free) NN scattering at the energies under consideration.

To exercise the possibility of exploring the nuclear medium effects on the parameters of the NN scattering amplitude (28), we vary σ_{NN}, $\rho_{NN,}$ and β^2_{NN} in such a way that it now includes only the simultaneous description of the elastic angular distribution and reaction cross section for ^{12}C-^{12}C system at energies under consideration. The results of these calculations are presented in Fig. 4. If we concentrate on the two terms in the NN amplitude (28), we find that the said variation in the NN amplitude parameters pushes theory closer to the experiment. However, the effects of such variations are found to be more prominent at 2.4 GeV. The values of σ_{NN}, ρ_{NN}, and β^2_{NN} obtained in this way are reported in Table II. Because in this case we have not imposed any condition of reproducing σ^f_{NN} [22] and ρ^f_{NN} [30], we may refer to these values as the effective values of the parameters of NN amplitude in the nuclear medium. Moreover, the corresponding (effective) values of the NN total cross section $\sigma^{\rm eff}_{NN}$ and the ratio of the real to the imaginary parts of the forward NN amplitude $\rho^{\rm eff}_{NN}$, which are obtained from Eq. (28) using the so-called effective values of σ_{NN}, ρ_{NN}, and β^2_{NN}, are also reported in Table II. It is seen that $\sigma^{\rm eff}_{NN}$ is less than (free) σ^f_{NN} in all the cases. Since $\sigma^{\rm eff}_{NN}$ is essentially an average over the nuclear volume involved during collision, our results, like the ones obtained by Ahmad *et al.* [20], provide only qualitative support to the results of the microscopic studies [23,24] in which the in-medium NN total cross sections are found to be less than the free ones. Further, if we compare the average values of $\sigma^{\rm eff}_{pp(nn)}$ and $\sigma^{\rm eff}_{pn(np)}$ as obtained in this work with the phenomenological ones [20], we find that the difference between the (free) σ^f_{NN} and our $\sigma^{\rm eff}_{NN}$ is not as large as reported

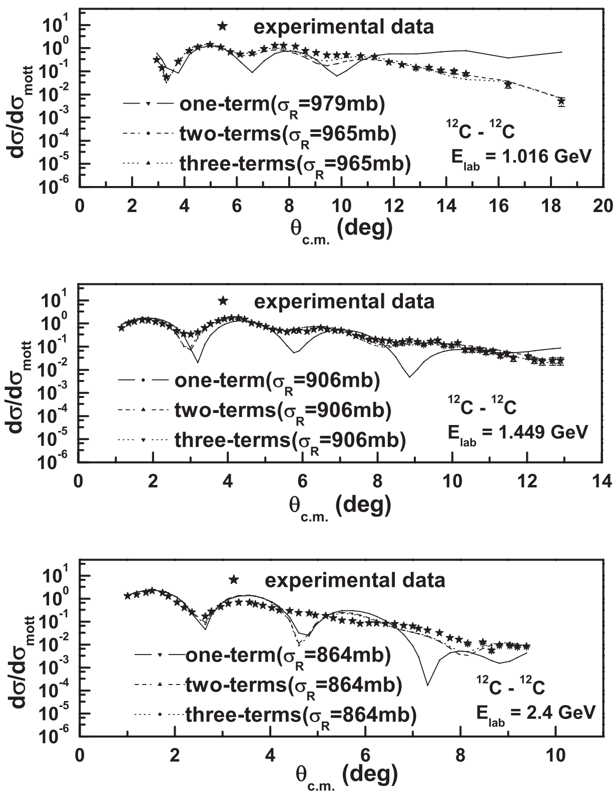

FIG. 1. Elastic angular distribution for ^{12}C-^{12}C at 1.016, 1.449, and 2.4 GeV using the parameters of the *NN* amplitude as reported in Table I. Curves correspond to one, two, and three terms in the *NN* amplitude (28). Theoretical values of reaction cross sections are given for each form of the *NN* amplitude. γ_{NN} is zero for all incident energies. Experimental data are from Ref. [19].

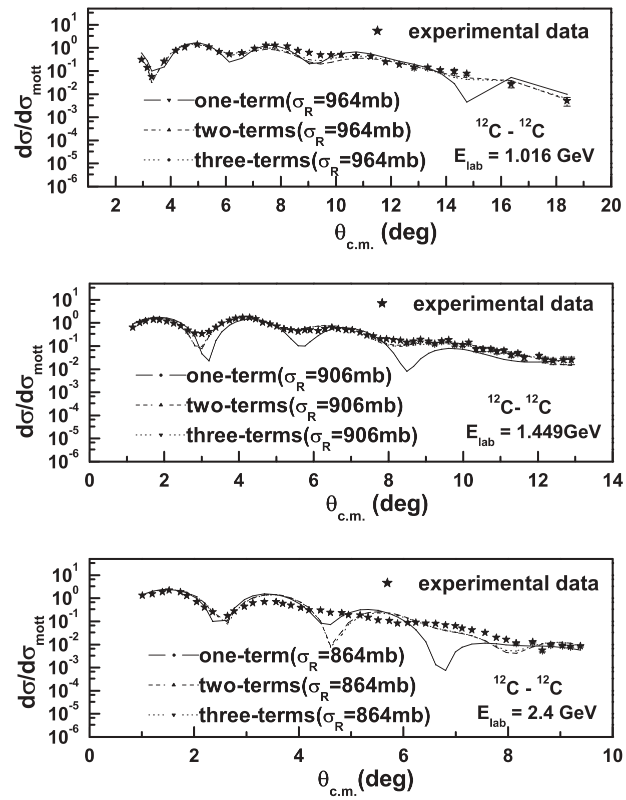

FIG. 2. Same as in Fig. 1, but with the optimum γ_{NN} for each incident energy (Table I).

in Ref. [20]. This might be because in phenomenological studies [20], the authors focused on reproducing the ^{12}C-^{12}C elastic angular distribution only; whereas in the present work, we also included the reaction cross section, which is one of the most fundamental quantities characterizing nuclear reactions. Moreover, if we compare the parameters (σ_{NN}, ρ_{NN} and β^2_{NN}) of the *NN* amplitude in Tables I and II, it is interesting to note that the so-called effective values are found to be different from their corresponding free ones. Thus, the results of the present analysis provide a qualitative description of the in-medium effects on the parameters of the (free) *NN* amplitude.

Figure 5 shows the effects of the phase of the *NN* amplitude γ_{NN}, which has been taken into account in the same way as in Fig. 2, but with the *NN* parameter values as given in Table II. Here also, we find that γ_{NN} does not provide any significant improvement over the results obtained in Fig. 4 in which γ_{NN} was fixed to be zero for all incident energies. The values of the optimum γ_{NN} obtained in this case are reported in the last column of Table II.

Finally in Fig. 6, we present a study of the effects of the c.m. correlations on the ^{12}C-^{12}C elastic scattering. As mentioned in Sec. II, the c.m. correlation effect appears in the two-body form factors of the colliding nuclei through the c.m. correlation correction factor $\theta(\vec{q})$. Thus, to study the effect of ignoring the c.m. correlation in the projectile and the target we simply take $\theta(\vec{q}) = 1$. Here, note that we calculated the ^{12}C-^{12}C elastic angular distribution by involving only two terms in the *NN* amplitude (28) and by taking its parameters to be the same as reported in Table I.

The dashed curve in Fig. 6 is the result of neglecting the c.m. correlations in both the projectile and the target. The dotted curve ignores the c.m. correlation in the target only, and it is obvious that due to the symmetrical nature of colliding nuclei, the effect of ignoring the c.m. correlation in the projectile only would be the same as observed in the dotted curve. The solid curve is obtained when the c.m. correlations are present in both the projectile and the target. It is found that, except for $\theta_{\text{c.m.}} \leqslant 3^\circ$, the c.m. correlation correction has an appreciable effect on the ^{12}C-^{12}C elastic scattering throughout the range of momentum transfer.

In summary, we have analyzed the ^{12}C-^{12}C elastic angular distribution and reaction cross section at 1.016, 1.449, and 2.4 GeV using the Coulomb modified correlation expansion for the Glauber amplitude based on the effective profile function approach as developed by Ahmad [27]. We emphasized the parametrization for the basic (input) *NN* amplitude that may be used for a wide range of angles. The calculations were performed in two steps. In the first step, we searched for the values of the parameters of *NN* amplitude that may provide the best possible description of the elastic angular distribution and reaction cross section for the ^{12}C-^{12}C system with the conditions that the σ^f_{NN} and ρ^f_{NN} be also reproduced at 85, 120, and 200 MeV/nucleon. Retaining up to the two-body density term in the correlation expansion for the Glauber

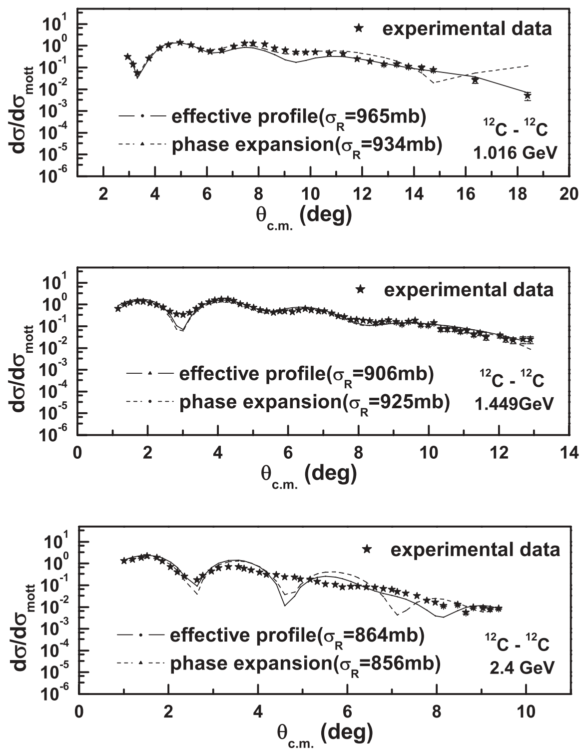

FIG. 3. Elastic angular distribution for ^{12}C-^{12}C at 1.016, 1.449, and 2.4 GeV using the effective profile function and phase expansion approaches. Theoretical values of the reaction cross sections are given for each approach. These calculations involve only two terms in the *NN* amplitude (28); parameter values are the same as in Table I; γ_{NN} is zero for all incident energies. Experimental data are from Ref. [19].

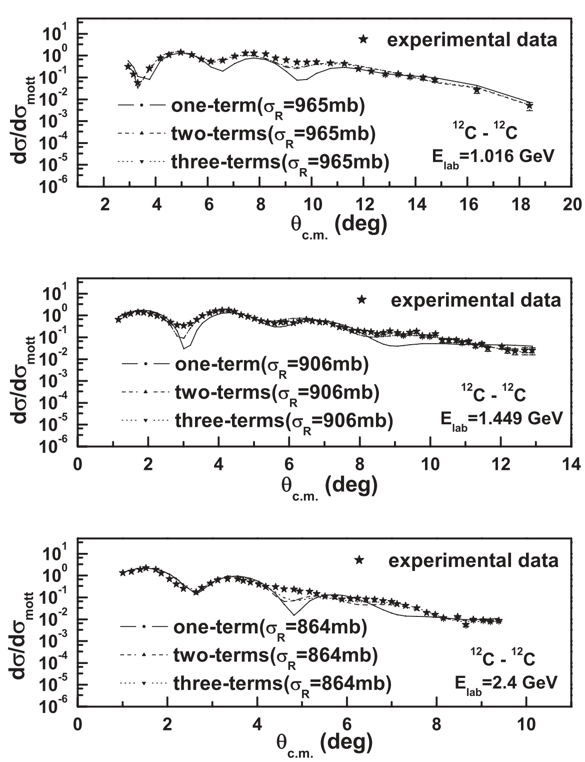

FIG. 4. Same as in Fig. 1, but with the parameters of the *NN* amplitude as reported in Table II.

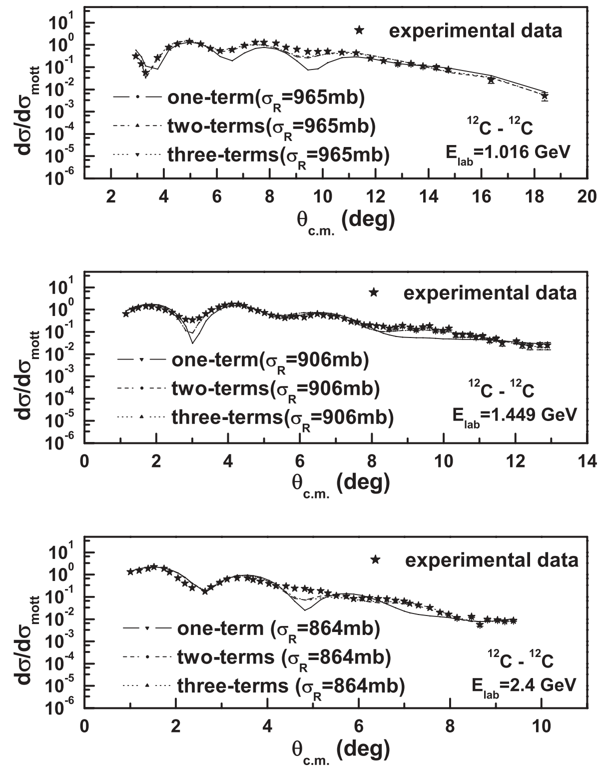

FIG. 5. Same as in Fig. 4, but with the optimum γ_{NN} for each incident energy (Table II).

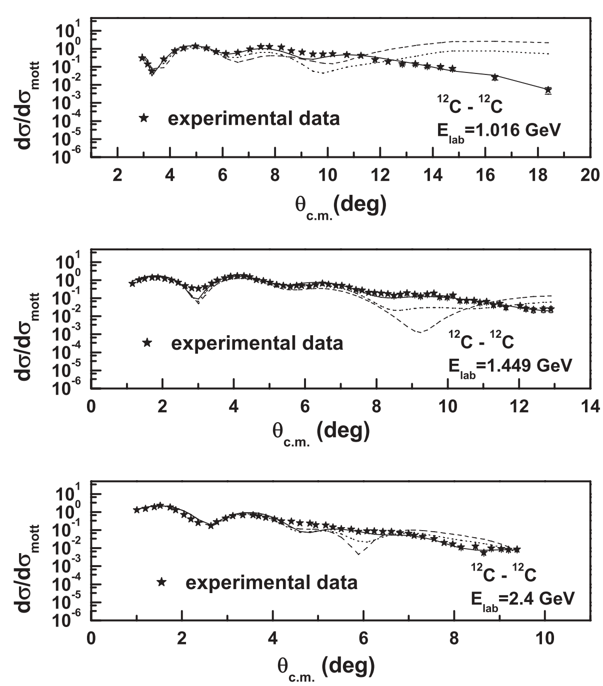

FIG. 6. Effects of c.m. correlations on ^{12}C-^{12}C elastic angular distribution at 1.016, 1.449, and 2.4 GeV. Dashed curve, without the c.m. correlation in both projectile and target; dotted curve, including the c.m. correlation in target only; solid curve, including the c.m. correlation in both projectile and target. Experimental data are from Ref. [19].

amplitude and using the realistic densities for the colliding nuclei, it is found that the consideration of the two terms in the *NN* amplitude (28) provides a significant improvement over the results obtained with the GNN, and we now have a quite satisfactory explanation of the data throughout the range of momentum transfer. This indicates the importance of large q components and hence the nondiffractive behavior of *NN* scattering at relatively lower energies. Furthermore, it has also been shown that between the effective profile and phase expansion approaches, the former one seems to be the better choice for the microscopic description of ^{12}C-^{12}C scattering at intermediate energies.

In the second step of our calculations, we attempted to see the in-medium effects on the (free) *NN* scattering amplitude. For this, we varied the paramaters of the *NN* amplitude (28) up to the extent of getting simultaneous (good) descriptions of the elastic angular distribution and reaction cross section without imposing any condition of reproducing σ_{NN}^{f} and ρ_{NN}^{f}. The results of such calculations support the microscopic findings [23,24] that the in-medium *NN* total cross sections are less than the free ones. However, the present study also sheds some light on the possible in-medium effects on the other parameters of the *NN* amplitude. As regards the effect of the phase of the *NN* amplitude, we find that it does not help in improving the results obtained by taking the phase of the *NN* amplitude to be zero for all incident energies. Finally, our calculations also highlight the importance of the c.m. correlations in nucleus-nucleus collisions at intermediate energies.

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