Quasiparticle time blocking approximation in coordinate space as a model for the damping of the giant dipole resonance

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The quasiparticle time blocking approximation (QTBA) is presented as a model for the description of natural-parity excitations in open-shell nuclei. Most attention is paid to the question of the damping of the giant dipole resonance. Within the model pairing correlations, two-quasiparticle ($2q$), and $2q\otimes$phonon configurations are included. Thus the QTBA is an extension of the quasiparticle random phase approximation to include quasiparticle-phonon coupling. Calculational formulas are presented for the case of neutral natural-parity excitations in spherical nuclei. The main equations are written in the coordinate representation that allows to take into account single-particle continuum completely. The QTBA is applied to describe $E1$ photoabsorption cross sections in nuclei ^{116,120,124}Sn. It has been obtained that the $2q\otimes$phonon configurations provide noticeable fragmentation of the giant dipole resonance resulting in the appearance of a significant spreading width. The results are compared with available experimental data.

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I. INTRODUCTION

Theoretical description of giant multipole resonances (GMRs) and resonance structures in magic and open-shell nuclei has a long history and remains a problem of current importance. The main tools in solving this problem within the framework of microscopic approach are the random phase approximation (RPA) and the quasiparticle RPA (QRPA) which is a straightforward generalization of the RPA to include pairing correlations. However, despite the fact that these models, in principle, enable one to reproduce experimental mean energies and total strengths of the GMRs, they fail to describe the damping of the resonances and their fine structure (see, e.g., Ref. [1]). One of the reasons is that RPA and QRPA do not provide a mechanism producing spreading width $\Gamma^{\downarrow}$ which gives a considerable contribution in the total widths of the GMRs.

The simplest mechanism of this type is the coupling of the quasiparticles to phonons being superpositions of the one-particle-one-hole ($1p1h$) or the two-quasiparticle ($2q$) configurations. As applied to structure of the even-even nuclei, the concept of the quasiparticle-phonon coupling (QPC) (see Ref. [2]) enables one to take into account $1p1h\otimes$phonon and $2q\otimes$phonon configurations in addition to $1p1h$ and $2q$ ones incorporated within the RPA and the QRPA. A series of models has been developed to go beyond the RPA and the QRPA by means of inclusion of this mechanism (see Refs. [1,3–6], and references therein). Recently, a new model has been formulated (see Ref. [7]) in which pairing correlations, $2q$, and $2q\otimes$phonon configurations are included. This model is based on the Green function (GF) formalism that mainly determines its name: quasiparticle time blocking approximation (QTBA). On one hand, the QTBA is an extension of the QRPA in which $2q\otimes$phonon configurations are incorporated, while more complicated intermediate states are blocked. On the other hand, it is a generalization of the method of chronological decoupling of diagrams (MCDD) developed in Ref. [4] to include $1p1h\otimes$phonon configurations in the case of the even-even nuclei without pairing.

The first aim of this paper is to present calculational formulas obtained from the general ones of the model [7] making use of certain approximations in the case of neutral natural-parity excitations in the spherically symmetric system. In particular, in the formulas presented zero-range forces are adopted as an effective interaction and the Bardeen-Cooper-Schrieffer (BCS) approximation is used to determine quasiparticle energies and wave functions. The equations obtained are a system of coupled equations corresponding to the excitations in particle-hole, particle-particle, and hole-hole channels. The basic equations are written in the coordinate representation that allows us to take into account the single-particle continuum completely. Notice that inclusion of the continuum together with the QPC mechanism is of particular importance to describe correctly the damping of the GMRs.

Our second aim is to test the QTBA in calculations of electric dipole excitations in the nuclei with pairing. With that end in view we have chosen tin isotopes ^{116,120,124}Sn. We present $E1$ photoabsorption cross sections calculated within QRPA and QTBA. Calculated integral characteristics of the giant dipole resonance (GDR) are compared with the experimental data.

The paper is organized as follows. In Sec. II the general scheme of the approach is presented. In Sec. III the basic equations of the QTBA are transformed to channel form in the coordinate space. Formulas for the correlated propagator of the QTBA in this representation are obtained in Sec. IV. In Sec. V we describe our calculational scheme and present the results obtained for the photoabsorption cross sections in three tin isotopes. Conclusions are drawn in the last section. In Appendix A we present derivation of the QTBA equations in terms of the partial components in the case of neutral natural-parity excitations. In Appendix B formulas for the correlated

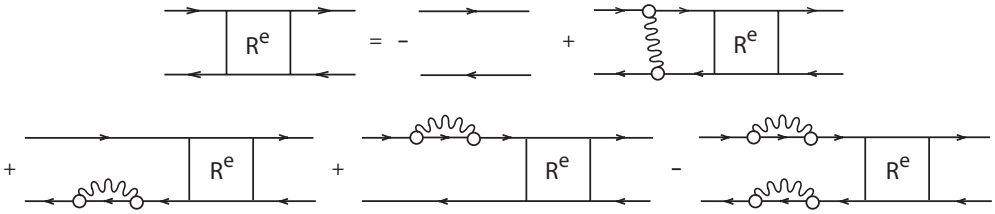

FIG. 1. Equation for the response function R^e determining correlated QTBA propagator. Here conventional notations are used: solid lines with arrows denote generalized single-particle Green functions (including both the normal and the anomalous ones), wavy lines represent phonon propagators, and small circles denote amplitudes of the quasiparticle-phonon interaction. The first term on the right-hand side corresponds to the uncorrelated QRPA propagator.

QTBA propagator in terms of the reduced matrix elements are drawn. In Appendix C the equations for the reduced matrix elements of the amplitude of quasiparticle-phonon interaction are obtained.

II. GENERAL FRAMEWORK

The basic quantity, which determines the physical observables in the QTBA, is the nuclear polarizability $\Pi(\omega)$. More precisely, it determines distribution of the transition strength caused by an external field V^0. In the representation of the single-quasiparticle basis functions, which will be specified below, $\Pi(\omega)$ is defined as

$$\Pi(\omega) = -\frac{1}{2}\sum_{1234}(eV^0)^*_{21}\, R^{\rm eff}_{12,34}(\omega)(eV^0)_{43}, \qquad (1)$$

where e is the effective charge operator, ω is complex energy variable, $R^{\rm eff}(\omega)$ is the effective response function. $R^{\rm eff}(\omega)$ is a solution of the following Bethe-Salpeter equation:

$$R^{\rm eff}_{12,34}(\omega) = A_{12,34}(\omega) - \sum_{5678} A_{12,56}(\omega)\mathcal{F}_{56,78}R^{\rm eff}_{78,34}(\omega), \qquad (2)$$

where $A(\omega)$ is a correlated propagator, $\mathcal{F}$ is an amplitude of the effective interaction. In particular, the strength function $S(E)$ which is frequently used for the description of nuclear excitations is expressed in terms of the polarizability as

$$S(E) = -\frac{1}{\pi}{\rm Im}\Pi(E + i\Delta), \qquad (3)$$

where Δ is a smearing parameter. Connection of this quantity with energies ω_n and probabilities $B_n(V^0) = |\langle n|V^{0\dagger}|0\rangle|^2$ of the excitations is determined by the relation

$$\lim_{\Delta\to+0} S(E) = \sum_{n\neq 0}[B_n(V^0)\delta(E-\omega_n) - B_n(V^{0\dagger})\delta(E+\omega_n)]. \qquad (4)$$

Equation (2) for the effective response function is quite general. In Ref. [7] it was shown that for the Fermi systems with and without pairing correlations it has the same form if we use generalized Green function formalism in which for the systems with pairing the normal and the anomalous GFs are treated in a unified way in terms of the components of generalized GFs in a doubled space. Within the framework of this approach the physical content of the model is determined by the choice of the propagator $A(\omega)$. In particular, the QRPA equation for the $R^{\rm eff}(\omega)$ has the same form Eq. (2) with uncorrelated propagator taken instead of $A(\omega)$. The equations defining $A(\omega)$ within the QTBA are given in Ref. [7]. In the present paper we consider the version of QTBA in which the correlated propagator includes contributions of the $2q$ and $2q\otimes$phonon configurations. Within this model $A(\omega)$ is determined by the response function R^e which is a solution of the equation shown in Fig. 1. Namely, $A(\omega)$ is the main term of the formal decomposition $R^e(\omega) = A(\omega) + B$, where B is the remainder term which is absorbed in the renormalization procedure (see [7] for details).

To obtain the calculational formulas corresponding general equations of the above-mentioned model we have to determine, as a first step, the single-quasiparticle basis functions $\psi_1(y)$ which form the matrix representation of the theory. For the Fermi systems with pairing these functions are defined in the doubled space spanned by the coordinates $y = \{x, \chi\}$, where the symbol $x = \{\mathbf{r}, \sigma, \tau\}$ includes the spatial coordinate $\boldsymbol{r}$, the spin σ, and the isospin τ variables, $\chi = \pm 1$ is an additional index introduced for denoting the different components of the single-quasiparticle functions in the doubled coordinate space. Index $1 = \{\lambda_1, \eta_1\}$ of the doubled configuration space includes index λ_1 of the usual single-particle configuration space and the index $\eta_1 = \pm 1$ which is the sign of the eigenvalue corresponding to $\psi_1(y)$, i.e., $\mathcal{H}\psi_1 = E_1\psi_1$ where $\mathcal{H}$ is single-quasiparticle Hamiltonian, $E_1 = \eta_1 E_{\lambda_1}$, $E_{\lambda_1} = |E_1|$. In the case of the spherically symmetric system, we are interested in, the index λ can be represented by the following set: $\lambda = \{(\lambda), m\}$ where $(\lambda) = \{\tau_\lambda, n, l, j\}$, m is the projection of the total angular momentum, and we have $E_\lambda = E_{(\lambda)}$.

We will use the approximation corresponding to so-called canonical basis representation of the functions $\psi_1(y)$ (see Ref. [8] for details). To determine functions $\psi_1(y)$ within this approximation let us note that in the matrix form the Hamiltonian $\mathcal{H}$ reads

$$\mathcal{H} = \begin{pmatrix} h-\mu & \Delta \\ -\Delta^* & \mu - h^* \end{pmatrix}, \qquad (5)$$

where $h = h(x, x')$ is the single-particle Hamiltonian, $\Delta = \Delta(x, x')$ is the operator of the pairing field, μ is the chemical potential. Let $\{\varphi_\lambda(x)\}$ be the complete set of orthonormal eigenfunctions of the Hamiltonian $h(x, x')$: $h\varphi_\lambda = \varepsilon_\lambda\varphi_\lambda$. We will assume that the operator $\Delta(x, x')$ has the canonical form in the same basis $\{\varphi_\lambda(x)\}$ that corresponds to the state-dependent version of the BCS approximation (see, e.g., Ref. [9]), or to the so-called approximation of the diagonal pairing. In this case

for the spherically symmetric system we have

$$\left.\begin{aligned}\psi_{\lambda,+}(x,+)&=u_\lambda\varphi_\lambda(x),\quad \psi_{\lambda,+}(x,-)=(-1)^{l+j+m}v_\lambda\varphi^*_{\bar\lambda}(x),\\ \psi_{\lambda,-}(x,-)&=u_\lambda\varphi^*_\lambda(x),\quad \psi_{\lambda,-}(x,+)=(-1)^{l+j+m}v_\lambda\varphi_{\bar\lambda}(x),\end{aligned}\right\}, \tag{6}$$

where $\bar\lambda=\{(\lambda),-m\}$, v_λ and u_λ are real numbers which satisfy the following conditions: $u_\lambda=\sqrt{1-v_\lambda^2}\geqslant 0$, $v_\lambda^2=v_{\bar\lambda}^2\leqslant 1$ [see Eqs. (A23)–(A26) in Appendix A for an explicit definition of v_λ and u_λ within the BCS approximation]. The choice of the phase factors is determined by the formulas

$$\varphi_\lambda(x)=\delta_{\tau_\lambda,\tau}R_{(\lambda)}(r)\,\phi_{jlm}(\boldsymbol{n},\sigma), \tag{7}$$

$$\phi_{jlm}(\boldsymbol{n},\sigma)=\sum_\mu(l\mu\tfrac{1}{2}\sigma|jm)Y_{l\mu}(\boldsymbol{n}), \tag{8}$$

$$\varphi_\lambda(x)=(-1)^{l+j+m+\frac{1}{2}+\sigma}\,\varphi^*_{\bar\lambda}(\bar x), \tag{9}$$

where $\boldsymbol{n}=\boldsymbol{r}/r$, $\bar x=\{\mathbf{r},-\sigma,\tau\}$.

In applications of the theory it is convenient to use another basis functions which differ from the functions (6) by a unitary transformation. Let us introduce a matrix

$$\begin{aligned}\mathcal{O}_{12}&=\mathcal{O}_{\lambda_1\eta_1,\lambda_2\eta_2}\\&=\delta_{\eta_1,\eta_2}\left[\delta_{\eta_1,+1}\,\delta_{\lambda_1,\lambda_2}+(-1)^{l_1+j_1-m_1}\,\delta_{\eta_1,-1}\,\delta_{\lambda_1,\bar\lambda_2}\right].\end{aligned} \tag{10}$$

This matrix is real and orthogonal, and consequently it is unitary. So wave functions $\tilde\psi_1(y)$ defined through the single-quasiparticle basis functions $\psi_1(y)$ by the formula

$$\tilde\psi_1(y)=\sum_2\mathcal{O}_{12}\psi_2(y) \tag{11}$$

also form a complete set of the orthonormal functions. We will use just the set $\{\tilde\psi_1(y)\}$ as the set of basis functions. This does not lead to an inconsistency since the single-particle GF $\tilde G(\varepsilon)=(\varepsilon-\mathcal{H})^{-1}$ is diagonal both in $\{\psi_1(y)\}$ and in $\{\tilde\psi_1(y)\}$ representation, and hence the formulas for the correlated propagator (see Ref. [7]) are the same in both representations.

To describe dynamics of the system and to calculate the polarizability and the strength function we start from the equation for linear response matrix (LRM) Λ. Notice that in the present work the term LRM is used instead of the frequently used one "density matrix variation in an external field" because it is more correct in our notations. The equation for Λ in the coordinate representation is obtained by the convolution of the equation for effective response function $R^{\rm eff}$ with an operator of the renormalized external field eV^0:

$$\begin{aligned}\Lambda(y_1,y_2;\omega)&=\Lambda^0(y_1,y_2;\omega)\\&\quad-\int dy_3\,dy_4\,dy_5\,dy_6\,A(y_1,y_2;y_3,y_4;\omega)\\&\quad\times\mathcal{F}(y_3,y_4;y_5,y_6)\,\Lambda(y_5,y_6;\omega),\end{aligned} \tag{12}$$

where

$$\Lambda(y_1,y_2;\omega)=-\sum_{1234}\tilde\psi_1^*(y_1)\tilde\psi_2(y_2)R^{\rm eff}_{12,34}(\omega)(eV^0)_{43}, \tag{13}$$

$$\begin{aligned}\Lambda^0(y_1,y_2;\omega)&=-\int dx_3\,dx_4[A(y_1,y_2;x_3+,x_4+;\omega)\\&\quad-A(y_1,y_2;x_4-,x_3-;\omega)]\tilde V^0(x_4,x_3).\end{aligned} \tag{14}$$

It is assumed that the correlated propagator of the model $A(\omega)$ is initially calculated in configuration space and then is transformed to coordinate space:

$$\begin{aligned}&A(y_1,y_2;y_3,y_4;\omega)\\&\quad=\sum_{1234}\tilde\psi_1^*(y_1)\tilde\psi_2(y_2)\,\tilde\psi_3(y_3)\,\tilde\psi_4^*(y_4)\,A_{12,34}(\omega).\end{aligned} \tag{15}$$

Components of the external field are

$$\begin{aligned}\tilde V^0(x_1,x_2)&=\tilde V^0(x_1+,x_2+)=-\tilde V^0(x_2-,x_1-)\\&=\sum_{12}\tilde\psi_1(x_1+)\tilde\psi_2^*(x_2+)(eV^0)_{12},\end{aligned} \tag{16}$$

and it is supposed that $\tilde V^0(x_1+,x_2-)=\tilde V^0(x_1-,x_2+)=0$.

In terms of the LRM the Eq. (1) for the polarizability reads

$$\Pi(\omega)=\int dx_1\,dx_2\,\tilde V^{0*}(x_2,x_1)\,\Lambda(x_1+,x_2+;\omega). \tag{17}$$

To determine the general form of the effective interaction in our approach let us note that within a self-consistent scheme the amplitude $\mathcal{F}$ in Eq. (12) can be defined as a second order functional derivative of some energy density functional $\mathcal{E}[\mathcal{R}]$ with respect to the extended density matrix $\mathcal{R}$ (see, e.g., Refs. [8,9]). If $\mathcal{E}[\mathcal{R}]$ is usual functional of the Hartree-Fock-Bogoliubov theory built up on the basis of the Hamiltonian which includes only two-particle density-independent interaction with the antisymmetrized amplitude $w^{(2)}$, we have

$$\begin{aligned}\mathcal{F}(y_1,y_2;y_3,y_4)&=\tfrac{1}{2}\delta_{\chi_1,\chi_2}\delta_{\chi_3,\chi_4}\\&\quad\times\big(\delta_{\chi_1,\chi_3}\left[\delta_{\chi_1,+1}\mathcal{F}^+(x_1,x_2;x_3,x_4)\right.\\&\quad\left.+\delta_{\chi_1,-1}\mathcal{F}^+(x_2,x_1;x_4,x_3)\right]\\&\quad-\delta_{\chi_1,-\chi_3}\left[\delta_{\chi_1,+1}\mathcal{F}^+(x_1,x_2;x_4,x_3)\right.\\&\quad\left.+\delta_{\chi_1,-1}\mathcal{F}^+(x_2,x_1;x_3,x_4)\right]\big)\\&\quad+\delta_{\chi_1,-\chi_2}\,\delta_{\chi_3,-\chi_4}\delta_{\chi_1,\chi_3}\\&\quad\times\left[\delta_{\chi_2,+1}\mathcal{F}^-(x_1,x_2;x_3,x_4)\right.\\&\quad\left.+\delta_{\chi_2,-1}\mathcal{F}^-(x_3,x_4;x_1,x_2)\right],\end{aligned} \tag{18}$$

where

$$\begin{aligned}\mathcal{F}^+(x_1,x_2;x_3,x_4)&=w^{(2)}(x_2,x_3;x_1,x_4),\\\mathcal{F}^-(x_1,x_2;x_3,x_4)&=\tfrac{1}{2}\,w^{(2)}(x_1,x_2;x_3,x_4).\end{aligned} \tag{19}$$

In what follows we assume that Eq. (18) is fulfilled for the interaction $\mathcal{F}$, however Eqs. (19) are not supposed to be fulfilled. In other words, the amplitudes $\mathcal{F}^+$ and $\mathcal{F}^-$ will be considered to be independent, and it will be supposed that no other independent components are contained in the amplitude $\mathcal{F}$. These assumptions correspond to the approximation adopted

in the Theory of Finite Fermi Systems with pairing correlations (TFFSPC) [10].

III. TRANSFORMATION OF THE EQUATION FOR LINEAR RESPONSE MATRIX TO CHANNEL FORM

The LRM defined by Eq. (13) contains information about excitations of the initial system in three different channels corresponding to the transitions to the states of the final systems with different numbers of particles. Suppose that the number of particles in the ground state of the initial system is conserved and is equal to N_0. Let N be the number of particles in the final system. Then, in accordance with the standard terminology, we have (i) ph channel if $N = N_0$; (ii) pp channel if $N = N_0 + 2$; (iii) hh channel if $N = N_0 - 2$. Notice that it is not necessary to introduce hp channel explicitly because of symmetry of the LRM and other quantities.

Let us introduce the channel index $c \in \{ph, pp, hh\}$ and define the projection operators $\Xi^{(c)}$:

$$\Xi^{(ph)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3,+1}\,\delta_{\chi_4,+1}\,\delta(x_1, x_3)\delta(x_2, x_4), \quad (20)$$

$$\Xi^{(pp)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3,+1}\delta_{\chi_4,-1}(-1)^{\frac{1}{2}+\sigma_2}\delta(x_1, x_3)\delta(\bar{x}_2, x_4), \quad (21)$$

$$\Xi^{(hh)}(x_1, x_2; y_3, y_4) = \delta_{\chi_3,-1}\delta_{\chi_4,+1}(-1)^{\frac{1}{2}+\sigma_1}\delta(\bar{x}_1, x_3)\delta(x_2, x_4). \quad (22)$$

The sense of these operators is obvious: acting on any quasiparticle operator they cut out its components with fixed χ values. Thus applying each of these projectors to LRM and correlated propagator we obtain the following components:

$$\Lambda^{(c)}(x_1, x_2; \omega) = \int dy_3\, dy_4\, \Xi^{(c)}(x_1, x_2; y_3, y_4)\, \Lambda(y_3, y_4; \omega), \quad (23)$$

$$\Lambda^{0\,(c)}(x_1, x_2; \omega) = \int dy_3\, dy_4\, \Xi^{(c)}(x_1, x_2; y_3, y_4)\, \Lambda^0(y_3, y_4; \omega), \quad (24)$$

$$A^{(c,ph)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5\, dy_6\, \Xi^{(c)}(x_1, x_2; y_5, y_6) \times [A(y_5, y_6; x_3+, x_4+; \omega) - A(y_5, y_6; x_4-, x_3-; \omega)], \quad (25)$$

$$A^{(c,pp)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5\, dy_6\, \Xi^{(c)}(x_1, x_2; y_5, y_6) \times A(y_5, y_6; x_3+, \bar{x}_4-; \omega)(-1)^{\frac{1}{2}+\sigma_4}, \quad (26)$$

$$A^{(c,hh)}(x_1, x_2; x_3, x_4; \omega) = \int dy_5\, dy_6\, \Xi^{(c)}(x_1, x_2; y_5, y_6) \times A(y_5, y_6; \bar{x}_3-, x_4+; \omega)(-1)^{\frac{1}{2}+\sigma_3}, \quad (27)$$

where the second channel indices in the Eqs. (25)–(27) are fixed by the χ-indices of the propagators on the right-hand sides of these equations. Let us also denote

$$\mathcal{F}^{(c,c')}(x_1, x_2; x_3, x_4) = \delta_{c,c'}\tilde{\mathcal{F}}^{(c)}(x_1, x_2; x_3, x_4) + \mathcal{F}^{rest\,(c,c')}(x_1, x_2; x_3, x_4), \quad (28)$$

$$\tilde{\mathcal{F}}^{(ph)}(x_1, x_2; x_3, x_4) = \mathcal{F}^{+}(x_1, x_2; x_3, x_4), \quad (29)$$

$$\tilde{\mathcal{F}}^{(pp)}(x_1, x_2; x_3, x_4) = (-1)^{\frac{1}{2}+\sigma_2+\frac{1}{2}+\sigma_4}\mathcal{F}^{-}(x_3, \bar{x}_4; x_1, \bar{x}_2), \quad (30)$$

$$\tilde{\mathcal{F}}^{(hh)}(x_1, x_2; x_3, x_4) = (-1)^{\frac{1}{2}+\sigma_1+\frac{1}{2}+\sigma_3}\mathcal{F}^{-}(\bar{x}_1, x_2; \bar{x}_3, x_4). \quad (31)$$

The additional restoring amplitude $\mathcal{F}^{rest\,(c,c')}$ is introduced in Eq. (28) for the purpose of "forced consistency" and will be specified in the following (see Appendix A).

Making use of the definitions (18), (20)–(31) one can rewrite Eq. (12) in the channel form

$$\Lambda^{(c)}(x_1, x_2; \omega) = \Lambda^{0\,(c)}(x_1, x_2; \omega) - \sum_{c'c''}\int dx_3\, dx_4\, dx_5\, dx_6 \times A^{(c,c')}(x_1, x_2; x_3, x_4; \omega) \times \mathcal{F}^{(c',c'')}(x_3, x_4; x_5, x_6)\, \Lambda^{(c'')}(x_5, x_6; \omega), \quad (32)$$

where a summation is performed over all the channels.

In what follows we shall restrict our consideration to the spherically symmetric systems with zero-range effective interaction. The latter restriction does not allow to apply some of the formulas presented here to the self-consistent calculations in general case. However, in practice the exact self-consistent residual interaction is often approximated in such calculations by the Landau-Migdal ansatz (see, e.g., Ref. [11]). In this approximation all our formulas remain valid. Within these limitations it is convenient to decompose all the quantities in terms of spherical tensor operators. In this case Eq. (32) for the neutral excitations with the total angular momentum J and the parity $(-1)^L$ can be reduced to the following equation for the partial components (see Appendix A and Sec. IV for the notations and details):

$$\Lambda^{(c)}_{JLS\tau}(r; \omega) = \Lambda^{0\,(c)}_{JLS\tau}(r; \omega) - \sum_{L'S'\tau'c'}\sum_{L''S''\tau''c''}\int_0^\infty dr'\, r'^2 \times \int_0^\infty dr''\, r''^2 A^{J\,(c,c')}_{LS\tau,L'S'\tau'}(r, r'; \omega) \times \mathcal{F}^{J\,(c',c'')}_{L'S'\tau',L''S''\tau''}(r', r'')\, \Lambda^{(c'')}_{JL''S''\tau''}(r''; \omega), \quad (33)$$

where

$$\Lambda^{0(c)}_{JLS\tau}(r; \omega) = -\sum_{L'S'c'}\int_0^\infty dr' r'^2 A^{J(c,c')}_{LS\tau,L'S'\tau}(r, r'; \omega)\tilde{V}^{0(c')}_{JL'S'\tau}(r'), \quad (34)$$

$$\tilde{V}^{0\,(c)}_{JLS\tau}(r) = \delta_{c,ph}\tilde{V}^{0}_{JLS\tau}(r). \quad (35)$$

In terms of the partial components the Eq. (17) for the polarizability takes the form

$$\Pi_J(\omega) = (2J+1)\sum_{LS\tau c}\int_0^\infty dr\, r^2 \tilde{V}^{0(c)}_{JLS\tau}(r)\Lambda^{(c)}_{JLS\tau}(r;\omega). \quad (36)$$

IV. CORRELATED PROPAGATOR IN THE COORDINATE SPACE

Formulas for the partial components of the effective interaction $\mathcal{F}^{J\,(c,c')}_{LS\tau,L'S'\tau'}(r,r')$ entering Eq. (33) in the case of the Landau-Migdal zero-range force, which is used in our calculations, are drawn in Appendix A. Let us obtain an explicit formula for the partial components of the correlated propagator $A^{J\,(c,c')}_{LS\tau,L'S'\tau'}(r,r';\omega)$ in terms of the reduced matrix elements $A^J_{[12,34]}(\omega)$ which in the case of the spherically symmetric system are determined by the formula

$$A^J_{[12,34]}(\omega) = \sum_{m_1m_2m_3m_4}(2J+1)A_{12,34}(\omega)(-1)^{j_2-m_2} \times \begin{pmatrix} j_1 & j_2 & J \\ m_1 & -m_2 & M\end{pmatrix}(-1)^{j_4-m_4}\begin{pmatrix} j_3 & j_4 & J \\ m_3 & -m_4 & M\end{pmatrix}, \quad (37)$$

where $1=\{[1],m_1\}$, $[1]=\{(1),\eta_1\}$, $(1)=(\lambda_1)=\{\tau_1,n_1,l_1,j_1\}$. Notice that with these abbreviated notations we have $u_{(1)}=u_{\lambda_1}$, $v_{(1)}=v_{\lambda_1}$, $\varepsilon_{(1)}=\varepsilon_{\lambda_1}$, $E_{(1)}=E_{\lambda_1}$. We will use the antisymmetric form of equations for the correlated propagator (see Ref. [7] for details) which allows to reduce the dimensions of matrices entering these equations. In this case the following relations are fulfilled:

$$A^J_{[12,34]}(\omega) = -\eta_1\eta_2(-1)^{J+l_1-l_2+j_1-j_2}A^J_{[\bar{2}\bar{1},34]}(\omega) = -\eta_3\eta_4(-1)^{J+l_3-l_4+j_3-j_4}A^J_{[12,\bar{4}\bar{3}]}(\omega), \quad (38)$$

where $[\bar{1}]=\{(1),-\eta_1\}$.

It is easy to show that functions $\tilde{\psi}_1(y)$ defined by Eqs. (10) and (11) obey the equalities

$$\tilde{\psi}_1(x;+) = w_1\varphi_{\lambda_1}(x), \quad (-1)^{\frac{1}{2}+\sigma}\tilde{\psi}_1(\bar{x};-) = \eta_1 w_{\bar{1}}\varphi_{\lambda_1}(x), \quad (39)$$

where

$$w_1 = w_{[1]} = \delta_{\eta_1,+1}u_{(1)} + \delta_{\eta_1,-1}v_{(1)}. \quad (40)$$

Making use of Eqs. (7), (8), (15), (20)–(22), (25)–(27), (38), and (39), we obtain from Eq. (A10) the following ansatz:

$$A^{J\,(c,c')}_{LS\tau,L'S'\tau'}(r,r';\omega) = (1+\delta_{c',ph})\delta_{\tau,\tau'} \times \sum_{[1234]}\delta_{\tau_1,\tau}\delta_{\tau_2,\tau}\delta_{\tau_3,\tau'}\delta_{\tau_4,\tau'}\theta_{(21)}\theta_{(43)} \times A^{J\,(c,c')\,LS,L'S'}_{[12,34]}(r,r';\omega), \quad (41)$$

where

$$A^{J\,(c,c')\,LS,L'S'}_{[12,34]}(r,r';\omega) = R_{(1)}(r)\,R_{(2)}(r)R_{(3)}(r')\,R_{(4)}(r')T^{(J)\,LS,L'S'}_{(12,34)} \times\left(\alpha^c_{[12]}-\eta_1\eta_2(-1)^S\alpha^c_{[\bar{2}\bar{1}]}\right)A^J_{[12,34]}(\omega) \times\left(\alpha^{c'}_{[34]}-\eta_3\eta_4(-1)^{S'}\alpha^{c'}_{[\bar{4}\bar{3}]}\right), \quad (42)$$

$$T^{(J)LS,L'S'}_{(12,34)} = \frac{1}{2J+1}\langle j_2l_2\|T_{JLS}\|j_1l_1\rangle\langle j_4l_4\|T_{JL'S'}\|j_3l_3\rangle, \quad (43)$$

$$\alpha^{ph}_{[12]} = w_1\,w_2, \quad \alpha^{pp}_{[12]} = \eta_2\,w_1\,w_{\bar{2}}, \quad \alpha^{hh}_{[12]} = \eta_1\,w_{\bar{1}}\,w_2. \quad (44)$$

The reduced matrix elements of the spherical tensor operators entering Eq. (43) are defined in Eq. (A4). Equations (41)–(44) have quite a general form. Explicit formulas for the reduced matrix elements $A^J_{[12,34]}(\omega)$ in the case of the QPC model are drawn in Appendix B.

Theoretically, summation in Eq. (41) is supposed to be over complete ordered set of the states forming the doubled configuration space. This summation is facilitated due to the symmetry defined by Eq. (38). The order-bounding factors are defined as follows: $\theta_{(21)}=1$ if the ordinal number of the state (1) is less than the number of (2) $[(1)<(2)]$, $\theta_{(21)}=\frac{1}{2}$ if $(1)=(2)$, $\theta_{(21)}=0$ if $(1)>(2)$.

However, in our calculational scheme, summation in Eq. (41) is restricted by the discrete and quasidiscrete states entering a valence zone near the Fermi level. It is supposed that the remaining part of the sum can be approximated fairly well by the RPA-like propagator $A^{\,cont}$, which contains transitions from the quasiparticle levels to the continuum. Thus, we use the scheme which is analogous to the ones described in Refs. [5,12]. According to this scheme the total correlated propagator is represented as a sum of two terms:

$$A^{J\,(c,c')}_{LS\tau,L'S'\tau'}(r,r';\omega) = A^{J\,(c,c')\,disc}_{LS\tau,L'S'\tau'}(r,r';\omega) + A^{J\,(c,c')\,cont}_{LS\tau,L'S'\tau'}(r,r';\omega), \quad (45)$$

where

$$\begin{aligned} A^{J\,(c,c')\,disc}_{LS\tau,L'S'\tau'}(r,r';\omega) &= \delta_{c,c'}\delta_{c,ph}\delta_{\tau,\tau'}\sum_{(12)}^{disc}\delta_{\tau_1,\tau}\,\delta_{\tau_2,\tau}\,\theta^b_{(1)}\left[1-\theta^b_{(2)}\right]T^{(J)\,LS,L'S'}_{(12,12)} \\ &\times R_{(1)}(r)\,R_{(2)}(r)R_{(1)}(r')\,R_{(2)}(r')\left[v^2_{(1)}\left(1-v^2_{(2)}\right)\right. \\ &\left.+(-1)^{S+S'}v^2_{(2)}\left(1-v^2_{(1)}\right)\right] \\ &\times\left[\frac{1}{\omega+E_{(1)}+E_{(2)}}-\frac{(-1)^{S+S'}}{\omega-E_{(1)}-E_{(2)}}\right] \\ &+(1+\delta_{c',ph})\,\delta_{\tau,\tau'}\sum_{[1234]}^{wind}\delta_{\tau_1,\tau}\delta_{\tau_2,\tau} \\ &\times\delta_{\tau_3,\tau'}\,\delta_{\tau_4,\tau'}\,\theta_{(21)}\,\theta_{(43)}A^{J\,(c,c')\,LS,L'S'}_{[12,34]}(r,r';\omega), \end{aligned} \quad (46)$$

$$A^{J\,(c,c')\,cont}_{LS\tau,L'S'\tau'}(r,r';\omega) = -\delta_{c,c'}\delta_{c,ph}\delta_{\tau,\tau'}\left\{\sum_{j_1l_1}\sum_{(2)}^{disc}\delta_{\tau_2,\tau}\,v^2_{(2)}R_{(2)}(r)\,R_{(2)}(r')\right.$$

$$\times T^{(J)LS,L'S'}_{(12,12)}\left[\tilde{G}^{nor}_{j_1 l_1 \tau}(r,r';\mu_\tau - E_{(2)} + \omega) + (-1)^{S+S'}\tilde{G}^{nor}_{j_1 l_1 \tau}(r,r';\mu_\tau - E_{(2)} - \omega)\right]$$

$$- \sum_{(12)}^{disc} \delta_{\tau_1,\tau}\,\delta_{\tau_2,\tau}\,\theta_{(21)} R_{(1)}(r)\,R_{(2)}(r) R_{(1)}(r')\,R_{(2)}(r')$$

$$\times T^{(J)LS,L'S'}_{(12,12)}\left[\frac{v^2_{(2)}}{\omega + \mu_\tau - E_{(2)} - \varepsilon_{(1)}} - \frac{v^2_{(1)}}{\omega - \mu_\tau + E_{(1)} + \varepsilon_{(2)}} + (-1)^{S+S'} \times \left(\frac{v^2_{(1)}}{\omega + \mu_\tau - E_{(1)} - \varepsilon_{(2)}} - \frac{v^2_{(2)}}{\omega - \mu_\tau + E_{(2)} + \varepsilon_{(1)}}\right)\right]\Bigg\}. \quad (47)$$

In Eq. (47), $\tilde{G}^{nor}_{jl\tau}(r,r';\varepsilon)$ is the partial component of the normal single-particle GF without pairing. It is calculated via the regular and irregular solutions of the Schrödinger equation that allows to take into account continuum completely on the RPA level (see, e.g., Ref. [13]).

In Eq. (46), $\theta^b_{(1)}$ are the bounding factors defined as $\theta^b_{(1)} = 1$ if $(1) < (\lambda_b)$, $\theta^b_{(1)} = 0$ if $(1) \geqslant (\lambda_b)$, where λ_b is the bottom level of the valence zone which coincides in our calculations with "pairing window", i.e., with zone where the energy gap is not equal to zero (see Sec. V for details). The superscript "*disc*" in the $\sum$ symbols means summation over all discrete and quasidiscrete states of the single-particle spectrum. The superscript "*wind*" means summation over the discrete and quasidiscrete states only inside the "pairing window", i.e., the same summation as in Eq. (A25).

V. CALCULATIONS: DETAILS AND DISCUSSION

As an application of our approach isovector electric dipole excitations in the GDR region have been calculated in semimagic isotopes ^{116,120,124}Sn. Notice that the GDR in these tin isotopes was calculated previously within the framework of different microscopic models (see, e.g., Refs. [5,6,12,14,15]). But only in Ref. [5] were the effects responsible for the spreading width (quasiparticle-phonon coupling) taken into account together with the single-particle continuum producing the escape width. As was mentioned above, both of these effects are important to describe the damping of the GDR. However, effects of the quasiparticle-phonon coupling were included in Ref. [5] within the framework of too restricted model. The treatment of these effects within the QTBA is more comprehensive.

In the present paper we use the non-self-consistent scheme of the QTBA including the single-particle continuum as described in Sec. IV. This calculational scheme is justified for the stable isotopes under consideration since phenomenological systematics of the mean field and of the residual interaction is known for them rather well. In addition, for these isotopes the experimental data on both single-particle and collective excitations are available. In our calculations we started from a description of the independent single-particle motion in the standard phenomenological Woods-Saxon potential. Then the gap equation (A25) was solved for neutron subsystem under the usual condition that the number of particles in valence zone is conserved on average. In our calculations for the tin isotopes this zone ("pairing window") consists of all discrete ($\varepsilon_\lambda < 0$) and quasidiscrete states above the chemical potential μ and of the discrete states below μ starting from $1f_{7/2}$ subshell for neutrons and $1d_{5/2}$ subshell for protons. We emphasize that the same valence zone restricts summations both in the gap equation (A25) and in the equations of Appendix B for the QTBA propagator.

As the quasidiscrete states we chose the discrete states with $\varepsilon_\lambda > 0$ calculated with a box boundary condition and having the last extremum of the radial wave function in the range of nonvanishing values of the discrete spectrum ($\varepsilon_\lambda < 0$) wave functions. According to this criterion the quasidiscrete states are selected quite well if size of the box is not too large. Notice that the partial components of the normal single-particle GF $\tilde{G}^{nor}_{jl\tau}(r,r';\varepsilon)$ entering Eq. (47) were calculated with outgoing wave boundary condition except for the small number of components for which jl-values coincide with jl-values of the quasidiscrete states. For such partial components the box boundary condition was used in order to avoid inconsistency in the calculations.

The parameter c_p in Eq. (A18) was chosen so as to obtain the averaged solution of the gap equation $\bar{\Delta}_\lambda$ to be equal to the averaged empirical value (see Ref. [16]): $\bar{\Delta} = 12\ \mathrm{MeV}/\sqrt{A}$. For ^{120}Sn we have obtained $\bar{\Delta} \approx 1.1$ MeV with $c_p = 0.719$ MeV. This parameter was used for the remaining two nuclei. The quantity ξ in Eq. (A18) is determined as follows: $\xi = \sqrt{\xi_1 \xi_2}$ where $\xi_1 = \mu - \min(\varepsilon_\lambda)$, $\xi_2 = \max(\varepsilon_\lambda) - \mu$.

We assumed that the observable energies of single-particle excitations in the neighboring odd nuclei have to be reproduced in the framework of the mean field plus BCS model. These observable energies (experimental energy differences) are defined by the following equation: $\varepsilon^{\mathrm{exp}}_\lambda = \pm[\mathcal{E}_\lambda(\mathrm{A} \pm 1) - \mathcal{E}_{\mathrm{g.s.}}(\mathrm{A})]$, where $\mathcal{E}_{\mathrm{g.s.}}$ is the ground state energy of the even-even nucleus (core) consisting of the A nucleons, $\mathcal{E}_\lambda$ is the energy of the ground or the excited state of the neighboring odd nucleus consisting of the $A \pm 1$ nucleons. In order to get an agreement with the experimental energies the well depth of central part of the Woods-Saxon potential was slightly varied so as to obtain $\varepsilon^{\mathrm{exp}}_\lambda = \varepsilon_\lambda$ for protons and $\varepsilon^{\mathrm{exp}}_\lambda = \mu_{\tau_\lambda} \pm E_\lambda$ for neutrons where E_λ is connected with ε_λ by Eq. (A24). Thus obtained energies and wave functions form the above mentioned basis set $\{\varepsilon_\lambda, \varphi_\lambda(x)\}$.

In the present calculations we included ground state correlations (GSC) only in the QRPA part of the correlated propagator. Another type of GSC caused by quasiparticle-phonon coupling (GSC/QPC) and originated from backward-going terms in time-ordered diagrams was not incorporated. This means that (i) only the term $\Phi^{J(\mathrm{res})}_{[12,34]}(\omega)$ in Eq. (B13) is accounted for; (ii) associated components of the propagator containing the functions $P^{J(++)}_{[12,34]}(\omega)$, $Q^{J(+-)}_{[12,34]}(\omega)$, and $Q^{J(-+)}_{[12,34]}(\omega)$ [see Eqs. (B1)–(B3), (B17)–(B19)] are excluded from the calculations.

Since our single-particle and single-quasiparticle energies are fitted to the experiment as it was mentioned above,

these energies and corresponding wave functions already contain effectively the admixture of phonons. This phonon contribution should be removed from the mean field, energy gap, and the effective interaction to avoid double counting if the quasiparticle-phonon coupling is included explicitly. To solve this problem we use the method which corresponds to the self-consistent scheme of the QTBA (see Ref. [7]) if the GSC/QPC are not included. In this case the method consists of the replacement of the amplitude $\Phi^{J}_{[12,34]}(\omega)$ in Eq. (B8) by the difference amplitude $\bar{\Phi}^{J(\mathrm{res})}_{[12,34]}(\omega)$ where

$$\bar{\Phi}^{J(\mathrm{res})}_{[12,34]}(\omega) = \Phi^{J(\mathrm{res})}_{[12,34]}(\omega) - \Phi^{J(\mathrm{res})}_{[12,34]}(0). \tag{48}$$

Making use of this subtraction procedure, statical contributions of the quasiparticle-phonon coupling defined by the quantity $\Phi^{J(\mathrm{res})}_{[12,34]}(0)$ are removed from both the mass operator and the effective interaction.

In the calculations of the GDR we neglected the contribution of pp and hh channels that is justified for this type of excitations (see Ref. [10] and discussion below). As the effective interaction in the ph channel we adopted the Landau-Migdal zero-range force [see Eqs. (A15) and (A17)] with microscopically determined nuclear ground state density ρ_0:

$$\rho_0(r) = \frac{1}{4\pi}\sum_{(\lambda)} v^2_\lambda\,(2j_\lambda+1)\,R^2_{(\lambda)}(r). \tag{49}$$

The parameters of this interaction were taken in accordance with the standard set which is usually employed in similar calculations (see, e.g., Ref. [12]), except for parameters f_{ex} and f'_{ex}. The parameter $f'_{ex} = 2.62$ used in our calculations has been changed as compared to one from Ref. [12] in order to reproduce the experimental mean energy of the GDR in ^{120}Sn within QTBA calculation. The parameter f_{ex} was slightly varied from -2.187 to -1.957 in the computation of the energies and amplitudes of low-lying collective 2^+ and 3^- phonons within QRPA in configuration space. This allowed us to fit the energies of these phonons to experimental values. Then the value of f_{ex} was averaged and used in the calculations of the remaining phonons. In the calculations of the GDR the averaged value $f_{ex} = -2.05$ was adopted for all three tin isotopes.

It is well known that the question about spurious isoscalar 1^- state arises in the calculations of E1 excitations if the consistency between the mean field and the residual interaction is broken down or even if the consistency conditions are fulfilled only approximately. In our non-self-consistent calculations the spurious 1^- state has been eliminated using the "forced consistency" method presented in Appendix A. This method was developed and successfully applied in calculations of both the isoscalar and the isovector $E1$ excitations in Refs. [12,17,18]. It turns out to be very efficient if it is necessary to restore the broken consistency as it takes place in our case.

The phonon characteristics were calculated within the QRPA by making use of Eqs. (C1)–(C9). The configuration space was restricted by the "pairing window" defined above. In this calculation the matrix elements of the interaction $\mathcal{F}^{(c)S}_{(12,34)}$ with $S = 1$ were omitted since they do not give significant contribution in the case of low-lying collective modes with natural parity. In addition, we have neglected contribution of pp and hh channels. Notice that the contribution of these channels (so-called dynamical pairing) can be important first of all for the states with positive parity (see, e.g., Refs. [19,20]). It should be taken into account in fully self-consistent QRPA calculations (see Refs. [20,21]), in particular, for proper handling spurious states. However, neglect of the dynamical pairing in our calculations is compensated to some extent by the fit of the phonon energies to the experimental data (see below). In the case of the GDR, the dynamical pairing contribution is effectively included into corrections introduced by our "forced consistency" procedure.

These simplifications decrease dimension of the QRPA matrix in the configuration space by a factor of two. For all three chosen tin isotopes the collective modes with spin and parity 2^+, 3^-, 4^+, 5^-, 6^+, and with energies below the neutron separation energy were included into the phonon space. A mode is assumed to enter the phonon space if its reduced transition probability is more than 10% of the maximal one for fixed spin and parity. This value was taken as an approximate criterion for phonons selection. The energies of the first 2^+ and 3^- phonons were fitted to the experimental values as described above. The so determined phonon space includes 23 phonons for ^{116}Sn and ^{124}Sn, and 22 phonons for ^{120}Sn (see Ref. [22] for more details).

The dipole photoabsorption cross section $\sigma_{E1}(E)$ is the basic observable computed in the present work. This value is expressed via the strength function $S_{E1}(E)$ according to the well known formula

$$\sigma_{E1}(E) = \frac{16\pi^3 e^2}{9\hbar c} E S_{E1}(E). \tag{50}$$

The strength function in turn is simply connected with the polarizability $\Pi_{E1}(\omega)$ [see Eq. (3)]:

$$S_{E1}(E) = -\frac{1}{\pi}\mathrm{Im}\Pi_{E1}(E + i\Delta). \tag{51}$$

As follows from Eq. (36), to determine the value of function $\Pi_{E1}(\omega)$ at a given complex energy variable ω and, consequently, to compute the values of $S_{E1}(E)$ and $\sigma_{E1}(E)$ one has to solve Eq. (33) for the LRM. This equation was solved within the framework of two models: QTBA and QRPA. Following calculational scheme which is usually used in the response function formalism, in the present work we introduced a smearing parameter Δ as an imaginary part of the energy variable ω which imitates contribution of the more complex configurations not incorporated in our approach. In the calculations of GDR the value $\Delta = 250$ keV was used.

Calculated photoabsorption cross sections for three above indicated tin isotopes as well as experimental data are shown in Figs. 2–4. The solid and the dashed curves represent the QTBA and the QRPA correspondingly. Since the smearing parameter taken is not large, our theoretical curves are not so smooth as a single Lorentz line. Nevertheless, in order to compare the calculated cross sections with experimental data we approximated them by the standard Lorentz function

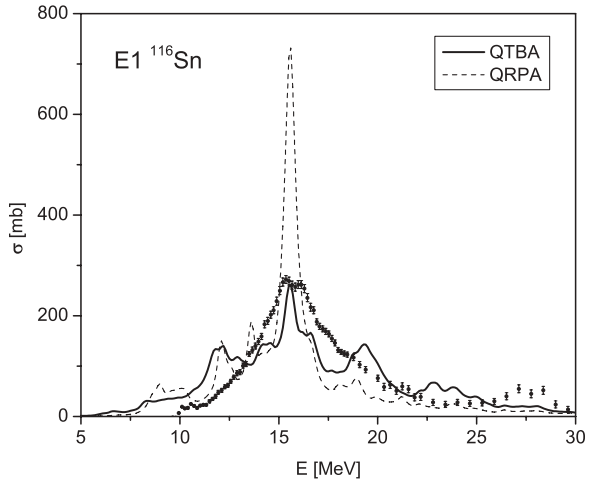

FIG. 2. $E1$ photoabsorption cross section for ^{116}Sn calculated within QRPA (dashed line) and QTBA (solid line). The smearing parameter Δ is equal to 250 keV. Experimental data from Ref. [24] are shown by the black circles.

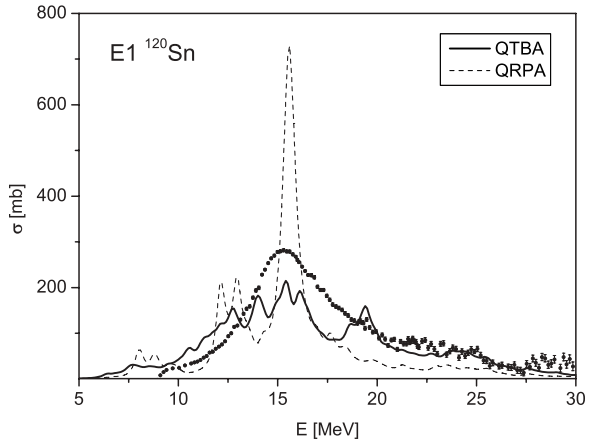

FIG. 3. Same as Fig. 2, but for ^{120}Sn.

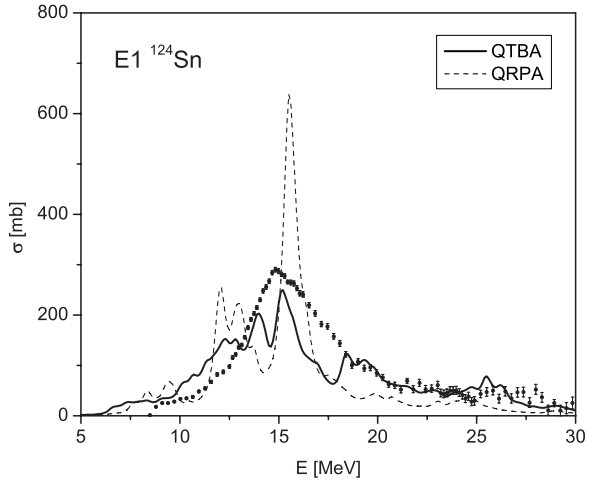

FIG. 4. Same as Fig. 2, but for ^{124}Sn.

TABLE I. Lorentz function parameters of the GDR in ^{116,120,124}Sn obtained within two microscopic approaches for 0–30 MeV energy interval. Values of the depletion of the EWSR are presented as percentages with respect to the corresponding TRK values (see text for more details).

	^{116}Sn		^{120}Sn		^{124}Sn	
	QRPA	QTBA	QRPA	QTBA	QRPA	QTBA
E_0 (MeV)	14.74	15.44	14.65	15.39	14.35	15.10
Γ (MeV)	2.4	4.0	2.7	4.4	2.6	4.4
σ_0 (mb)	452	302	423	288	452	298
EWSR-L (%)	100	112	103	114	104	116
EWSR-I (%)	94	99	97	102	97	102

$\sigma_L(E)$:

$$\sigma_L(E) = \sigma_0 \frac{\Gamma^2 E^2}{\left(E^2 - E_0^2\right)^2 + \Gamma^2 E^2}. \quad (52)$$

The parameters E_0, Γ, and σ_0 in Eq. (52) were obtained by making use of the following condition: the energy-weighted moments m_0, m_{-1}, and m_{-2} of the functions (50) and (52) should coincide. This method is analogous to the one developed in Ref. [23] but in contrast to Ref. [23] we calculated the moments in the finite energy interval 0–30 MeV for which experimental data are available. Parameters of the Lorentz fit are compiled in the Table I. In this table values of the depletion of the energy weighted sum rule (EWSR) are also presented in two forms. Namely, the total areas under Lorentz lines (EWSR-L) and the integrated cross sections in the finite interval (EWSR-I) are given as percentages with respect to the corresponding Thomas-Reiche-Kuhn (TRK) values, i.e.,

$$\text{EWSR-L} = 100\frac{\pi}{2}\sigma_0\Gamma/(59.74\ NZ/A\ \text{MeV mb}), \quad (53)$$

$$\text{EWSR-I} = 100\int_0^{30\text{MeV}} \sigma_{E1}(E)\,dE/(59.74\ NZ/A\ \text{MeV mb}). \quad (54)$$

In Table II experimental Lorentz function parameters of the GDR are shown. Experimental mean energies demonstrate the well known property to decrease against neutron excess. Our QTBA results drawn in Table I reproduce quite well these mean energies. The remaining parameters of the Lorentz function calculated within QTBA, i.e., Γ and σ_0 (and consequently EWSR-L), are also in a reasonable agreement with experimental values. It is worth noting that the EWSR-L values obtained in the QTBA are much more close to the experimental ones as compared to the EWSR-I values which are close to the TRK ones (see below). Obviously, the EWSR-L and the EWSR-I values will coincide with each other only for infinite energy interval.

Consider the results obtained within QTBA and QRPA to analyze effect of the QPC on the integral characteristics of resonances. As one can see from Table I, QTBA gives

TABLE II. Experimental Lorentz function parameters of the GDR. The results are taken from Refs. [24] and [25].

	^{116}Sn		^{120}Sn		^{124}Sn	
	[24]	[25]	[24]	[25]	[24]	[25]
E_0 (MeV)	15.67 ± 0.04	15.57 ± 0.1	15.40 ± 0.04	15.38 ± 0.1	15.18 ± 0.04	15.29 ± 0.1
Γ (MeV)	4.19 ± 0.06	5.21 ± 0.1	4.88 ± 0.06	5.25 ± 0.1	4.81 ± 0.06	4.96 ± 0.1
σ_0 (mb)	266 ± 7	270 ± 5	280 ± 8	284 ± 5	283 ± 8	275 ± 5
EWSR-L (%)	103 ± 3	130 ± 3	123 ± 4	134 ± 3	120 ± 4	120 ± 3

significant increase of the total width as compared to QRPA ($\Gamma_{\mathrm{QTBA}} \gtrsim 1.6\Gamma_{\mathrm{QRPA}}$) owing to contribution of the spreading width $\Gamma^{\downarrow}$. Clearly this result could be expected from physical point of view. Comparison of the calculated values of the widths and the experimental data from Table II demonstrates that inclusion of the QPC within the QTBA is sufficient to describe the damping of the GDR in the considered tin isotopes fairly well.

The EWSR-I values obtained within the QTBA for the investigated energy interval 0–30 MeV are rather close to the TRK ones but again there is a noticeable difference between the QTBA and the QRPA values. However this difference has another source which is the subtraction procedure described above [see Eq. (48)]. It can be rigorously proved that for the version of QTBA in which GSC/QPC are not taken into account and the subtraction procedure is not applied the equality $\mathrm{EWSR\text{-}I}_{\mathrm{QTBA}} = \mathrm{EWSR\text{-}I}_{\mathrm{QRPA}}$ is fulfilled exactly for the infinite energy interval. Notice that the analogous equality is fulfilled between the values of EWSR defined within the MCDD (Ref. [4]) and within the RPA. In our calculations just the subtraction procedure determined by Eq. (48) gives rise to increment of EWSR in the QTBA. Switching off this procedure we have obtained that the values of EWSR-I calculated within the QTBA in the "infinite" energy interval (0–200 MeV in our calculations) are equal to the corresponding QRPA values with sufficiently high accuracy. This result can be considered as a test of our calculational scheme.

There is one more source which leads to the enhancement of the EWSR in our approach. It is the fit of the single-quasiparticle energies to the experimental values which effectively introduces the nontrivial dependence on the angular momentum operator into the mean field. The use of the fit changes EWSR even in the continuum RPA with zero-range forces and local single-particle potential.

The mean energy shift of about 0.7–0.8 MeV obtained in QTBA with respect to QRPA arises also mainly due to the subtraction procedure. Without this subtraction the QTBA mean energies decrease as compared to the QRPA ones by about of 0.2 MeV. Thus, the subtraction procedure results in the significant change of the averaged characteristics of the excited states calculated in the QTBA as compared to the QRPA. On the other hand, as it was indicated above, it ensures elimination of the statical QPC contributions from the mean field and the residual interaction.

Finally, notice that although we have taken into account the most important effects of the QPC, the neglected contributions of the GSC/QPC and of the dynamical coupling to the pp and hh channels may also affect the results. In the case of magic nuclei, the role of the GSC/QPC in the description of nuclear excitations was investigated in a series of papers (see, e.g., Ref. [1] and references therein). The study of these effects in the nuclei with pairing is in progress.

VI. CONCLUSIONS

The quasiparticle time blocking approximation (QTBA) is applied to describe $E1$ excitations in the even-even open-shell spherical nuclei. The QTBA is an extension of the QRPA, in which two-quasiparticle⊗phonon configurations, arising due to the quasiparticle-phonon coupling, are included in addition to pairing correlations and simple two-quasiparticle configurations incorporated in the QRPA. To determine response of the spherically symmetric system against an external field within the QTBA the integral equation for the partial components of the linear response matrix in the coordinate space is solved. Configurations with a particle in the continuum are included into the QRPA part of the response function. This enables us to describe both spreading and escape widths of nuclear excited states. Thus, all the main effects responsible for the damping of the giant resonances in the open-shell nuclei are taken into account in our calculations.

Using the method developed, the isovector $E1$ strength distribution in nuclei ^{116,120,124}Sn has been calculated. Since our main purpose was to test new approach, these first calculations have been performed assuming some additional simplifications of the model: ground state correlations caused by quasiparticle-phonon coupling and dynamical coupling to pp and hh channels were ignored. Moreover, in our calculations the non-self-consistent scheme of the QTBA was used, so the additional procedure to eliminate spurious dipole state has been implemented. Noticeably fragmented giant dipole resonance in the photoabsorption cross section has been obtained for all three investigated tin isotopes. The total width, determining the damping of the resonance, and other integral characteristics of the $E1$ strength distribution have been calculated. The results obtained within the QTBA are in a reasonable agreement with experimental data.

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APPENDIX A: EQUATION FOR PARTIAL COMPONENTS OF THE LRM IN THE CASE OF NEUTRAL NATURAL-PARITY EXCITATIONS

In the present paper we solve Eq. (32) for the LRM in the case of natural-parity excitations in the neutral channel. To separate the angular dependence in a spherically symmetric system we use decompositions in terms of spherical tensor operators T_{JLSM} defined as

$$T_{JLSM}(\boldsymbol{n})_{\sigma_1\sigma_2} = \sum_{m\mu}(LmS\mu|JM)Y_{Lm}(\boldsymbol{n})(\sigma_{S\mu})_{\sigma_1\sigma_2}, \quad \text{(A1)}$$

where $\sigma_{S\mu}$ are the Pauli matrices in the tensor representation:

$$(\sigma_{S\mu})_{\sigma_1\sigma_2} = \sqrt{2\,(2\,S+1)}\,(-1)^{\frac{1}{2}-\sigma_1}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & S\\ \sigma_2 & -\sigma_1 & \mu\end{pmatrix}. \quad \text{(A2)}$$

The reduced matrix elements of the operator T_{JLSM} are defined by the conventional relation

$$\langle j_1 l_1 m_1|T_{JLSM}|j_2 l_2 m_2\rangle = (-1)^{j_1-m_1}\begin{pmatrix}j_1 & j_2 & J\\ m_1 & -m_2 & -M\end{pmatrix}\langle j_1 l_1\|T_{JLS}\|j_2 l_2\rangle. \quad \text{(A3)}$$

In the explicit form we have

$$\begin{aligned}\langle j_1 l_1\|T_{JLS}\|j_2 l_2\rangle &= \frac{1}{2}\left[1+(-1)^{L+l_1+l_2}\right](-1)^{S+j_2-\frac{1}{2}}\\ &\times\sqrt{\frac{(2J+1)\,(2L+1)\,(2j_1+1)\,(2j_2+1)}{4\pi}}\begin{pmatrix}j_1 & j_2 & J\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\\ &\times\left\{\begin{pmatrix}J & L & S\\ 0 & 0 & 0\end{pmatrix}+\sqrt{\frac{S\,(S+1)}{J\,(J+1)}}\begin{pmatrix}J & L & S\\ 1 & 0 & -1\end{pmatrix}\right.\\ &\left.\times\left[(l_1-j_1)\,(2j_1+1)+(-1)^{J+L+S}\,(l_2-j_2)\,(2j_2+1)\right]\right\}.\end{aligned} \quad \text{(A4)}$$

It is assumed that the operator of the renormalized external field in Eqs. (14), (16), (17) has the form

$$\tilde{V}^0(x_1,x_2) = \tilde{V}^0_{JM}(x_1,x_2) = \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\,\delta_{\tau_1,\tau_2} \times\sum_{LS}\tilde{V}^0_{JLS\tau_1}(r_1)T_{JLSM}(\boldsymbol{n}_1)_{\sigma_1\sigma_2}. \quad \text{(A5)}$$

For the functions $\tilde{V}^0_{JLS\tau}(r)$ in the case of electric multipole excitations we use the standard ansatz

$$\tilde{V}^0_{JLS\tau}(r) = \delta_{JL}\delta_{S0}\,r^L e^{(L)}_\tau, \quad \text{(A6)}$$

where $e^{(L)}_\tau$ is an effective charge in the center-of-mass reference frame: $e^{(L)}_n = Z\,(-A^{-1})^L$, $e^{(L)}_p = (1-A^{-1})^L + (Z-1)\,(-A^{-1})^L$.

Let us denote in accordance with Eqs. (14), (24), (25), and (A5)

$$\Lambda^{0(c)}_{JM}(x_1,x_2;\omega) = -\int dx_3\,dx_4 \times A^{(c,ph)}(x_1,x_2;x_3,x_4;\omega)\,\tilde{V}^0_{JM}(x_4,x_3). \quad \text{(A7)}$$

Let $\Lambda^{(c)}_{JM}$ be the solution of Eq. (32) with $\Lambda^{0(c)} = \Lambda^{0(c)}_{JM}$. In this case the partial components of the LRM $\Lambda^{(c)}_{JM}$ and of the related quantities are defined as

$$\Lambda^{(c)}_{JLS\tau}(r;\omega) = \int d\boldsymbol{n}\,dx_1\,dx_2 T^\dagger_{JLSM}(\boldsymbol{n})_{\sigma_1\sigma_2} \times\delta(x_1,x_2;\boldsymbol{r},\tau)\,\Lambda^{(c)}_{JM}(x_1,x_2;\omega), \quad \text{(A8)}$$

$$\Lambda^{0(c)}_{JLS\tau}(r;\omega) = \int d\boldsymbol{n}\,dx_1\,dx_2 T^\dagger_{JLSM}(\boldsymbol{n})_{\sigma_1\sigma_2} \times\delta(x_1,x_2;\boldsymbol{r},\tau)\,\Lambda^{0(c)}_{JM}(x_1,x_2;\omega), \quad \text{(A9)}$$

$$\begin{aligned}A^{J(c,c')}_{LS\tau,L'S'\tau'}(r,r';\omega) = \delta_{\tau,\tau'}&\int d\boldsymbol{n}\,d\boldsymbol{n}'\,dx_1\,dx_2\,dx_3\,dx_4\\ &\times\delta(x_1,x_2;\boldsymbol{r},\tau)\delta(x_3,x_4;\boldsymbol{r}',\tau')\\ &\times T^\dagger_{JLSM}(\boldsymbol{n})_{\sigma_1\sigma_2}A^{(c,c')}(x_1,x_2;x_3,x_4;\omega)\\ &\times T_{JL'S'M}(\boldsymbol{n}')_{\sigma_4\sigma_3},\end{aligned} \quad \text{(A10)}$$

where

$$T^\dagger_{JLSM}(\boldsymbol{n})_{\sigma_1\sigma_2} = (-1)^{J+L+S+M}T_{JLS-M}(\boldsymbol{n})_{\sigma_1\sigma_2}, \quad \text{(A11)}$$

$$\delta(x_1,x_2;\boldsymbol{r},\tau) = \delta_{\tau_1,\tau}\delta_{\tau_2,\tau}\,\delta(\boldsymbol{r}_1-\boldsymbol{r})\delta(\boldsymbol{r}_2-\boldsymbol{r}). \quad \text{(A12)}$$

Further, we assume that the effective interaction is determined by the following decomposition:

$$\mathcal{F}^{(c,c')}(x_1,x_2;x_3,x_4) = \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}_3-\boldsymbol{r}_4)\,\delta_{\tau_1,\tau_2}\,\delta_{\tau_3,\tau_4} \times\sum_{LSL'S'JM}T_{JLSM}(\boldsymbol{n}_1)_{\sigma_2\sigma_1}\,T^\dagger_{JL'S'M}(\boldsymbol{n}_3)_{\sigma_3\sigma_4} \times\mathcal{F}^{J(c,c')}_{LS\tau_1,L'S'\tau_3}(r_1,r_3), \quad \text{(A13)}$$

where

$$\mathcal{F}^{J(c,c')}_{LS\tau,L'S'\tau'}(r,r') = \delta_{c,c'}\delta_{LL'}\delta_{SS'}\,\frac{\delta(r-r')}{rr'}\mathcal{F}^{(c)}_{S,\tau\tau'}(r) + \delta_{LL'}\delta_{L1}\delta_{J1}\sum_{k=1}^{2}\varkappa_k\,F^{(c)}_{S\tau,k}(r)F^{(c')}_{S'\tau',k}(r'), \quad \text{(A14)}$$

$$\mathcal{F}^{(ph)}_{S,\tau\tau'} = C_0(\delta_{S0}[f + (2\delta_{\tau,\tau'} - 1)\, f'] + \delta_{S1}[g + (2\delta_{\tau,\tau'} - 1)\, g']), \tag{A15}$$

$$\mathcal{F}^{(pp)}_{S,\tau\tau'} = \mathcal{F}^{(hh)}_{S,\tau\tau'} = \delta_{S0}\delta_{\tau,\tau'}\,\mathcal{F}^{\xi}. \tag{A16}$$

In Eq. (A14) the first term represents Landau-Migdal zero-range interaction of the TFFSPC. In the standard parametrization, the quantities C_0, g, and g' in Eq. (A15) are constants. The functions $f(r)$ and $f'(r)$ are determined by the parameters f_{ex}, f_{in}, f'_{ex}, f'_{in}, and by the nuclear density in the ground state $\rho_0(r)$ by means of the ansatz

$$f(r) = f_{ex} + (f_{in} - f_{ex})\rho_0(r)/\rho_0(0), \tag{A17}$$

and analogously for $f'(r)$. For the interaction in the pp and hh channels we have

$$\mathcal{F}^{\xi} = C_0/\ln(c_p/\xi), \tag{A18}$$

where c_p is a constant, ξ is an energy cutoff parameter.

The second term in Eq. (A14) is a correction corresponding to the amplitude $\mathcal{F}^{rest\,(c,c')}$ in Eq. (28). The similar correction arises in the calculational scheme with "forced consistency" which was developed in Ref. [12] to eliminate the spurious isoscalar 1^- state in the non-self-consistent approach for the case when only ph channel is taken into account. Here the straightforward generalization of this scheme is presented in which the pp and hh channels are also included. Making use of the same arguments as in Ref. [12] one can show that setting

$$F^{(c)}_{S\tau,1}(r) = \delta_{c,ph}\delta_{S0}\frac{dU^{\tau}(r)}{dr}, \qquad F^{(c)}_{S\tau,2}(r) = \sum_{\tau'}\mathcal{F}^{(c)}_{S,\tau\tau'}(r)Q^{(c)}_{S\tau'}(r), \tag{A19}$$

$$\varkappa_k^{-1} = -\sum_{cS\tau}\int_0^{\infty} dr\, r^2\, F^{(c)}_{S\tau,k}(r)\, Q^{(c)}_{S\tau}(r), \tag{A20}$$

where

$$Q^{(c)}_{S\tau}(r) = \sum_{c'S'}\int_0^{\infty} dr'\, r'^2 A^{1\,(c,c')}_{1S\tau,1S'\tau}(r,r';\omega_0)F^{(c')}_{S'\tau,1}(r'), \qquad \omega_0 \to 0, \tag{A21}$$

we obtain the spurious state exactly at zero energy. In Eqs. (A19), $U^{\tau}(r)$ is an auxiliary potential, the well depth of which is chosen to satisfy the condition

$$\int_0^{\infty} dr\, r^3\left[Z\, Q^{(ph)}_{0n}(r) - N\, Q^{(ph)}_{0p}(r)\right] = 0, \tag{A22}$$

which ensures the spurious state excitation probability to be equal to zero.

Making use of the above definitions we can reduce Eq. (32) for the LRM in the case of the spherically symmetric system to Eq. (33) for its partial components.

It is important to note that the amplitudes u_λ and v_λ, which define the single-quasiparticle basis functions according to Eq. (6), have to be determined from the solution of the gap equation with the same interaction $\mathcal{F}^{\xi}$ which enters Eq. (33) for the LRM through Eqs. (A14) and (A16). Namely, we have

$$u_\lambda = \sqrt{\frac{1}{2}\left(1 + \frac{\varepsilon_\lambda - \mu_{\tau_\lambda}}{E_\lambda}\right)}, \qquad v_\lambda = \mathrm{sgn}(\Delta_\lambda)\sqrt{\frac{1}{2}\left(1 - \frac{\varepsilon_\lambda - \mu_{\tau_\lambda}}{E_\lambda}\right)}, \tag{A23}$$

where ε_λ is the eigenvalue of the single-particle Hamiltonian corresponding to the eigenfunction $\varphi_\lambda(x)$, μ_τ is the chemical potential,

$$E_\lambda = \sqrt{(\varepsilon_\lambda - \mu_{\tau_\lambda})^2 + \Delta_\lambda^2}. \tag{A24}$$

The values of the energy gap Δ_λ are determined within the BCS approximation from the equation:

$$\Delta_\lambda = -\sum_{(\lambda')}\frac{2j_{\lambda'}+1}{4\pi}\mathcal{F}^{\xi}_{(\lambda\lambda')}\frac{\Delta_{\lambda'}}{2E_{\lambda'}}, \tag{A25}$$

where

$$\mathcal{F}^{\xi}_{(\lambda\lambda')} = \delta_{\tau_\lambda,\tau_{\lambda'}}\int_0^{\infty} dr\, r^2 R^2_{(\lambda)}(r)\, R^2_{(\lambda')}(r)\mathcal{F}^{\xi}(r). \tag{A26}$$

APPENDIX B: CORRELATED PROPAGATOR OF THE QTBA IN TERMS OF THE REDUCED MATRIX ELEMENTS

In this appendix we draw the formulas for the reduced matrix elements of the correlated propagator $A^J_{[12,34]}(\omega)$ which correspond to the formulas of Sec. 3.2 of Ref. [7] obtained within the QTBA in the case of the QPC model. In what follows, the summation over single-quasiparticle indices means summation over the discrete states inside the "pairing window" (see Sec. V for details). The general formula for the propagator of the QTBA satisfying Eq. (38) reads

$$A^J_{[12,34]}(\omega) = \sum_{[5678]}\left[\delta_{[15]}\delta_{[26]} + Q^{J\,(+-)\,a}_{[12,56]}(\omega)\right]A^{J\,(--)}_{[56,78]}(\omega) \times\left[\delta_{[73]}\delta_{[84]} + Q^{J\,(-+)\,a}_{[78,34]}(\omega)\right] + \tfrac{1}{2}\left[P^{J\,(++)}_{[12,34]}(\omega) - (-1)^{J+l_1-l_2+j_1-j_2}P^{J\,(++)}_{[\bar{2}\bar{1},34]}(\omega)\right], \tag{B1}$$

where

$$Q^{J\,(+-)\,a}_{[12,34]}(\omega) = \theta_{(43)}\left[Q^{J\,(+-)}_{[12,34]}(\omega) + (-1)^{J+l_3-l_4+j_3-j_4}Q^{J\,(+-)}_{[12,\bar{4}\bar{3}]}(\omega)\right], \tag{B2}$$

$$Q^{J\,(-+)\,a}_{[12,34]}(\omega) = \theta_{(21)}\left[Q^{J\,(-+)}_{[12,34]}(\omega) + (-1)^{J+l_1-l_2+j_1-j_2}Q^{J\,(-+)}_{[\bar{2}\bar{1},34]}(\omega)\right], \tag{B3}$$

$$A^{J\,(--)}_{[12,34]}(\omega) = \delta_{\eta_1,-\eta_2}\delta_{\eta_3,-\eta_4}A^{J\,(--)}_{(12)\eta_1,(34)\eta_3}(\omega). \tag{B4}$$

The quantity $A^{J(--)}_{(12)\eta,(34)\eta'}(\omega)$ is a solution of the equation

$$A^{J(--)}_{(12)\eta,(34)\eta'}(\omega) = \tilde{A}^{J}_{(12)\eta,(34)\eta'}(\omega) + \sum_{\eta''}\sum_{(56)} \theta_{(65)} \mathcal{K}^{J}_{(12)\eta,(56)\eta''}(\omega) A^{J(--)}_{(56)\eta'',(34)\eta'}(\omega), \tag{B5}$$

where

$$\tilde{A}^{J}_{(12)\eta,(34)\eta'}(\omega) = -\frac{\eta\delta_{\eta,\eta'}\left[\delta_{(13)}\,\delta_{(24)} + (-1)^{J+l_1-l_2+j_1-j_2}\,\delta_{(14)}\,\delta_{(23)}\right]}{2\left(\omega - \eta\left[E_{(1)} + E_{(2)}\right]\right)}, \tag{B6}$$

$$\mathcal{K}^{J}_{(12)\eta,(34)\eta'}(\omega) = \frac{\eta\left[\Phi^{J}_{(12)\eta,(34)\eta'}(\omega) + (-1)^{J+l_1-l_2+j_1-j_2}\,\Phi^{J}_{(21)\eta,(34)\eta'}(\omega)\right]}{\omega - \eta\left[E_{(1)} + E_{(2)}\right]}, \tag{B7}$$

$$\Phi^{J}_{(12)\eta,(34)\eta'}(\omega) = \sum_{\eta_1\eta_2\eta_3\eta_4} \delta_{\eta_1,\eta}\,\delta_{\eta_2,-\eta}\,\delta_{\eta_3,\eta'}\,\delta_{\eta_4,-\eta'}\,\Phi^{J}_{[12,34]}(\omega). \tag{B8}$$

It is easy to see that solution of the Eq. (B5) possesses the following symmetry:

$$A^{J(--)}_{(12)\eta,(34)\eta'}(\omega) = (-1)^{J+l_1-l_2+j_1-j_2} A^{J(--)}_{(21)\eta,(34)\eta'}(\omega) = (-1)^{J+l_3-l_4+j_3-j_4} A^{J(--)}_{(12)\eta,(43)\eta'}(\omega). \tag{B9}$$

It enables one to determine all the elements of the matrix $A^{J(--)}_{(12)\eta,(34)\eta'}(\omega)$ by solving Eq. (B5) for the nonzero block of the matrix $\theta_{(21)} A^{J(--)}_{(12)\eta,(34)\eta'}(\omega)\,\theta_{(43)}$. However, as follows from Eqs. (42), (46), (B1)–(B4), to construct the propagator in the coordinate space it is sufficient to determine the elements only of this block.

In order to define the remaining quantities which enter Eqs. (B1)–(B3), (B8) in the case of the quasiparticle-phonon coupling model let us introduce notations

$$\Omega_{12\,q} = E_{12} + \eta_1\omega_q, \quad E_{12} = \eta_1\left[E_{(1)} + E_{(2)}\right], \tag{B10}$$

$$D^{q}_{[12,34]\eta} = \delta_{\eta,+1} g^{q}_{[13]}\, g^{q*}_{[24]} + \delta_{\eta,-1}(-1)^{j_1+j_2+j_3+j_4} g^{q*}_{[31]}\, g^{q}_{[42]}, \tag{B11}$$

$$X^{J\,q}_{[12,34]\eta} = (-1)^{J+J_q+j_2-j_3} \begin{Bmatrix} j_1 & j_2 & J \\ j_4 & j_3 & J_q \end{Bmatrix} D^{q}_{[12,34]\eta},$$

$$Y^{q}_{[12,3]} = \frac{\delta_{j_1 j_2}\,\delta_{l_1 l_2}}{2j_1+1} D^{q}_{[12,33]\eta_3}, \tag{B12}$$

where the reduced matrix elements of the amplitude of quasiparticle-phonon interaction $g^{q}_{[12]}$ and the phonon energies ω_q are defined by the formulas of Appendix C. Making use of Eqs. (37), (B10)–(B12) we obtain

$$\Phi^{J}_{[12,34]}(\omega) = \Phi^{J(\mathrm{res})}_{[12,34]}(\omega) + \Phi^{J(\mathrm{GSC})}_{[12,34]}(\omega) + \Phi^{(\mathrm{comp})}_{[12,34]}(\omega), \tag{B13}$$

$$\Phi^{J(\mathrm{res})}_{[12,34]}(\omega) = \eta_1\,\delta_{\eta_1,-\eta_2}\,\delta_{\eta_3,-\eta_4} \times \sum_q \left[\delta_{\eta_1,\eta_3}\left(\frac{X^{J\,q}_{[12,34]\eta_1}}{\omega - \Omega_{32\,q}} + \frac{X^{J\,q}_{[12,34]\eta_2}}{\omega - \Omega_{14\,q}}\right) + \delta_{[24]}\sum_{[5]}\frac{\delta_{\eta_5,\eta_1}\,Y^{q}_{[13,5]}}{\omega - \Omega_{52\,q}} + \delta_{[13]}\sum_{[6]}\frac{\delta_{\eta_6,\eta_2}\,Y^{q}_{[42,6]}}{\omega - \Omega_{16\,q}}\right], \tag{B14}$$

$$\Phi^{J(\mathrm{GSC})}_{[12,34]}(\omega) = -\eta_1\,\delta_{\eta_1,-\eta_2}\,\delta_{\eta_3,-\eta_4} \times \sum_q \left[\delta_{\eta_1,-\eta_3}\left(\frac{X^{J\,q}_{[12,34]\eta_1}}{\Omega_{42\,q}} + \frac{X^{J\,q}_{[12,34]\eta_2}}{\Omega_{13\,q}}\right) + \delta_{[24]}\sum_{[5]}\frac{\delta_{\eta_5,\eta_2}\,Y^{q}_{[13,5]}\,(\omega - E_{12} - \Omega_{35\,q})}{\Omega_{15\,q}\Omega_{35\,q}} + \delta_{[13]}\sum_{[6]}\frac{\delta_{\eta_6,\eta_1}\,Y^{q}_{[42,6]}\,(\omega - E_{12} - \Omega_{64\,q})}{\Omega_{62\,q}\Omega_{64\,q}}\right], \tag{B15}$$

$$\Phi^{(\mathrm{comp})}_{[12,34]}(\omega) = -\eta_1\,\delta_{\eta_1,-\eta_2}\,\delta_{\eta_3,-\eta_4}\,\delta_{\eta_1,\eta_3} \times \sum_{[56]qq'}\frac{\delta_{\eta_5,\eta_2}\,\delta_{\eta_6,\eta_1}\,Y^{q}_{[13,5]}\,Y^{q'}_{[42,6]}}{\Omega_{15\,q}\Omega_{35\,q}\,\Omega_{62\,q'}\Omega_{64\,q'}} \times (\omega + E_{56} - \Omega_{12\,q} - \Omega_{34\,q'}), \tag{B16}$$

$$P^{J(++)}_{[12,34]}(\omega) = \eta_1\,\delta_{\eta_1,\eta_2}\,\delta_{\eta_3,\eta_4}\sum_q\left[\frac{\delta_{\eta_1,-\eta_3}\,X^{J\,q}_{[12,34]\eta_4}}{\Omega_{13\,q}\Omega_{24\,q}} \times \left(\frac{1}{\omega - \Omega_{32\,q}} - \frac{1}{\omega - \Omega_{14\,q}}\right) + \delta_{[24]}\sum_{[5]}\frac{\delta_{\eta_5,-\eta_1}\,Y^{q}_{[13,5]}}{\Omega_{51\,q}\Omega_{53\,q}(\omega - \Omega_{52\,q})} - \delta_{[13]}\sum_{[6]}\frac{\delta_{\eta_6,-\eta_2}\,Y^{q}_{[42,6]}}{\Omega_{26\,q}\Omega_{46\,q}(\omega - \Omega_{16\,q})}\right], \tag{B17}$$

$$Q^{J(+-)}_{[12,34]}(\omega) = \delta_{\eta_1,\eta_2}\,\delta_{\eta_3,-\eta_4}\sum_q\left\{\frac{\delta_{\eta_1,-\eta_3}\,X^{J\,q}_{[12,34]\eta_3}}{\Omega_{31\,q}(\omega - \Omega_{32\,q})} + \frac{\delta_{\eta_2,-\eta_4}\,X^{J\,q}_{[12,34]\eta_4}}{\Omega_{24\,q}(\omega - \Omega_{14\,q})} + \delta_{[24]}\sum_{[5]}\left[\frac{\delta_{\eta_5,\eta_1}\,Y^{q}_{[13,5]}}{E_{13}\Omega_{53\,q}}\right.\right.$$

$$+\frac{\delta_{\eta_5,-\eta_1}Y^q_{[13,5]}}{\Omega_{51\,q}}\left(\frac{1}{E_{13}}+\frac{1}{\omega-\Omega_{52\,q}}\right)\Bigg]$$
$$+\delta_{[13]}\sum_{[6]}\Bigg[\frac{\delta_{\eta_6,\eta_2}Y^q_{[42,6]}}{E_{42}\Omega_{46\,q}}+\frac{\delta_{\eta_6,-\eta_2}Y^q_{[42,6]}}{\Omega_{26\,q}}$$
$$\times\left(\frac{1}{E_{42}}+\frac{1}{\omega-\Omega_{16\,q}}\right)\Bigg]\Bigg\}, \tag{B18}$$

$$Q^{J\,(-+)}_{[12,34]}(\omega)=(-1)^{j_1+j_2+j_3+j_4}\,Q^{J\,(+-)}_{[43,21]}(-\omega)$$
$$=\delta_{\eta_1,-\eta_2}\,\delta_{\eta_3,\eta_4}\sum_q\Bigg\{\frac{\delta_{\eta_4,-\eta_2}X^{J\,q}_{[12,34]\eta_1}}{\Omega_{42\,q}(\omega-\Omega_{32\,q})}$$
$$+\frac{\delta_{\eta_3,-\eta_1}X^{J\,q}_{[12,34]\eta_2}}{\Omega_{13\,q}(\omega-\Omega_{14\,q})}+\delta_{[24]}\sum_{[5]}\Bigg[\frac{\delta_{\eta_5,\eta_3}Y^q_{[13,5]}}{E_{31}\Omega_{51\,q}}$$
$$+\frac{\delta_{\eta_5,-\eta_3}Y^q_{[13,5]}}{\Omega_{53\,q}}\left(\frac{1}{E_{31}}+\frac{1}{\omega-\Omega_{52\,q}}\right)\Bigg]$$
$$+\delta_{[13]}\sum_{[6]}\Bigg[\frac{\delta_{\eta_6,\eta_4}Y^q_{[42,6]}}{E_{24}\Omega_{26\,q}}+\frac{\delta_{\eta_6,-\eta_4}Y^q_{[42,6]}}{\Omega_{46\,q}}$$
$$\times\left(\frac{1}{E_{24}}+\frac{1}{\omega-\Omega_{36\,q}}\right)\Bigg]\Bigg\}. \tag{B19}$$

Equations (B1)–(B19) completely define the reduced matrix elements $A^J_{[12,34]}(\omega)$ which enter Eqs. (42) and (46) for the "discrete" part of the total correlated propagator in the coordinate representation. Notice that in Eq. (42) the matrix elements $A^J_{[12,34]}(\omega)$ are the same for all the channels which differ from each other only by the amplitudes $\alpha^c_{[12]}$. Notice also that the form of the above equations for the reduced matrix elements $A^J_{[12,34]}(\omega)$ are the same both for magic and for open-shell nuclei. In the former case, however, the index η_1 in the set $[1]=\{(1),\eta_1\}$ is not independent, but is determined by the occupation number $n_{(1)}=0$ or 1 of the state $(1)=(\lambda_1)$ as $\eta_1=1-2\,n_{(1)}$.

APPENDIX C: PHONON INPUT IN THE QTBA

In the present calculations the reduced matrix elements of the amplitude of quasiparticle-phonon interaction $g^q_{[12]}$ and the phonon energies ω_q entering formulas for the correlated propagator of the QTBA were determined within QRPA. As a first step, the QRPA equation for the transition amplitudes is solved making use of the basis restricted by discrete and quasidiscrete single-particle states entering a valence zone near the Fermi level. In terms of the reduced matrix elements this equation reads

$$(\omega_q-\eta[E_{(1)}+E_{(2)}])\rho^q_{(12)\eta}$$
$$=\sum_{\eta'}\sum_{(34)}^{wind}\eta\mathcal{F}^{J_q}_{(12)\eta,(34)\eta'}\,\theta_{(43)}\,\rho^q_{(34)\eta'}, \tag{C1}$$

where

$$\mathcal{F}^J_{(12)\eta,(34)\eta'}=\sum_{\eta_1\eta_2\eta_3\eta_4}\delta_{\eta_1,\eta}\,\delta_{\eta_2,-\eta}\,\delta_{\eta_3,\eta'}\,\delta_{\eta_4,-\eta'}\mathcal{F}^J_{[12,34]}, \tag{C2}$$

$$\mathcal{F}^J_{[12,34]}=\frac{1}{2}\sum_{LSc}\left(\alpha^c_{[12]}-\eta_1\eta_2(-1)^S\alpha^c_{[\bar{2}\bar{1}]}\right)$$
$$\times\left(\alpha^c_{[34]}-\eta_3\eta_4(-1)^S\alpha^c_{[\bar{4}\bar{3}]}\right)$$
$$\times(1+\delta_{c,ph})T^{(J)\,LS,LS}_{(12,34)}\mathcal{F}^{(c)S}_{(12,34)}, \tag{C3}$$

$$\mathcal{F}^{(c)S}_{(12,34)}=\delta_{\tau_1,\tau_2}\,\delta_{\tau_3,\tau_4}$$
$$\times\int_0^\infty dr\,r^2R_{(1)}(r)\,R_{(2)}(r)R_{(3)}(r)\,R_{(4)}(r)\mathcal{F}^{(c)}_{S,\tau_1\tau_3}(r). \tag{C4}$$

The quantities entering these formulas are defined by Eqs. (A15), (A16), (43), (44). Notice that QRPA equation (C1) includes contributions of all the channels: ph, pp, and hh, though in the present calculations only ph channel was included. Normalization condition for the transition amplitudes $\rho^q_{(12)\eta}$ has the form

$$\sum_\eta\sum_{(12)}^{wind}\theta_{(21)}\eta\left|\rho^q_{(12)\eta}\right|^2=2J_q+1. \tag{C5}$$

The reduced matrix elements $g^q_{[12]}$ and the reduced probability $B(q)\uparrow$ of excitation induced by the external field (A5) are determined by the found values of $\rho^q_{(12)\eta}$ through the formulas

$$g^q_{[12]}=\sum_{[34]}^{wind}\mathcal{F}^{J_q}_{[12,34]}\,\theta_{(43)}\,\delta_{\eta_3,-\eta_4}\,\rho^q_{(34)\eta_3}, \tag{C6}$$

$$B(q)\uparrow=\frac{1}{2J_q+1}\left|\sum_\eta\sum_{(12)}^{wind}\theta_{(21)}\left(\tilde{V}^0_{J_q}\right)_{(21)\eta}\rho^q_{(12)\eta}\right|^2, \tag{C7}$$

where

$$\left(\tilde{V}^0_J\right)_{(21)\eta}=\sum_{LS}\eta^S[u_{(1)}\,v_{(2)}+(-1)^S\,v_{(1)}\,u_{(2)}]$$
$$\times\langle\,j_2l_2\|T_{JLS}\|j_1l_1\rangle\left(\tilde{V}^0_{JLS}\right)_{(21)}, \tag{C8}$$

$$\left(\tilde{V}^0_{JLS}\right)_{(12)}=\delta_{\tau_1,\tau_2}\int_0^\infty dr\,r^2\,R_{(1)}(r)R_{(2)}(r)\,\tilde{V}^0_{JLS\tau_1}(r). \tag{C9}$$

More general formulas for the phonon amplitudes and energies are drawn in Ref. [22].

[1] S. P. Kamerdzhiev, G. Ya. Tertychny, and V. I. Tselyaev, Fiz. Elem. Chastits At. Yadra **28**, 333 (1997) [Phys. Part. Nuclei **28**, 134 (1997)].

[2] A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, 1975).

[3] V. G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and*

Phonons (Institute of Physics, Bristol and Philadelphia, USA, 1992).
[4] V. I. Tselyaev, Yad. Fiz. **50**, 1252 (1989) [Sov. J. Nucl. Phys. **50**, 780 (1989)].
[5] S. P. Kamerdzhiev and E. V. Litvinova, Yad. Fiz. **67**, 180 (2004) [Phys. At. Nucl. **67**, 183 (2004)].
[6] D. Sarchi, P. F. Bortignon, and G. Colò, Phys. Lett. **B601**, 27 (2004).
[7] V. I. Tselyaev, Phys. Rev. C **75**, 024306 (2007).
[8] S. Krewald, V. B. Soubbotin, V. I. Tselyaev, and X. Viñas, Phys. Rev. C **74**, 064310 (2006).
[9] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New York, 1980).
[10] A. B. Migdal, *Theory of Finite Fermi Systems and Applications to Atomic Nuclei* (Interscience, New York, 1967).
[11] E. Khan, N. Sandulescu, M. Grasso, and N. Van Giai, Phys. Rev. C **66**, 024309 (2002).
[12] S. Kamerdzhiev, R. J. Liotta, E. Litvinova, and V. Tselyaev, Phys. Rev. C **58**, 172 (1998).
[13] S. Shlomo and G. Bertsch, Nucl. Phys. **A243**, 507 (1975).
[14] K. Hagino and H. Sagawa, Nucl. Phys. **A695**, 82 (2001).
[15] G. A. Lalazissis, T. Nikšić, D. Vretenar, and P. Ring, Phys. Rev. C **71**, 024312 (2005).
[16] A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 1 (Benjamin, New York, 1969).
[17] S. P. Kamerdzhiev, R. J. Liotta, and V. I. Tselyaev, Phys. Rev. C **63**, 034304 (2001).
[18] S. P. Kamerdzhiev and V. I. Tselyaev, Phys. Rev. C **66**, 044304 (2002).
[19] M. Matsuo, Nucl. Phys. **A696**, 371 (2001).
[20] N. Paar, P. Ring, T. Nikšić, and D. Vretenar, Phys. Rev. C **67**, 034312 (2003).
[21] J. Terasaki, J. Engel, M. Bender, J. Dobaczewski, W. Nazarewicz, and M. Stoitsov, Phys. Rev. C **71**, 034310 (2005).
[22] E. V. Litvinova and V. I. Tselyaev, arXiv:nucl-th/0512030.
[23] V. I. Tselyaev, Izv. Ross. Akad. Nauk, Ser. Fiz. **64**, 541 (2000) [Bull. Russ. Acad. Sci., Phys. (USA) **64**, 434 (2000)].
[24] S. C. Fultz, B. L. Berman, J. T. Caldwell, R. L. Bramblett, and M. A. Kelly, Phys. Rev. **186**, 1255 (1969).
[25] A. Leprêtre, H. Beil, R. Bergère, P. Carlos, A. De Miniac, A. Veyssière, and K. Kernbach, Nucl. Phys. **A219**, 39 (1974).