Center-of-mass correction in a relativistic Hartree approximation including meson degrees of freedom

P. Alberto,[1] S. S. Avancini,[2] M. Fiolhais,[1] and J. R. Marinelli[2]
[1]*Center for Computational Physics and Physics Department, University of Coimbra, P-3004-516 Coimbra, Portugal*
[2]*Depto de Física, CFM, Universidade Federal de Santa Catarina, Florianópolis, SC, CP. 476, CEP 88.040-900, Brazil*
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We use the Peierls-Yoccoz projection method to study the motion of a relativistic system of nucleons interacting with sigma and omega mesons, generalizing a method developed for the alpha particle. The nuclear system is described in a mean-field Hartree approach, including explicitly the meson contribution. The formalism is applied to ^{4}He, ^{16}O, and ^{40}Ca. The center-of-mass correction makes the system too much bounded. It turns out that a new set of model parameters is needed when the center-of-mass motion is consistently treated with respect to the traditional approaches. An appropriate refitting of the model brings the radii and binding energies to reasonable values for the oxygen and calcium.

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I. INTRODUCTION

Relativistic models for finite nuclei, with nucleons and mesons are usually treated in the Hartree or Hartree-Fock approximations. In these approximations the total linear momentum is no longer a conserved quantity and the spurious center of mass (c.m.) motion gives rise to unphysical contributions in the calculated nuclear observables. As already discussed for nonrelativistic nuclear models, using the same kind of mean-field approximation, the correct treatment for the c.m. motion introduces a modest modification in the total binding energy for intermediate mass nuclei, but relatively large contributions in other observables, e.g., charge distributions and spectral functions [1,2]. In relativistic treatments, the c.m. correction is up to now limited to the harmonic approximation for the energy or to the subtraction of $\frac{\langle \hat{\vec{P}}_A^2 \rangle}{2AM}$ from the total energy, where $\hat{\vec{P}}_A$ is the total nucleus momentum operator, M is the nucleon mass and the mean-value is taken using the Hartree self-consistent state for A nucleons.

More recently [3], the c.m. energy correction was estimated within the σ-ω model, using the Peierls-Yoccoz projection procedure [4], for $N = Z$ spherical nuclei within the Hartree approximation. Although the correction obtained in this way is of the same order of magnitude of the harmonic approximation, only the nucleonic degrees of freedom were taken into account in that calculation. In Ref. [5], a formalism has been developed to include the mesonic degrees of freedom in the c.m. projection within σ-ω models and an application was then made to the ^{4}He nucleus. In the present paper we generalize the results obtained in [5] in order to extend the calculations to heavier spherical nuclei, allowing us to draw more systematic conclusions. In Sec. II, we review the main results from Ref. [5]. Then, in Sec. III, the linear momentum projection within the model is presented and the total energy functional is worked out. Since most of the model parametrizations within σ-ω models are based on fits to the experimental data of both binding energy and charge radius, we take the same point of view. The nuclear charge mean square radius is discussed in Sec. IV. The numerical results for ^{4}He, ^{16}O, and ^{40}Ca are shown and discussed in Sec. V. Finally the conclusions and perspectives are summarized in Sec. VI.

II. THE MEAN-FIELD HAMILTONIAN WITH MESONS AS COHERENT STATES

In this section we summarize the main aspects of the relativistic nucleon-meson models of nuclei. The model used in this work is restricted to σ and ω mesons, without self-interactions (the inclusion of such interactions is straightforward using the method presented in this paper).

The Lagrangian density for a system of nucleons interacting with sigma and omega mesons reads [6]

$$L = L_N^{\text{free}} + L_\sigma^{\text{free}} + L_\omega^{\text{free}} + L_{NN\sigma}^{\text{int}} + L_{NN\omega}^{\text{int}}, \tag{1}$$

where N denotes the nucleon and σ, ω the mesons. The Lagrangians for the free fields are

$$L_N^{\text{free}} = \overline{\psi}(x)(\mathrm{i}\gamma^\mu \partial_\mu + M)\psi(x), \tag{2}$$
$$L_\sigma^{\text{free}} = -\tfrac{1}{2}\left[m_\sigma^2 \sigma^2(x) - \partial_\mu \sigma(x)\partial^\mu \sigma(x)\right], \tag{3}$$
$$L_\omega^{\text{free}} = -\tfrac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \tfrac{1}{2} m_\omega^2 \omega_\nu(x)\omega^\nu(x), \tag{4}$$

where

$$F_{\mu\nu} \equiv \partial_\mu \omega_\nu(x) - \partial_\nu \omega_\mu(x),$$

M is the rest mass of the nucleon, and m_σ and m_ω are the meson masses. The sigma and omega fields are denoted respectively by $\sigma(x)$ and $\omega^\nu(x)$, and the nucleon field by $\psi(x)$. The interaction parts of the Lagrangian are

$$L_{NN\sigma}^{\text{int}} = g_\sigma \overline{\psi}(x)\sigma(x)\psi(x), \tag{5}$$
$$L_{NN\omega}^{\text{int}} = -g_\omega \overline{\psi}(x)\omega^\nu(x)\gamma_\nu \psi(x). \tag{6}$$

From the above Lagrangian density one derives the following Hamiltonian density:

$$\mathcal{H} = \mathcal{H}_N + \mathcal{H}_\omega + \mathcal{H}_\sigma, \tag{7}$$

where the fermionic term is

$$\mathcal{H}_N = \psi^\dagger(x)\Big\{-\mathrm{i}\vec{\alpha}\cdot\vec{\nabla} + \beta[M - g_\sigma\sigma(x)] - g_\omega\vec{\alpha}\cdot\vec{\omega}(x) + \frac{g_\omega}{m_\omega^2}\vec{\nabla}\cdot\vec{P}_\omega\Big\}\psi(x) + \frac{g_\omega^2}{2m_\omega^2}[\psi^\dagger(x)\psi(x)]^2, \quad (8)$$

and the free meson terms are

$$\mathcal{H}_\omega = \frac{1}{2}\left[\vec{P}_\omega\cdot\vec{P}_\omega + \frac{(\vec{\nabla}\cdot\vec{P}_\omega)^2}{m_\omega^2} + (\vec{\nabla}\times\vec{\omega})^2 + m_\omega^2\vec{\omega}^2\right], \quad (9)$$

$$\mathcal{H}_\sigma = \tfrac{1}{2}\left[P_\sigma^2 + \vec{\nabla}\sigma\cdot\vec{\nabla}\sigma + m_\sigma^2\sigma^2\right]. \quad (10)$$

Note that we have used the definitions $P_\omega^i = F^{0i}$ with $i = 1, 2, 3$ and $P_\sigma = \partial_0\sigma$. The quantization of the model follows the usual procedure described, for instance, in [7]. The most important steps of the quantization are described in [5], but the procedure can be outlined here by saying that, for the canonical quantization of massive vector fields, one cannot use the field ω_0, because its canonical conjugate field is identically zero. For conserved four-vector sources (as it is the case of the nucleon vector current) the four-divergence of ω_μ is zero, and therefore one can use the full Klein-Gordon equation for ω_0 to write this field in terms of the divergence of the conjugate field of the spatial components ω^i and of the zeroth component of the vector current, $g_\omega\psi^\dagger\psi$ (see eq. (12) in Ref. [5]). The Hamiltonian is then built in the usual way by using only the spatial components, ω^i, and their respective conjugate fields, P_ω^i. The two-body contact term in Eq. (8) arises from the quadratic term in ω_0 in the Lagrangian.

The nucleon field operators can be expanded as

$$\hat{\psi}(x) = \sum_\alpha u_\alpha(\vec{r})\mathrm{e}^{-\mathrm{i}E_\alpha t}b_\alpha + \sum_\alpha v_\alpha(\vec{r})\mathrm{e}^{\mathrm{i}E_\alpha t}d_\alpha^\dagger, \quad (11)$$

$$\hat{\psi}^\dagger(x) = \sum_\alpha u_\alpha^\dagger(\vec{r})\mathrm{e}^{\mathrm{i}E_\alpha t}b_\alpha^\dagger + \sum_\alpha v_\alpha^\dagger(\vec{r})\mathrm{e}^{-\mathrm{i}E_\alpha t}d_\alpha, \quad (12)$$

where $u_\alpha(\vec{r})$ and $v_\alpha(\vec{r})$ form a complete set of Dirac spinors in the coordinate space, and b_α and $b_\alpha^\dagger$ are the creation and annihilation operators of a nucleon in the state α. By d_α and $d_\alpha^\dagger$ we denote the creation and the annihilation operators for the antinucleons in the state α. Similarly, the σ meson field may also be expanded in the following form:

$$\hat{\sigma} = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2\omega_\sigma(k)}}[c(\vec{k})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}} + c^\dagger(\vec{k})\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}]. \quad (13)$$

The omega field expansion, considering longitudinal and transverse waves relative to the wave vector $\vec{k}$, reads

$$\hat{\vec{\omega}} = \frac{1}{(2\pi)6^{3/2}}\int\frac{d^3k}{\sqrt{2\omega_\omega(k)}}\left\{\left[\frac{\omega_\omega}{m_\omega}\vec{k}\,a_l(\vec{k}) + \sum_{t=1,2}\hat{e}_t(\vec{k})a_t(\vec{k})\right]\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}} + \mathrm{h.c}\right\}. \quad (14)$$

All creation and annihilation operators $(c, c^\dagger)$, $(a_l, a_l^\dagger)$ and $(a_t, a_t^\dagger)$ obey canonical boson commutation relations, and we have introduced the frequencies $\omega_\sigma = \sqrt{m_\sigma^2 + k^2}$ and $\omega_\omega = \sqrt{m_\omega^2 + k^2}$. Using the above expansions, the free meson field Hamiltonians can be cast in the form

$$H_\sigma = \int d^3r\mathcal{H}_\sigma = \int d^3k\,\omega_\sigma(k)c^\dagger(\vec{k})c(\vec{k}), \quad (15)$$

and

$$H_\omega = \int d^3k\,\omega_\omega(k)\left[a_l^\dagger(\vec{k})a_l(\vec{k}) + \sum_{t=1,2}a_t^\dagger(\vec{k})a_t(\vec{k})\right]. \quad (16)$$

The nucleus state is assumed to be described by $|\psi\rangle = |A\rangle|\sigma\rangle|\omega\rangle$, with $|A\rangle$ representing an A fermion Slater determinant with the lowest energy states occupied (valence or no-sea approximation), i.e.,

$$|A\rangle = b_{\alpha_1}^\dagger b_{\alpha_2}^\dagger\ldots b_{\alpha_A}^\dagger|0\rangle, \quad (17)$$

where $\alpha_1, \ldots, \alpha_A$ are sets of single-particle quantum numbers and $|0\rangle$ is the bare vacuum. As it is usual in σ-ω models, we work in the valence approximation, which means that the polarization of the negative energy single particle states is neglected. This is also a common approximation in quark-meson chiral soliton models, such as the linear sigma model or the chromodielectric model [8]. The approximation is even more justifiable here since the binding energy per nucleon is small compared with the rest mass of the nucleon. However, this might not be the case in other approximations (see, for instance, Ref. [9]).

In the above product state, $|\sigma\rangle$ represents a coherent state describing the σ mesons and $|\omega\rangle$ a coherent state describing the ω mesons. For instance, for the σ meson cloud:

$$|\sigma\rangle = N_\sigma\exp\left[\int d^3k\,\eta(\vec{k})c^\dagger(\vec{k})\right]|0\rangle, \quad (18)$$

with $c(\vec{k})|\sigma\rangle = \eta(\vec{k})|\sigma\rangle$ and, from the normalization of the state

$$N_\sigma = \exp\left[-\frac{1}{2}\int d^3k\,|\eta(\vec{k})|^2\right]. \quad (19)$$

We now enforce the mean value of the σ field operator in the coherent state to be equal to the potential obtained in the mean-field Hartree approximation, i.e., we demand

$$\langle\sigma|\hat{\sigma}|\sigma\rangle = \phi_0(r), \quad (20)$$

for a spherical symmetric potential. This condition allows us to determine, in an unique way, the function $\eta(\vec{k})$ in Eq. (18). Exploiting the spherical symmetry of the scalar potential, one finds

$$\eta(k) = \sqrt{\frac{\omega_\sigma(k)}{\pi}}\int dr\,r^2\,j_0(kr)\phi_0(r), \quad (21)$$

where j_0 is the spherical Bessel function of zeroth order. A similar procedure can be carried out for the ω meson field. In this case,

$$|\omega\rangle = N_\omega \exp\left[\int d^3k\,[\Omega_l(\vec{k})a_l^\dagger(\vec{k}) + \sum_{t=1,2}\Omega_t(\vec{k})a_t^\dagger(\vec{k})\right]|0\rangle, \tag{22}$$

and, using again the normalization and the properties of the vector potential in the Hartree approximation, such as $\langle\omega|\hat{\vec{\omega}}|\omega\rangle = 0$, $\langle\omega|\hat{\omega}^0|\omega\rangle = \omega_0(r)$ and $\langle\omega|\hat{\vec{P}}_\omega|\omega\rangle = \hat{r}\frac{d\omega_0(r)}{dr}$, one finds

$$\Omega_l(k) = \frac{1}{m_\omega}\sqrt{\frac{\omega_\omega(k)}{\pi}}\int dr\; r^2 j_1(kr)\frac{d\omega_0(r)}{dr}, \tag{23}$$

and $\Omega_t(\vec{k}) = 0$. If we now take the states defined in Eqs. (17), (18), and (22) and calculate the mean value of the Hamiltonian obtained from Eq. (7), we exactly recover the nucleus energy obtained in the usual Hartree approximation.

Let us stress that the coherent state is a multiparticle state and the description of meson clouds by coherent states introduces many-body correlations.

III. THE CENTER-OF-MASS APPROXIMATE PROJECTION

Next, we want to obtain the center-of-mass (c.m.) correction to the energy using the model described in Sec. II. It is well known, from the nuclear many-body theory, that mean-field approximations break translational invariance (see Ref. [4]) and that the broken symmetry can be recovered by applying the Peierls-Yoccoz projection to the symmetry-breaking state. The projection operator

$$\mathcal{P}_{\vec{p}} = \int \exp\left[i(\hat{\vec{P}} - \vec{p})\cdot\vec{a}\right]d^3\vec{a}, \tag{24}$$

exhibits the property

$$\mathcal{P}_{\vec{p}}\mathcal{P}_{\vec{p}\,'} = \delta(\vec{p} - \vec{p}\,')\mathcal{P}_{\vec{p}}. \tag{25}$$

In Eq. (24), $\hat{\vec{P}} = \hat{\vec{P}}_A + \hat{\vec{P}}_\sigma + \hat{\vec{P}}_\omega$ is the total linear momentum operator and $\vec{p}$ the corresponding eigenvalue. Our approach consists in assuming that the model state representing the physical nucleus is obtained by projecting the product mean-field Hartree state onto a zero momentum ($\vec{p} = \vec{0}$) state (the procedure is known as projection after variation). Since the Hamiltonian $H = \int d^3r\mathcal{H}$, with $\mathcal{H}$ given by Eq. (7), commutes with the projection operator, we may write the total energy, already corrected for the c.m. spurious motion, as

$$E_{\vec{p}=0} = \frac{\langle\psi|H\mathcal{P}_{\vec{p}=\vec{0}}|\psi\rangle}{\langle\psi|\mathcal{P}_{\vec{p}=\vec{0}}|\psi\rangle}. \tag{26}$$

We emphasize that, in the valence approximation, the projection operator acts on the mesons and on the positive energy fermions. The vacuum single-particle states are unperturbed and the vacuum is invariant under translations, so that the shifted states have the same energy as the unshifted ones in the so-called variation before projection method [10] that we are using here.

In order to compute the projected energy let us first consider the norm overlap:

$$\langle\psi\mid\mathcal{P}_{\vec{p}=\vec{0}}\mid\psi\rangle = \int d\vec{a}\,\langle\sigma|e^{i\hat{\vec{P}}_\sigma\cdot\vec{a}}|\sigma\rangle \times\langle\omega|e^{i\hat{\vec{P}}_\omega\cdot\vec{a}}|\omega\rangle\langle A|e^{i\hat{\vec{P}}_A\cdot\vec{a}}|A\rangle, \tag{27}$$

and begin with the σ field contribution. Its norm overlap reads:

$$N_\sigma(a) = \langle\sigma|e^{i\vec{P}_\sigma\cdot\vec{a}}|\sigma\rangle = \exp\left\{4\pi\int dk k^2|\eta(k)|^2[j_0(ka) - 1]\right\}, \tag{28}$$

where $\eta(k)$ is defined by Eq. (21). We then find

$$N_\sigma(a) = \exp\left\{4\pi\int dk k^2\frac{(m_\sigma^2 + k^2)^{1/2}}{2}\tilde{\phi}_0^2(k)[j_0(ka) - 1]\right\}, \tag{29}$$

where

$$\tilde{\phi}_0(k) = \int dr r^2 j_0(kr)\phi_0(r). \tag{30}$$

Similarly, for the ω meson norm overlap

$$N_\omega(a) = \exp\left\{\frac{4\pi}{m_\omega}\int dk\,k^2\frac{(m_\omega^2 + k^2)^{1/2}}{2}\tilde{\omega}_0^2(k)[j_0(ka) - 1]\right\}, \tag{31}$$

where

$$\tilde{\omega}_0(k) = \int dr r^2 j_0(kr)\omega_0(r). \tag{32}$$

The calculation of the fermionic part of the norm overlap is more involved, and we just quote here the main result in a compact form:

$$N_A(a) = \langle A|e^{i\hat{\vec{P}}_A\cdot\vec{a}}|A\rangle = \det B, \tag{33}$$

where the B matrix is defined by

$$B_{\alpha\beta} = \langle\alpha|\beta(a)\rangle. \tag{34}$$

Each label (α and β stands for the set of particle quantum numbers (n, l, j, m) as well as for the isospin projection quantum number necessary to classify the state. The ket $|\beta(a)\rangle$ means a single-particle (four-component) state for which the spatial coordinate $\vec{r}$ is changed to $\vec{r} + \vec{a}$.

Next, we move our attention to the energy kernel calculation. The total Hamiltonian is written in the form

$$H = H_N + H_\sigma + H_\omega, \tag{35}$$

where the first term contains the free fermion part as well as their interaction with the $\sigma - \omega$ mesons. The second and third terms are given by Eqs. (15) and (16) and represent the free mesonic terms. Let us consider the free σ field energy kernel. Using Eqs. (18),(21) and the result

$$|\sigma(a)\rangle = \mathrm{e}^{\mathrm{i}\hat{\vec{P}}_\sigma\cdot\vec{a}}|\sigma\rangle = N_\sigma \exp\left[\int d\vec{k}\,\eta'(\vec{k})b(\vec{k})\right]|0\rangle, \quad (36)$$

with $\eta'(\vec{k}) = \eta(\vec{k})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{a}}$, we obtain

$$\begin{aligned}\varepsilon_\sigma(a) &= \langle\sigma|H_\sigma|\sigma(a)\rangle \\ &= \frac{1}{2}\left[\int dk\,k^2\left(m_\sigma^2+k^2\right)\tilde{\phi}_0^2(k)j_0(ka)\right]N_\sigma(a). \end{aligned} \quad (37)$$

For the free ω meson energy kernel, a similar analysis leads us to the following result:

$$\begin{aligned}\varepsilon_\omega(a) &= \langle\omega|H_\omega|\omega(a)\rangle \\ &= \frac{1}{2m_\omega^2}\left[\int dkk^4\left(m_\omega^2+k^2\right)\tilde{\omega}_0^2(k)j_0(ka)\right]N_\omega(a). \end{aligned} \quad (38)$$

For the fermionic part of the energy kernel, it is more convenient to rewrite the corresponding original Hamiltonian. From Eq. (8), in the Hartree mean-field, we can read off the fermionic Hamiltonian written in second quantized form:

$$\begin{aligned}H_N &= \hat{h}^{(1)} + \hat{h}^{(1\,2)} = \sum_{\alpha,\beta} h_{\alpha\beta}^{(1)} b_\alpha^\dagger b_\beta \\ &\quad + \sum_{\alpha,\beta\gamma\delta} h_{\alpha\beta\gamma\delta}^{(1\,2)} : b_\alpha^\dagger b_\gamma b_\beta^\dagger b_\delta :, \end{aligned} \quad (39)$$

with

$$\begin{aligned}h_{\alpha\beta}^{(1)} &= \int d\vec{r}\,u_\alpha^\dagger(\vec{r})\left\{-\mathrm{i}\vec{\alpha}\cdot\vec{\nabla} + \beta[M - g_\sigma\sigma(x)]\right. \\ &\quad \left. + \frac{g_\omega}{m_\omega^2}\vec{\nabla}\cdot\vec{P}_\omega\right\}u_\beta(\vec{r}), \end{aligned} \quad (40)$$

and

$$h_{\alpha\beta\gamma\delta}^{(1\,2)} = \int\int d\vec{r}d\vec{r}'u_\alpha^\dagger(\vec{r})u_\beta^\dagger(\vec{r}')\frac{g_\omega^2}{m_\omega^2}\delta(\vec{r}-\vec{r}')u_\gamma(\vec{r})u_\delta(\vec{r}'). \quad (41)$$

In the above equations, $u_{\alpha,\beta}$ represents the Dirac single-particle spinor, which we choose to be the Hartree mean-field solution. Observing now that, the ω_0 field should obey the Klein-Gordon equation:

$$\nabla^2\omega_0(r) = -g_\omega\rho_B(r) + m_\omega^2\omega_0(r), \quad (42)$$

and that $\vec{\nabla}\cdot\vec{P}_\omega = -\nabla^2\omega_0(r)$, we may rewrite the one-body part in Eq. (39) as

$$\hat{h}^{(1)} = h_{\mathrm{MFA}} - \frac{g_\omega^2}{m_\omega^2}\rho_B(\vec{r}), \quad (43)$$

with

$$h_{\mathrm{MFA}}u_\alpha = \epsilon_\alpha u_\alpha. \quad (44)$$

We are now in position to perform the calculation of the fermionic part of the energy kernel, which reads [5]

$$\begin{aligned}\varepsilon_N(a) &= \langle A|H_N\mathrm{e}^{\mathrm{i}\vec{P}_A\cdot\vec{a}}|A\rangle = \sum_\alpha \epsilon_\alpha N_A - \langle A|V^{(1)}\mathrm{e}^{\mathrm{i}\vec{P}_A\cdot\vec{a}}|A\rangle \\ &\quad + \langle A|h^{(1\,2)}\mathrm{e}^{\mathrm{i}\vec{P}_A\cdot\vec{a}}|A\rangle, \end{aligned} \quad (45)$$

where we have defined $V^{(1)} = \frac{g_\omega^2}{m_\omega^2}\rho_B(\vec{r})$. The second and third terms in Eq. (45) can then be obtained with the help of the well-known results (see, e.g., Ref. [11]):

$$\langle A|V^{(1)}\mathrm{e}^{\mathrm{i}\vec{P}_A\cdot\vec{a}}|A\rangle = N_A(a)\sum_{\alpha\beta}\langle\alpha|V^{(1)}|\beta(a)\rangle B_{\beta\alpha}^{-1}, \quad (46)$$

and

$$\begin{aligned}\langle A|h^{(1\,2)}\mathrm{e}^{\mathrm{i}\vec{P}_A\cdot\vec{a}}|A\rangle &= \frac{1}{2}N_A(a)\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|h^{(1\,2)} \\ &\quad \times|\gamma(a)\delta(a)\rangle B_{\gamma\alpha}^{-1}B_{\delta\beta}^{-1}, \end{aligned} \quad (47)$$

where the exchange term has been neglected. Putting everything together, we finally obtain the total nucleus energy corrected for the spurious c.m. motion

$$E_{\vec{p}=0} = \frac{\int d\vec{a}[\varepsilon_N(a)N_\sigma(a)N_\omega(a) + \varepsilon_\sigma(a)N_A(a)N_\omega(a) + \varepsilon_\omega(a)N_A(a)N_\sigma(a)]}{\int d\vec{a}[\langle\psi|\psi(a)\rangle]}. \quad (48)$$

We stress that both the nucleons and the mesons were taken into account in the evaluation of this projected energy.

IV. THE NUCLEAR CHARGE ROOT-MEAN-SQUARE RADIUS

We now turn to the evaluation of the nuclear root-mean-square (rms) radius in the formalism. Most of the measurements refer to the proton charge rms radius so we restrict ourselves to that case (measurements for the neutron rms radius are under way and are receiving an increasing interest [12]). On the other hand, since the mesons in the model are all neutral we have to consider just the nucleon (proton) contribution. Finally, in the discussion below, we consider point-particle nucleons, though nucleon form factors can be included without major difficulties.

The (translationally invariant) nuclear radius operator is

$$R_{\mathrm{TI}}^2 = \sum_{i=1}^{A} e_i(\vec{r}_i - \vec{R}_{\mathrm{c.m.}})^2, \tag{49}$$

where $\vec{R}_{\mathrm{c.m.}}$ is the center of mass coordinate and e_i is the charge of the ith particle. The above operator can be rewritten, for $N = Z$, as

$$R_{\mathrm{TI}}^2 = \frac{(A-1)}{A}\sum_{i=1}^{A} e_i r_i^2 - \frac{2}{A}\sum_{i<j}^{A} e_i \vec{r}_i \cdot \vec{r}_j. \tag{50}$$

As for the energy calculation, the above radius operator commutes with the total linear momentum, but our model wave function $|\psi\rangle$ is not a total momentum eigenfunction, so the mean radius is then given by

$$\langle r^2\rangle_{\mathrm{proj}} = \frac{1}{Ze}\frac{\int d\vec{a}\langle\psi|R_{\mathrm{TI}}^2 \exp^{i\vec{P}\cdot\vec{a}}|\psi\rangle}{\int d\vec{a}\langle\psi|\psi(a)\rangle}. \tag{51}$$

Noting that the radius operator contains an one-body and a two-body term, the numerator of the above equation can be worked out with the help of equations like Eqs. (46) and (47), respectively.

V. NUMERICAL APPLICATIONS FOR $N = Z$ CLOSED SHELL NUCLEI

In order to perform applications to specific nuclei, we must solve first the $\sigma - \omega$ model above described in the Hartree approximation, disregarding the c.m. motion effects. This is totally equivalent to solve the model treating the mesons as classical fields [6]. We choose to follow the method described in reference [13], where both the nucleon Dirac spinors and the fields are expanded in three-dimensional harmonic oscillator functions, $R_{kl}(r)$, and treat the expansion coefficients as variational parameters. As we are dealing here with closed shell nuclei only, we have

$$g_{nlj}(r) = \sum_{k=0}^{N} C_k^{(nlj)} R_{kl}(r), \tag{52}$$

$$f_{nlj}(r) = \sum_{k=0}^{N'} \tilde{C}_k^{(nlj)} R_{kl}(r), \tag{53}$$

with g and f being the upper and lower radial components for the single-particle wave function. For the meson fields:

$$B(r) = \sum_{k=0}^{N_B} C_k^B R_{k0}(r), \tag{54}$$

where B stands for ϕ_0 or ω_0 and R_{kl} for the radial harmonic oscillator function. Those expansions can be introduced in the Dirac and Klein-Gordon equations and solved self-consistently for the expansions coefficients C_k, $\tilde{C}_k$, and C_k^B. After that, it is straightforward to implement the calculation of the energy and rms radius as presented in the above sections, including the c.m. motion correction due to the nucleons and mesons.

TABLE I. Ground-state energy without (E) and with (E_{proj}) the c.m. correction for the three double-closed shell nuclei considered in this work and the root-mean-square charge radius without ($\langle r^2\rangle$) and with ($\langle r^2\rangle_{\mathrm{proj}}^{1/2}$) the same corrections.

Nucleus	E[MeV]	E_{proj}[MeV]	$\langle r^2\rangle^{1/2}$[fm]	$\langle r^2\rangle_{\mathrm{proj}}^{1/2}$[fm]
^{4}He	−4.85	−68.95	2.06	1.84
^{16}O	−94.63	−190.67	2.59	2.51
^{40}Ca	−331.32	−420.17	3.33	3.28

In Table I we show our results for the energy and for the root-mean-square charge radius without and with the c.m. projection (the set of parameters for the nucleon and meson masses and for the coupling constants are taken from Ref. [14], but disregarding the ρ meson and the eletromagnetic field). In Table II, we show the effect of the c.m. correction over the total energy, without the meson contributions, i.e., only the nucleonic degrees of freedom are taken in to account [3], together with the usual harmonic oscillator approximation [13], and also including the correction computed from $\langle P_A^2/2AM\rangle$. From Table II it is clear that the last two corrections are similar and not very different from the Peierls-Yoccoz correction without the meson degrees of freedom. Let us remember that the Peierls-Yoccoz method gives us not only the energy correction but also a translationally invariant wave function for the system.

It is worthwhile to note that the inclusion of the mesonic contribution makes the system too much bounded in comparison with the case where just the fermionic contribution is explicitly taken in to account. However, with a slight modification of model parameters, we are able to obtain reasonable results for the energy and charge radius, as shown in Table III, in which the experimental results are also displayed. For comparison within our calculation, we have extracted the proton form factor contribution from the experimental charge radius using the prescription given in equation (6.2), Ref. [13].

We must stress that the results shown in Table III are not obtained from a careful fitting of the model parameters, which should be done only after the inclusion of other mesons, as well as nonlinear terms in the original Lagrangian. Formally, these terms can be readily included but then the calculations become more involved.

TABLE II. Ground-state energy, E, without the c.m. correction for the three double-closed shell nuclei considered in this work and with the c.m. correction E_{proj} but not considering the meson degrees of freedom. Also shown is the energy with the c.m. correction, E_{harm}, calculated in the harmonic oscillator approximation and the energy corrected just by the subtraction of $\langle P_A^2/2AM\rangle$.

Nucleus	E[MeV]	E_{proj}[MeV]	E_{harm}[MeV]	$E_{P_A^2/2AM}$[MeV]
^{4}He	−4.85	−18.07	−24.22	−16.35
^{16}O	−94.63	−107.87	−106.83	−104.92
^{40}Ca	−331.32	−342.56	−340.31	−339.84

TABLE III. Ground-state energy E_{proj} for the three double-closed shell nuclei considered in this work and charge radius $\langle r^2\rangle^{1/2}_{\text{proj}}$ with the c.m. corrections included, compared to the experimental results. The figures were obtained using the values $g_s = 10.45$, $g_v = 13.82$, and $m_s = 522$ MeV, as compared to the values $g_s = 10.47$, $g_v = 13.80$, and $m_s = 520$ MeV from [14].

Nucleus	E_{proj}[MeV]	E_{exp}[MeV]	$\langle r^2\rangle^{1/2}_{\text{proj}}$[fm]	$\langle r^2\rangle^{1/2}_{\text{exp}}$[fm]
^{4}He	−53.50	−28.30	2.01	1.57
^{16}O	−158.50	−127.68	2.60	2.61
^{40}Ca	−339.14	−338.00	3.36	3.39

VI. CONCLUSIONS

We have computed the center-of-mass correction in the binding energy and charge radius for spherical $N = Z$ nuclei using the well known Peierls-Yoccoz projection method applied to the Hartree solution of the Walecka σ-ω model. Although no explicit reference has to be made to the mesonic states in the Hartree approximation, we have chosen coherent states to describe meson degrees of freedom. Those states are then completely determined in this approximation and this allows us to obtain the nucleonic as well as the mesonic center-of-mass motion correction. The numerical results show a very important contribution from the mesons to the final binding energies and a modest but still noticeable contribution to the charge radius, as compared to the case where only the nucleonic c.m. correction is taken into account or to the situation where no correction is done. It is known that the Peierls-Yoccoz projection suffers from the so-called mass parameter problem which can be circumvented by using the Peierls-Thouless or the so-called variation-after-projection method [10]. Both are technically difficult to implement but the latter might be feasible in systems of nucleons and mesons, at least approximately. However, it was shown in Ref. [15] that some observables, calculated in $\vec{p} = 0$ states, do not suffer from the Peierls-Yoccoz mass problem. We intend to perform a partial variation-after-projection in a restricted meson space but do not expect large discrepancies for oxygen and calcium, whose number of particles is already large, so that quantum fluctuations are expected to be smaller.

Another important feature of our result is the fact that the c.m. correction, including the mesons, makes the system too much bounded. This is expected as long as the model parameters were chosen to reproduce some aspects of finite nuclei without that correction. We have also shown that a few percent change in the coupling constants can bring the total energy and rms radius close to the experimental values, at least for the ^{16}O and ^{40}Ca cases. For ^{4}He the results are still too far from the desirable using our proposed values, but this is true even when no c.m. correction is included and using the original parametrization for that nucleus. Furthermore, the energy correction in the ^{4}He case is relatively large irrespective to the approximation used to extract the CM motion, so we believe that for this light mass region the projection after variation procedure may not be applicable.

In short, we may say that, if we want to take into account the mesons in the center-of-mass correction applied to a relativistic model for the nucleus, a new set of parameters must be found in order to reproduce some basic nuclear properties as the binding energy and radius. Once this is achieved, it would be interesting to obtain other important nuclear properties as, e.g., the eletromagnetic form factors and spectroscopic factors. The model and the techniques explored in this paper would provide a good opportunity to obtain those observables. This work is in progress.

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