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Properties of the 0_2^+ state and isospin excitation in the $N = Z$ nucleus ^{68}Se

Falih H. Al-Khudair, Y. S. Li, and G. L. Long
Key Laboratory for Quantum Information and Measurements and Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China and
Center of Nuclear Theory, Lanzhou National Laboratory of Accelerators, Lanzhou 730000, People's Republic of China
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Band structure and electromagnetic transition properties of the low-lying states in the $N = Z$ ^{68}Se nucleus were studied within the framework of interacting boson model 3. The isospin excitation states with $T > T_Z$ are identified. The $M1$ and $E2$ matrix elements for low-lying states have been investigated and were used to identify the low-lying mixed symmetry states. Special attention is given to the occurrence of 0_2^+ state, recently predicted by the projected shell-model (PSM) calculation. The present predicted spectrum for ^{68}Se is close to the recent PSM results and confirms the results for the 0_2^+ state. The calculated results are compared with available experimental data, and they are in general good agreement.

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I. INTRODUCTION

During recent years comprehensive new experimental information on energy levels in the $N \simeq Z$ nuclei have been collected [1–5]. Because isospin excitations have been observed at modest excitation energies in these nuclei, this region is of specific interest because different microscopic theoretical approaches can be compared and tested. These nuclei are expected to give new insight into the neutron-proton correlation. The studies [6–8] have demonstrated that mixing between $T = 0$ and $T = 1$ modes and between $T_z = \pm 1$ and $T_z = 0$ in the $T = 1$ mode are necessary in an isospin invariant approach. In even-even $Z = N$ nuclei, the 0_1^+, $T = 0$ ground state is separated from the excited $T = 1$ states by a relatively large energy gap and isospin symmetry forces the isovector pairing to be identical in all three $T = 1$ pairing channels. However, the relative dominance of $T = 0$ versus $T = 1$ pairing in $N = Z$ nuclei has been shown to be linked to the energy separation of the two types of pair and hence to the separation of the $T = 0$ and $T = 1$ states in odd-odd $N = Z$ nuclei. Over the past few years, the structure of these nuclei have received intensive attention [9–17]. Many detailed theoretical description of these nuclei were available, due largely to the fact that the structure of these nuclei provides a sensitive test for the isospin symmetry of nuclear force. The isospin effect were studied in Refs. [18–23].

In this work, we shall examine an alternative description of the band structure and isospin excitation states ($T > T_z$) of ^{68}Se in the interacting boson model 3 (IBM-3). As we shall see, this model can provide a relatively simple, yet accurate, description for the states. This work can be considered a continuation of the work in Ref. [24] and a partial effort to the comprehensive understanding of the nuclear structure in this region. Before going to the IBM-3 treatment, it is worth mentioning the following work. The excitation states of ^{68}Se were investigated in Refs. [25,26], where the levels were inferred from the ^{12}C(^{58}Ni,$2n$) and ^{40}Ca(^{36}Ar,2α) reactions. The nuclear structure of coexistence of differently deformed shapes has been found in ^{68}Se, where the ground-state band has an oblate deformation whereas an excited band has prolate deformation [27]. Some effort has been made to provide a more sophisticated description of these low-lying states. However, these calculations are generally restricted to the study of nuclear shape and band structure with isospin ($T_z = 0$).

More recently, several theoretical investigations of ^{68}Se isotope have been carried out:

(i) Sun [28], has performed systematic analysis of even-even ^{68}Se isotope in the projected shell model, satisfactorily reproduced the excitation energy levels of this nucleus, including the coexisting oblate and prolate minima, the backbending at $J = 8^+$ and 16^+ and a number of high K isomers at approximately 5 MeV above the ground state.

(ii) Kaneko *et al.* [29] have considered the configuration space ($2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2}$) and performed shell model and constrained Hartree-Fock calculations for some $N = Z$ nuclei. The authors found shape transition from prolate to oblate deformation in these $N = Z$ nuclei and oblate-prolate coexistence at ^{68}Se. The ground state of ^{68}Se has an oblate shape, whereas the shape of ^{60}Zn and ^{64}Ge are prolate.

(iii) Afanasjev and Frauendorf [30], have performed calculations using the cranked relativistic mean field, cranked relativistic Hartree-Bogoliubov theories, and cranked Nilsson-Strutinsky approach for ^{68}Se isotope; their calculations strongly suggest that the presence of strong isovector *np* pairing at low spin and the strength is restricted by the isospin symmetry.

(iv) Sun *et al.* [31] have investigated shape isomeric states using a multi-mass-zone x-ray burst model, given a two waiting point nuclei ^{68}Se and ^{72}Kr that are characterized by shape coexistence. The ground state takes an oblate shape with ($\varepsilon_2 = 0.25$) and another local minimum with a prolate shape ($\varepsilon_2 = 0.4$) is found to be 1.1 MeV(^{68}Se) and 0.7 MeV (^{72}Kr), which were interpreted as isomeric states.

II. THE MODEL HAMILTONIAN AND THE PARAMETER

The IBM describes the low-lying energy levels in the even-even nuclei, starting from the symmetric coupling of bosons. In this model one can describe collective states by a system of N identical bosons. These bosons have angular momentum $L = 0$ (s-boson) and $L = 2$ (d-boson), respectively [32–34]. The neutron-proton extension of the model (IBM-2) predicts a new class of states [35] having mixed symmetry in the proton and neutron degrees of freedom. The mixed symmetry states have been successfully observed in various experiments [36–38]. The main advantage of this description, in which the proton-neutron degree of freedom is included explicitly, is that it is closely related to the microscopic shell model, allowing the microscopic derivation of the parameters of the model [39].

In lighter nuclei the valence protons and neutrons are filling the same major shell, isospin must be introduced. Within the IBM with isospin (IBM-3) [40], the neutron-proton pair must be included in addition to the two other types of bosons in the IBM-2 to form an isospin triplet. The microscopic picture of the interacting boson model is given in terms of collective pairs of nucleon. The building blocks of the IBM-3 are the neutron-neutron boson (s_ν, d_ν), neutron-proton boson (s_δ, d_δ) and proton-proton boson (s_π, d_π). The three s-bosons and three d-bosons form a $T = 1$ multiplet, respectively. The IBM-3 has $U(18)$ group as its dynamical symmetry group, and the dynamical group chain must contain the $O_L(3)$ and $SU_T(2)$ groups, because the angular momentum and the isospin are good quantum numbers [41–44]. One class of group chain is the $U(6) \times U(3)$ limit, which includes [45]

$$\begin{aligned} &\mathrm{U}(18) \supset (\mathrm{U}_c(3) \supset \mathrm{SU}_T(2)) \\ &\qquad \times (\mathrm{U}_{sd}(6) \supset \mathrm{U}_d(5) \supset \mathrm{O}_d(5) \supset \mathrm{O}_d(3)) \\ &\mathrm{U}(18) \supset (\mathrm{U}_c(3) \supset \mathrm{SU}_T(2)) \\ &\qquad \times (\mathrm{U}_{sd}(6) \supset \mathrm{O}_{sd}(6) \supset \mathrm{O}_d(5) \supset \mathrm{O}_d(3)) \\ &\mathrm{U}(18) \supset (\mathrm{U}_c(3) \supset \mathrm{SU}_T(2)) \\ &\qquad \times (\mathrm{U}_{sd}(6) \supset \mathrm{SU}_{sd}(3) \supset \mathrm{O}_d(3)). \end{aligned} \tag{1}$$

These group chains are called the U(5), O(6), and SU(3) limits, and they describe the vibrational, γ-unstable, and rotational motion, respectively. These limits have been used in the microscopic study of the IBM-3 [41,46].

The IBM-3 Hamiltonian can be written as

$$H = \epsilon_s \hat{n}_s + \epsilon_d \hat{n}_d + H_2, \tag{2}$$

where

$$\begin{aligned} H_2 = &\frac{1}{2}\sum_{L_2T_2} C_{L_2T_2}[(d^\dagger d^\dagger)^{L_2T_2} \cdot (\tilde{d}\tilde{d})^{L_2T_2}] \\ &+ \frac{1}{2}\sum_{T_2} B_{0T_2}[(s^\dagger s^\dagger)^{0T_2} \cdot (\tilde{s}\tilde{s})^{0T_2}] \\ &+ \sum_{T_2} A_{2T_2}[(s^\dagger d^\dagger)^{2T_2} \cdot (\tilde{d}\tilde{s})^{2T_2}] \\ &+ \frac{1}{\sqrt{2}}\sum_{T_2} D_{2T_2}[(s^\dagger d^\dagger)^{2T_2} \cdot (\tilde{d}\tilde{d})^{2T_2}] \\ &+ \frac{1}{2}\sum_{T_2} G_{0T_2}[(s^\dagger s^\dagger)^{0T_2} \cdot (\tilde{d}\tilde{d})^{0T_2}]. \end{aligned} \tag{3}$$

The symbols T_2 and L_2 represent the two-boson isospin and angular momentum, respectively. The parameters A, B, C, D, and G are the two-body matrix elements. There has been microscopic study of these parameters [47–49]. The normal product form of the Hamiltonian (2) and the Casimir operators of IBM-3 limits are related. They can be given, after some tedious algebra. We have rewritten the Hamiltonian in terms of linear combination of Casimir operators, which is convenient to analyze the dynamical symmetry nature. The expressions of the Casimir operators can be found in Ref. [45]. In Casimir operator form, the Hamiltonians is

$$\begin{aligned} H = &\lambda C_{2\mathrm{U}_{sd}(6)} + a_T T(T+1) + \varepsilon C_{1\mathrm{U}_d(5)} \\ &+ \gamma C_{2\mathrm{O}_{sd}(6)} + \eta C_{2\mathrm{SU}_{sd}(3)} \\ &+ \beta C_{2\mathrm{U}_d(5)} + \delta C_{2\mathrm{O}_d(5)} + a_L C_{\mathrm{O}_d(3)}. \end{aligned} \tag{4}$$

The $\hat{C}_{nG}$ denotes the nth order Casimir operator of the algebra G. The definition of all operators and the full Hamiltonian of the IBM-3 in terms of Casimir operators can be found in Ref. [46].

Although this nucleus has large deformations, the ratio $E4_1^+/E2_1^+ = 2.27$ is close to 2.5 rather than 3.3, suggesting γ unstable collective motion. Furthermore, the ratio $E4_1^+/E2_2^+ = 1.21$ and until now no experimental data for 0_2^+ state were available. Within IBM-3, the features exhibiting the isospin excitation have not been investigated previously. We expect neutron-proton boson to play a crucial role in this spectra, especially in the isospin excitation states. To describe the energy spectra we use the Hamiltonian form in terms of the Casimir operators of dynamical symmetry groups given in Eq. (4). In performing the energy fit, the a_T parameter was fitted to relative position of $0^+_{T=2}$, i.e., the shift between the $T = 0$ ground state and the first $T = 2$ state. Here we have assumed that the ground-state energy of ^{68}Ge is equal to that of the IBM-3 calculated $0^+_{T=2}$ state in ^{68}Se. We estimate the energy of the isospin analog state in ^{68}Ge by considering the binding energy difference of ^{68}Se and ^{68}Ge and then subtracting the Coulomb energy difference. This estimation is crude because Coulomb energy depends on the shape of the nucleus sensitively. By using the data in Ref. [50] and the following Coulomb energy formula

$$E_{\mathrm{Coulomb}} = 0.70\frac{Z^2}{A^{1/3}}(1 - 0.76\,Z^{-2/3}), \tag{5}$$

we obtained the energy of the $T = 2$ isospin analog state in ^{68}Ge to be 6.054 MeV, which is close to the energy of the $0^+_{T=2}$ at 6.060 MeV in our IBM-3 calculation. The λ parameter determines the position of the mixed symmetry states. They occur when the motions of the protons and neutrons are not in phase. The mixed-symmetry states lie usually high in the energy, and therefore the Majorana interaction is varied so as to push up these states at about 5 MeV. Using the available experimental and theoretical information as a guide we have derived "initial" values of input parameters. We use a perturbed U(5) Hamiltonian together with a perturbation predominantly of O(6) type. The spectrum is dominated by the vibrational $C_{1\mathrm{U}_d(5)}$ term, where the coefficient of $C_{1\mathrm{U}_d(5)}$ is very large. The low-lying levels of ^{68}Se can be described by the following

TABLE I. The parameters of the IBM-3 Hamiltonian used for the description of the ^{68}Se isotope. The $\epsilon_{d\rho} - \epsilon_{s\rho} = 0.748$ MeV, where $\rho = \pi, \nu$.

$A_i(i = 0, 1, 2)$	$C_{i0}(i = 0, 2, 4)$	$C_{i2}(i = 0, 2, 4)$	$C_{i1}(i = 1, 3)$	$B_i(i = 0, 2)$	$D_i(i = 0, 2)$	$G_i(i = 0, 2)$
−4.380,−1.680,1.680	−4.980,−4.432,-4.110	1.080,1.628,1.950	−1.996,−1.766	−4.400 1.660	0.000,0.000	0.045,0.045

Hamiltonian:

$$\begin{aligned} H = & -0.18C_{2\mathrm{U}_{sd}(6)} + 1.010T(T+1) + 0.430C_{1\mathrm{U}_d(5)} \\ & + 0.010C_{2\mathrm{O}_{sd}(6)} + 0.012C_{2\mathrm{U}_d(5)} \\ & + 0.031C_{2\mathrm{O}_d(5)} + 0.023C_{\mathrm{O}_d(3)}. \end{aligned} \quad (6)$$

The corresponding parameters in the form of Eq. (3) are also given in Table I. In order to have a good understanding how these parameters affect the collective feature of the states we vary the parameters around the optimal values. The procedure was to vary one parameter in small steps while keeping the other fixed. The $C_{L_2T_2}$, with $L_2 = 0, 2, 4$ and $T_2 = 0, 2$, are well known to affect only the details of the energy spectrum, in particular the splitting of the two-phonon states. The parameters A_1, C_{11}, and C_{31} are similar to the Majorana interactions in the IBM-2, which will be referred also as Majorana interactions here, and they are important to shift the states with mixed symmetry with respect to the total symmetric ones. The B_0 and B_2 parameters have a very large effect on the energy levels, whereas the G_0 and G_2 parameters have very small effect, especially on the ground-state band. The D_0 and D_2 parameters are adopted zero because in the SU(3) Casimir operator is absent in the Casimir form of the Hamiltonian, reflecting the fact that the nucleus is γ soft.

III. EXCITATION ENERGY

The ^{68}Se isotope ($N = Z = 34$) has $N_\pi = N_\nu = 3$ bosons. In the ground state of this nucleus the $f_{7/2}$ shell is filled, and the six protons and six neutrons outside $N = Z = 28$ closed shell core occupy the lowest-lying orbitals ranging from $p_{3/2}$ to $f_{5/2}$. Accordingly, both proton and neutron boson are of particle type. The $T = 0$ states were observed up to $J^\pi = 26^+$ [25]. Experimental data [51] and calculations from three models, the IBM-3, PSM [31] and SM [29], are presented in the Fig. 1 and discussed in the text. The energy levels have been ordered into groups according to isospin and U(6) symmetry labels. In Fig. 1, we show the fits to the data for the $2_1^+, 4_1^+, 6_1^+, 8_1^+, 10_1^+$, and 12_1^+ states of ground-state band, as well as the $2_2^+, 4_2^+$, and 6_2^+ states of the first excitation band, respectively. The splitting of 2_2^+ and 4_1^+ in the "two phonon states" is well reproduced. Clear reproduction of the low-lying structural features observed in the experimental data can be seen, especially those of the ground-state band in the calculations of all three models as shown in Fig. 1. We have a very reasonable fit to the experimental data about these states up to 10_1^+. The maximum deviation in the first excitation band occurs at the 6_2^+ state with an error about 0.5 MeV. It is noticed that the isotope exhibits backbanding in the ground band, which can be explained by the collective backbanding proposed in Ref. [52]. Recently, Sun *et al.* [28] studied the property of the 8^+ state of ground band for ^{68}Se and observed the first and second sharp backbending at $J = 8^+$ and $J = 16^+$, respectively, and they considered these states as isomeric states because no allowed low multipolarity γ-transition matrix elements can connect these states to the nearby ground-state bands. We see a good agreement between the calculated energy of $J = 8_2^+$ in the PSM result with experimental ones, and this is outside of the IBM-3 model space. The state at 3.073 MeV in the experimental data with a transition $E_\gamma = 2.220$ MeV to the 2_1^+ state [51] is closed to the full symmetry state with $J = 4_3^+$ at 3.013 MeV in our IBM-3 results.

We have analyzed the wave function of the low lying states, and found that the main components of the wave function for the states in the ground-state band are all basically $s^N, s^{N-1}d, s^{N-2}d^2, s^{N-3}d^3$ and so on configurations. For instance

$$\begin{aligned} |2_1^+\rangle = & -0.472\{|s_\nu^3 s_\pi^2 d_\pi^1\rangle + |s_\pi^3 s_\nu^2 d_\nu^1\rangle\} + 0.272\{|s_\nu^2 s_\pi^1 s_\delta^2 d_\pi^1\rangle \\ & + |s_\nu^1 s_\pi^2 s_\delta^2 d_\nu^1\rangle + |s_\nu^2 s_\pi^2 s_\delta^1 d_\delta^1\rangle\} + 0.373|s_\delta^5 d_\delta^1\rangle \\ & - 0.334|s_\nu^1 s_\pi^1 s_\delta^3 d_\delta^1\rangle + \cdots, \\ |4_1^+\rangle = & +0.518|s_\pi^2 s_\nu^2 d_\nu^1 d_\pi^1\rangle - 0.244\{|s_\nu^2 s_\pi^1 s_\delta^1 d_\pi^1 d_\delta^1\rangle \\ & + |s_\nu^1 s_\pi^2 s_\delta^1 d_\nu^1 d_\delta^1\rangle\} + 0.292\{|s_\nu^3 s_\pi^1 d_\pi^2\rangle + |s_\nu^1 s_\pi^3 d_\nu^2\rangle\} \\ & + 0.211\{|s_\nu^1 s_\delta^3 d_\pi^1 d_\delta^1\rangle + |s_\pi^1 s_\delta^3 d_\nu^1 d_\delta^1\rangle\} \\ & + 0.259|s_\delta^2 s_\nu^1 s_\pi^1 d_\delta^2\rangle - 0.374|s_\delta^4 d_\delta^2\rangle + \cdots, \\ |6_1^+\rangle = & +0.451\{|s_\nu^2 s_\pi^1 d_\nu^1 d_\pi^2\rangle + |s_\nu^1 s_\pi^2 d_\nu^2 d_\pi^1\rangle\} + 0.225\{|s_\pi^1 s_\delta^2 d_\nu^1 d_\delta^2\rangle \\ & + |s_\nu^1 s_\delta^2 d_\pi^1 d_\delta^2\rangle\} + 0.1502\{|s_\nu^3 d_\pi^3\rangle + |s_\pi^3 d_\nu^3\rangle\} \\ & + 0.184|s_\nu^1 s_\pi^1 s_\delta^1 d_\delta^3\rangle - 0.300|s_\pi^1 s_\nu^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle \\ & - 0.375|s_\delta^3 d_\delta^3\rangle + \cdots. \end{aligned}$$

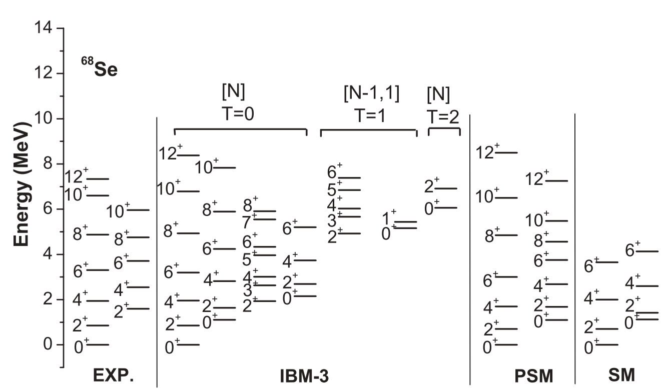

FIG. 1. Comparison between lowest excitation energy bands ($T = T_z, T_z + 1$, and $T_z + 2$) of the IBM-3 calculation, PSM from Ref. [31], SM from Ref. [29], and experimental data from Ref. [51] for ^{68}Se.

The first and second $J = 3^+$ states are both full symmetry states at 2.630 and 3.645 MeV, respectively. The IBM-3 wave functions of the first excitation states with $J = 0^+$ and $T = T_Z, T_Z + 1$, and $T_Z + 2$ are

$$\begin{aligned}|0^+_{T=0}\rangle &= -0.509|s_\nu^2 s_\pi^2 d_\pi^1 d_\nu^1\rangle + 0.367|s_\delta^4 d_\delta^2\rangle - 0.293\{|s_\nu^3 s_\pi^1 d_\pi^2\rangle \\ &\quad + |s_\pi^3 s_\nu^1 d_\nu^2\rangle\} - 0.255|s_\pi^1 s_\nu^1 s_\delta^2 d_\delta^2\rangle \\ &\quad + 0.240\{|s_\nu^1 s_\pi^1 s_\delta^2 d_\pi^1 d_\nu^1\rangle + |s_\nu^2 s_\pi^1 s_\delta^1 d_\pi^1 d_\delta^1\rangle \\ &\quad + |s_\nu^1 s_\pi^2 s_\delta^1 d_\nu^1 d_\delta^1\rangle\} + \cdots, \\ |0^+_{T=1}\rangle &= +0.583\{|s_\nu^3 s_\pi^1 d_\pi^2\rangle - |s_\pi^3 s_\nu^1 d_\nu^2\rangle\} + 0.238\{|s_\pi^2 s_\delta^2 d_\nu^2\rangle \\ &\quad - |s_\nu^2 s_\delta^2 d_\pi^2\rangle + |s_\nu^1 s_\pi^2 s_\delta^1 d_\nu^1 d_\delta^1\rangle - |s_\nu^2 s_\pi^1 s_\delta^1 d_\pi^1 d_\delta^1\rangle\} \\ &\quad + 0.206\{|s_\nu^1 s_\delta^3 d_\pi^1 d_\delta^1\rangle - |s_\pi^1 s_\delta^3 d_\nu^1 d_\delta^1\rangle\} \cdots, \\ |0^+_{T=2}\rangle &= +0.682|s_\delta^6\rangle - 0.373|s_\nu^3 s_\pi^3\rangle - 0.107|s_\delta^4 d_\delta^2\rangle \\ &\quad - 0.075|s_\nu^2 s_\pi^2 d_\nu^1 d_\pi^1\rangle + \cdots .\end{aligned}$$

The 0_2^+ is a remarkably pure two d-boson state containing a significant amount of d_δ^2 component, and the expectation values of s- and d- boson numbers of this state are 3.987 and 2.013, respectively. Although the $0^+_{T=2}$ state is mainly a s^N state with some admixture of the two-d-boson state. The first calculated $T = 1$ and 2 states are identified according to the analog states in ^{68}As and ^{68}Ge, respectively. The excitation energy from the $T = 0$ ground-state bands to the first isospin excitation $T = 1$ band states is well reproduced, where the calculated energy of the $3^+_{T=1}$ state equals 5.67 MeV, which is close to the energy of the $T = 1$ isospin analog ground state in ^{68}As at 4.86 MeV.

From Fig. 1, we see that the mixed $J = 2^+$ state with $[N - 1, 1]$ partition is lower in energy than the 1^+ state, i.e., the lowest mixed symmetry state is a $J = 2^+$. Among the low-lying states, the 1^+ state is of particular interest. It is predicted by the IBM [53,54] as a mixed symmetry state and is called the *scissors mode* [55], characterized by large values of $B(M1)$ and small $B(E2)$ values. Classically, these states can be regarded as small amplitude oscillations of the angle between symmetry axes of the deformed valence neutrons and valence protons [56]. It has been discovered in high-resolution electron-scattering experiment [57]. The 1^+ $T = 1$ state has been observed in $N = Z$ ^{44}Ti nucleus at 7.216 MeV. In the case of $Z = N$ nucleus the first calculated 1^+ state comes from $[N - 1, 1]$ partition with $T = 1$ at a relatively high excitation energy of 5.540 MeV for ^{68}Se. No experimental data are available about this state at the moment. In the IBM-3 Hamiltonian one can fit these energy levels by changing the Majorana interaction with $L_2 = 1$. The main components of the mixed symmetry 1^+ state are $0.8215|s_\pi^2 s_\delta^2 d_\nu^1 d_\pi^1\rangle - 0.3873|s_\pi^1 s_\nu^1 s_\delta^2 d_\nu^1 d_\pi^1\rangle$. The IBM-3 calculation has well reproduced the staggering of odd-even angular momentum levels in the γ band, i.e., $(3_1^+, 4_2^+)$,.... The calculation predicted a 0_3^+ level at 2.153 MeV in ^{68}Se, and this remains to be seen in experiment.

IV. ELECTROMETRIC TRANSITIONS

In this section the $M1$ and $E2$ transitions in ^{68}Se are investigated. The $E2$ transition can be calculated by the following transition operators [44]

$$T(E2) = T^0(E2) + T^1(E2), \tag{7}$$

where

$$T^0(E2) = \alpha_0\sqrt{3}[(s^\dagger\tilde{d})^{20} + (d^\dagger\tilde{s})^{20}] + \beta_0\sqrt{3}[(d^\dagger\tilde{d})^{20}], \tag{8}$$

$$T^1(E2) = \alpha_1\sqrt{2}[(s^\dagger\tilde{d})^{21} + (d^\dagger\tilde{s})^{21}] + \beta_1\sqrt{2}[(d^\dagger\tilde{d})^{21}]. \tag{9}$$

The $M1$ transition is also a one boson operator with an isoscalar part and an isovector part

$$T(M1) = T^0(M1) + T^1(M1), \tag{10}$$

where,

$$T^0(M1) = g_0\sqrt{3}(d^\dagger\tilde{d})^{10} = g_0 L/\sqrt{10}, \tag{11}$$

$$T^1(M1) = g_1\sqrt{2}(d^\dagger\tilde{d})^{11}, \tag{12}$$

and g_1 and g_0 are the isovector and isoscalar g factor, respectively, and L is the angular momentum operator.

$E2$ transitions are calculated with parameters close to the values used in the recent work in Ref. [24], where $\alpha_0 = \beta_0 = 0.076$ eb and $\alpha_1 = \beta_1 = 0.05$ eb. Due to the predominantly isovector nature of $T(M1)$ operator, the g factors are $g_1 = 2.7\mu_N$ and $g_0 = 0.0\mu_N$, respectively. In addition, the isovector $M1$ transition between the $T = 0$ states are forbidden. The $E2$ and $M1$ transitions are shown in Fig. 2 and listed in Table II.

To see the contributions from the isoscalar and isovector parts in the $M1$ and $E2$ transitions directly, we write the terms in the zero isospin z component of the transition operators as follows

$$\begin{aligned}T^0_{sd}(E2) &= [(s^\dagger\tilde{d})^2 + (d^\dagger\tilde{s})^2]_\pi + [(s^\dagger\tilde{d})^2 + (d^\dagger\tilde{s})^2]_\delta \\ &\quad + [(s^\dagger\tilde{d})^2 + (d^\dagger\tilde{s})^2]_\nu,\end{aligned} \tag{13}$$

$$T^0_{dd}(E2) = [(d^\dagger\tilde{d})^2]_\pi + [(d^\dagger\tilde{d})^2]_\delta + [(d^\dagger\tilde{d})^2]_\nu, \tag{14}$$

$$T^1_{sd}(E2) = [(s^\dagger\tilde{d})^2 + (d^\dagger\tilde{s})^2]_\pi - [(s^\dagger\tilde{d})^2 + (d^\dagger\tilde{s})^2]_\nu, \tag{15}$$

$$T^1_{dd}(E2) = [(d^\dagger\tilde{d})^2]_\pi - [(d^\dagger\tilde{d})^2]_\nu, \tag{16}$$

$$T^0_{dd}(M1) = [(d^\dagger\tilde{d})^1]_\pi + [(d^\dagger\tilde{d})^1]_\delta + [(d^\dagger\tilde{d})^1]_\nu, \tag{17}$$

$$T^1_{dd}(M1) = [(d^\dagger\tilde{d})^1]_\pi - [(d^\dagger\tilde{d})^1]_\nu. \tag{18}$$

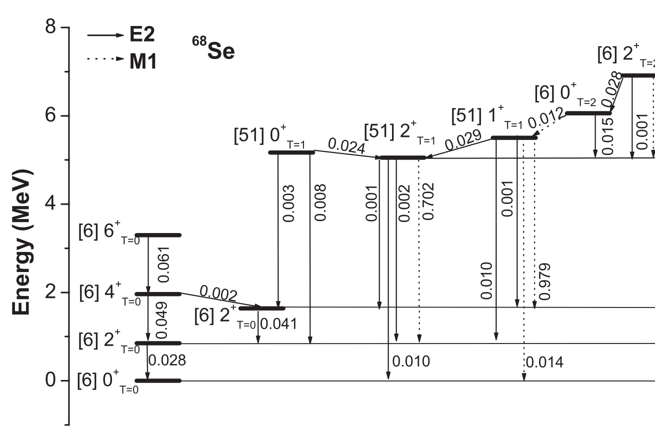

FIG. 2. Energy levels $(T = T_z, T_z + 1, T_z + 2)$ and the $B(E2)$(solid line) and $B(M1)$(dotted line) values are in units of $e^2 \cdot b^2$ and μ_N^2, respectively. The U(6) partition is given in parentheses for each level.

TABLE II. The $E2$ and $M1$ transition properties. Columns 2–6 are the reduced matrix elements for the various terms in the $E2$ transition operator. Column 7 is the reduced $E2$ transition probability. Columns 8 are the reduced matrix elements for the isovector $M1$ transition operator, and column 9 is that for the total $M1$ transition operator. Column 10 is the $B(M1)$ value. Here the $B(E2)$ and $B(M1)$ are given relative percentages and the reduced values are shown in Fig. 2.

$J_i^+ \to J_f^+$	$T_{sd}^0(E2)$	$T_{dd}^0(E2)$	$T_{sd}^1(E2)$	$T_{dd}^1(E2)$	$T(E2)$	$B(E2)$	$T_{dd}^1(M1)$	$T(M1)$	$B(M1)$
$2_1^+ \to 0_1^+$	−4.9772				−0.3783	100			
$2_2^+ \to 2_1^+$	2.9191		−0.0002		0.2218	146			
$1_{T=1}^+ \to 2_1^+$			−1.5416		−0.0770	35			
$1_{T=1}^+ \to 2_2^+$		0.0001		−0.4908	−0.0245	4	−0.5809	−0.7662	100
$1_{T=1}^+ \to 0_1^+$							0.1541	0.2033	1.43
$1_{T=1}^+ \to 2_{T=1}^+$	1.7523				0.1331	105			
$2_{T=1}^+ \to 0_1^+$			−4.5318		−0.2265	35			
$2_{T=1}^+ \to 2_1^+$				0.8176	0.0408	7	0.6352	0.8377	71.71
$2_{T=1}^+ \to 2_2^+$	−0.0002		−0.4776		−0.0239	2			
$0_{T=1}^+ \to 2_{T=1}^+$	0.9092				0.0691	86			
$0_{T=1}^+ \to 2_1^+$			0.8192		0.0409	28			
$0_{T=1}^+ \to 2_2^+$		−0.0001	−0.0001	0.4600	0.0229	11			
$0_{T=2}^+ \to 1_{T=1}^+$							−0.0487	−0.0643	1.23
$0_{T=2}^+ \to 2_{T=1}^+$			−1.1100		−0.0555	53			
$2_{T=2}^+ \to 2_{T=1}^+$				−0.4478	−0.0224	2	−0.3479	−0.4588	21.45
$2_{T=2}^+ \to 1_{T=1}^+$			−1.0901		−0.0545	6			
$2_{T=2}^+ \to 0_{T=2}^+$	4.9775				0.3783	100			

The total transition operators are

$$T(E2) = \alpha_0 T_{sd}^0(E2) + \beta_0 T_{dd}^0(E2) + \alpha_1 T_{sd}^1(E2) + \beta_1 T_{dd}^1(E2), \tag{19}$$

$$T(M1) = \sqrt{\frac{3}{4\pi}}\left\{g_0 T_{dd}^0(M1) + g_1 T_{dd}^1(M1)\right\}. \tag{20}$$

The reduced $E2$ and $M1$ transition probabilities are

$$B(E2; J_i \to J_f) = \frac{2J_f+1}{2J_i+1}|\langle J_f\|T(E2)\|J_i\rangle|^2, \tag{21}$$

$$B(M1; J_i \to J_f) = \frac{2J_f+1}{2J_i+1}|\langle J_f\|T(M1)\|J_i\rangle|^2, \tag{22}$$

where we have used the Brink-Satchler convention [58] for the reduced density matrix element $\langle J_f\|T(E2)\|J_i\rangle$.

For this $N = Z$ nucleus, it is found that the transition between the $T = 1$ states does not have an isovector component as shown in Table II. A very peculiar phenomenon is the decay of the first isospin excitation $J^\pi = 2^+$ state. This state decays preferentially to the 2_1^+ state through an $M1$ transition with $B(M1) = 0.702\mu_N^2$, and it does not decay to the 2_2^+ state. The latter transition is almost forbidden. When looking at the individual $E2$ transition terms as shown in Table II, we found that the isoscalar $E2$ components are large for both $2_1^+ \to 0_1^+$ and $2_2^+ \to 2_1^+$ transitions. The $1_1^+ \to 2_{T=1}^+$ and $0_{T=1}^+ \to 2_{T=1}^+$ $E2$ transitions with $\Delta T = 0$ are similarly isoscalar dominant. The $M1$ transition from the $1_1^+ \to 2_2^+$ is of special interest. It is of allowed $\Delta T = 1$ nature with $B(M1)$ =0.9790 μ_N^2, whereas the transition $1_1^+, T = 1 \to 2^+, T = 1$ is isospin forbidden. The quadrupole moments of the 2_1^+ and 4_1^+ states are $Q(2_1^+) = 0.1306$ *eb* and $Q(4_1^+) = 0.1947$ *eb*, respectively.

V. CONCLUSION

This calculation has yielded several interesting results. First, the excitation energy of the second 0_2^+ state in ^{68}Se is identified as 1.106 MeV. The present calculated energy is consistent with the SM and PSM results as shown in Fig. 1, and it will be highly desirable to substantiate this model predictions in future experiment.

Second, when we did the calculation for this nucleus, we did not take into account of the experimental 2_2^+ states in the fitting. However, the calculated 2_2^+ state result agrees very well with the recent SM and PSM calculation and agrees with the experimental data at 1.594 MeV. In all theoretical studies, no $J = 2^+$ state are closed to the experimental $J = 2^+$ at 1.197 MeV [26]. The nature of this level needs further study.

Third, based on isospin analog state in ^{68}Ge, the calculation suggests that the first and second isospin excited $J = 2^+$ and $J = 0^+$ states with $T = 1$ are at 4.922 and 5.163 MeV, respectively, with $[N-1, 1]$ U(6) label. These suggestions do not contradict the experimental data [51].

Fourth, because IBM-3 has three charge states, it is possible to have U(6) partitions into three rows, namely the $[N1, N2, N3]$ states that are characteristic of IBM-3. We found that such state comes high in energy, up at about 7.5 MeV, and the lowest such example being a scissor mode at 7.486 MeV, which is predominantly the [4, 1, 1] partition with $T = 1$. The first state coming from [2, 2, 2] partition is a $J = 0^+$ at 8.223 MeV with $T = 0$. It is very significant if these properties are observed in experiment.

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