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Improved short-range correlations and $0\nu\beta\beta$ nuclear matrix elements of ^{76}Ge and ^{82}Se

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We calculate the nuclear matrix elements of the neutrinoless double beta ($0\nu\beta\beta$) decays of ^{76}Ge and ^{82}Se for the light neutrino exchange mechanism. The nuclear wave functions are obtained by using realistic two-body forces within the proton-neutron quasiparticle random-phase approximation (pnQRPA). We include the effects that come from the finite size of a nucleon, from the higher-order terms of nucleonic weak currents, and from the nucleon-nucleon short-range correlations. Most importantly, we improve on the presently available calculations by replacing the rudimentary Jastrow short-range correlations by the more advanced unitary correlation operator method (UCOM). The UCOM-corrected matrix elements turn out to be notably larger in magnitude than the Jastrow-corrected ones. This has drastic consequences for the detectability of $0\nu\beta\beta$ decay in present and future double beta experiments.

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The nuclear matrix elements of neutrinoless double beta ($0\nu\beta\beta$) decay have become an important issue in present-day neutrino physics (see, e.g., Refs. [1–4]). This has been boosted by the verification of the existence of neutrino mass by the oscillation experiments [5] and the claimed discovery of the $0\nu\beta\beta$ decay [6,7]. Further incentive to produce reliable nuclear matrix elements comes from the needs of the running NEMO 3 [8] and CUORICINO [9] experiments, as well as future large-scale experiments under R&D planning and construction (see, e.g., Ref. [10]). For these experiments the nuclear matrix elements are an essential prerequisite in the extraction of reliable values for the absolute mass scale of the neutrino [11] and possibly the CP phases of the neutrino-mixing matrix [12].

The most popular nuclear model to treat the structure of medium-heavy and heavy double beta decaying nuclei is the proton-neutron quasiparticle random-phase approximation (pnQRPA) [13–15]. Some recent shell model results are also available [16,17]. The pnQRPA model is tailored to efficiently describe the energy levels of odd-odd nuclei and their beta decays to the neighboring even-even nuclei [18]. Its derivative, renormalized pnQRPA [19], has also been used to compute double beta matrix elements [20,21], although its use has been heavily criticized (see, e.g., Ref. [22] and references therein). A problem with the use of the pnQRPA (and the renormalized pnQRPA) is that it contains a free parameter, the so-called particle-particle strength parameter, g_{pp}, that controls the magnitude of the proton-neutron two-body interaction matrix elements in the $T = 0$ pairing channel [23,24]. There are basically two ways to fix the value of this parameter, either by using the data on two-neutrino double beta ($2\nu\beta\beta$) decay [21] or the data on single beta decay [25,26]. In the case of ^{76}Ge and ^{82}Se there is no available data on single beta decays, so in this work we have chosen to use the $2\nu\beta\beta$ data to fix the possible values of g_{pp}.

In this article we address the mass mode of the $0\nu\beta\beta$ decay where a light virtual Majorana neutrino is exchanged by the two decaying neutrons of the initial nucleus. Typically the exchanged momentum is so large as to force the two neutrons to overlap unless steps are taken to prevent the occurrence of such a spurious event. The traditional way [27,28] to remove this spuriosity is to introduce an explicit Jastrow-type correlation function into the involved two-body transition matrix elements in the parametrization of Miller and Spencer [29]. This method, although microscopically inspired, is just a phenomenological way to introduce short-range correlations into the two-nucleon relative wave function. A conspicuous flaw of the Jastrow method is that the Jastrow function effectively cuts out the small r [30] part from the relative wave function of the two nucleons. For this reason, the traditionally adopted Jastrow procedure [13] does not conserve the norm of the relative wave function [31].

In the present calculations we improve on the Jastrow method and adopt the more sophisticated microscopic approach of the unitary correlation operator method (UCOM) [32]. In the UCOM one obtains the correlated many-particle state $|\tilde{\Psi}\rangle$ from the uncorrelated one as

$$|\tilde{\Psi}\rangle = C|\Psi\rangle, \tag{1}$$

where C is the unitary correlation operator [32]. Due to the unitarity of the operator C, the norm of the correlated state is conserved and no amplitude is lost in the relative wave function. In the $0\nu\beta\beta$ calculations this leads to a more complete description of the relative wave function at small distances r, as was demonstrated in [33]. It should be stressed that no extra free parameters are introduced by the use of the UCOM at the level of double beta decay calculations. All the needed parameters have been fixed by minimization procedures for $\beta\beta$-independent observables [34]. The UCOM method has been demonstrated [35] to produce good results for the binding energies of nuclei over a wide mass range already at the Hartree-Fock level. It was also shown that the UCOM renders a good starting point for inclusion of long-range correlations by means of many-body perturbation theory.

In Ref. [33] it was demonstrated for the ^{48}Ca and ^{76}Ge $0\nu\beta\beta$ decays that the Jastrow procedure leads to the excessive reduction of 30–40% in the magnitudes of the $0\nu\beta\beta$ nuclear matrix elements. At the same time the UCOM reduces the magnitudes of the matrix elements only by 7–16%. This explains the large short-range correlation corrections to the matrix elements of Ref. [21].

The issue about the magnitude of the short-range corrections is an extremely important one since it directly affects the magnitudes of the relevant nuclear matrix elements used to extract the neutrino masses from potentially succesful future double beta experiments. There are large differences between the Jastrow and UCOM corrections, the Jastrow corrected matrix elements being substantially smaller that the UCOM corrected ones. This difference would severely alter the predicted sensitivities of future neutrino experiments, the UCOM corrected matrix elements being more favorable for the detection of neutrinoless double beta decay.

The double beta decays of ^{76}Ge and ^{82}Se proceed through the virtual states of the intermediate nuclei ^{76}As and ^{82}Br to the ground states of the final nuclei ^{76}Se and ^{82}Kr. By assuming the neutrino-mass mechanism to be the dominant one, we can write the inverse of the half-life as [14]

$$\left[t_{1/2}^{(0\nu)}\right]^{-1} = G_1^{(0\nu)} \left(\frac{\langle m_\nu \rangle}{m_{\rm e}}\right)^2 (M^{(0\nu)})^2, \qquad (2)$$

where $m_{\rm e}$ is the electron mass and $G_1^{(0\nu)}$ is the leptonic phase-space factor. The $0\nu\beta\beta$ nuclear matrix element $M^{(0\nu)}$ consists of the Gamow–Teller, Fermi, and tensor parts as

$$M^{(0\nu)} = M_{\rm GT}^{(0\nu)} - \left(\frac{g_{\rm V}}{g_{\rm A}}\right)^2 M_{\rm F}^{(0\nu)} + M_{\rm T}^{(0\nu)}. \qquad (3)$$

Numerical calculations show that the tensor part in Eq. (3) is quite small and its contribution can be safely neglected in what follows. The expressions for the phase-space factor, the effective neutrino mass $\langle m_\nu \rangle$ and the matrix elements of Eq. (3) are given, e.g., in Refs. [11,13,14]. To the "bare" matrix elements we have applied the Jastrow short-range correlation corrections, together with the *higher-order terms of nucleonic weak currents* and the *nucleon's finite-size* corrections in the way described in Refs. [21,36]. In addition, we have computed the corrected matrix elements by replacing the Jastrow correlations with the UCOM correlations.

We calculated the wave functions of all the nuclear states in the intermediate nuclei by the use of the pnQRPA framework in the model space of 1p-0f-2s-1d-0g-0h$_{11/2}$ single-particle orbitals, both for protons and neutrons. The single-particle energies were obtained from a spherical Coulomb-corrected Woods–Saxon potential with a standard parametrization, optimized for nuclei near the line of beta stability. Slight adjustments were done for some of the energies at the vicinity of the proton and neutron Fermi surfaces to better reproduce the low-energy spectra of the neighboring odd-A nuclei and those of the intermediate nuclei.

The Bonn-A G-matrix was used as a two-body interaction and was renormalized in the standard way, as discussed, e.g., in Refs. [37,38]. Due to this phenomenological renormalization we did not perform an additional UCOM renormalization [34] of the two-body interaction. After fixing all the Hamiltonian parameters, the only free parameter left was the $g_{\rm pp}$ parameter mentioned earlier. In Fig. 1 we have studied the $g_{\rm pp}$ dependence of the matrix element $M^{(0\nu)}$ of Eq. (3) for both ^{76}Ge and ^{82}Se. We have used $R = 1.2A^{1/3}$ fm as the nuclear radius, and the finite-size, higher-order term and UCOM corrections were taken into account. Here one can see the typical breakdown of the pnQRPA at large values of $g_{\rm pp}$. Moreover, the corresponding breakdown points for the two nuclei are close to each other.

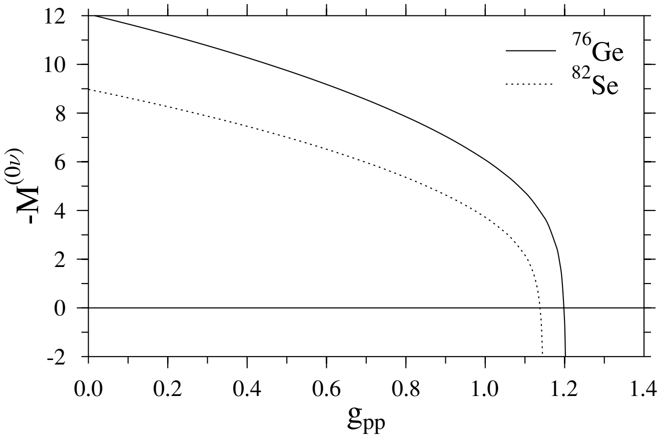

FIG. 1. The $0\nu\beta\beta$ nuclear matrix elements $M^{(0\nu)}$ of Eq. (3) for the decays of ^{76}Ge and ^{82}Se as functions of the particle-particle interaction parameter $g_{\rm pp}$.

We obtained the physical values of $g_{\rm pp}$ by using the method of Ref. [21]. Consequently, we used the recommended data [39] on $2\nu\beta\beta$-decay half-lives of ^{76}Ge and ^{82}Se by including the experimental error limits and the uncertainty in the value of the axial-vector coupling constant $1.0 \leqslant g_{\rm A} \leqslant 1.254$. The resulting intervals [40] of "experimental matrix elements" were then converted to the following intervals of $g_{\rm pp}$ values:

$$\begin{aligned} 1.02 \leqslant g_{\rm pp} \leqslant 1.06 \quad &\text{for} \quad ^{76}\text{Ge}, \\ 0.96 \leqslant g_{\rm pp} \leqslant 1.00 \quad &\text{for} \quad ^{82}\text{Se}. \end{aligned} \qquad (4)$$

In Fig. 2 we show for the ^{82}Se decay the decomposition of the total matrix element Eq. (3) into multipoles. The bare matrix element contains no short-range, finite-size or higher-order-term corrections, whereas the UCOM and Jastrow matrix elements include all these corrections. The spread in the multipole contributions corresponds to the $g_{\rm pp}$ interval for ^{82}Se in Eq. (4). More specifically, the upper end of the bar represents the case $g_{\rm pp} = 0.96$ with $g_{\rm A} = 1.0$ and the lower

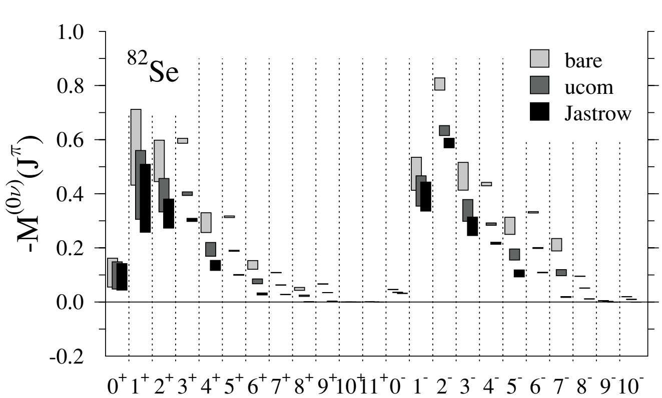

FIG. 2. Multipole decomposition of the matrix element $M^{(0\nu)}$ of Eq. (3) for the decay of ^{82}Se. The spread in the multipole contributions corresponds to the $g_{\rm pp}$ interval for ^{82}Se in Eq. (4).

TABLE I. Matrix element $M^{(0\nu)}$ of Eq. (3) computed by correcting successively the bare matrix element (b.m.e.) by the higher-order terms in the nucleonic current (A), by the nucleon finite-size effect (B), and by either the Jastrow (C) or UCOM (D) correlations. The values $g_{pp} = 1.0$ and $g_A = 1.254$ were used in the calculations.

Nucleus	b.m.e.	+A	+A+B	+A+B+C	+A+B+D
^{76}Ge	−8.529	−7.720	−6.356	−4.723	−6.080
^{82}Se	−5.398	−4.862	−3.914	−2.771	−3.722

end the case $g_{pp} = 1.00$ with $g_A = 1.254$. In the figure we see that the corrections substantially reduce the magnitude of a given multipole contribution. The differences between the Jastrow and UCOM-corrected multipole contributions increase with increasing multipole. This pattern is reminiscent of the one shown in Fig. 3 of Ref. [33] where no finite-size or higher-order term corrections were taken into account. All this bears evidence of the fact that the rudimentary Jastrow method overestimates the effect of nuclear short-range correlations in $0\nu\beta\beta$-decay calculations.

In Table I we display the effects of the various corrections to the matrix elements $M^{(0\nu)}$ of ^{76}Ge and ^{82}Se. There we show in the second column the bare matrix elements (b.m.e.), then in the third column we show the b.m.e. corrected for the higher-order terms in the nucleonic current (A). In the fourth column we have added the nucleon finite-size effect (B) to the previous matrix elements (b.m.e.+A), and finally, in the last two columns, we have added to the previous matrix elements (b.m.e.+A+B) either the Jastrow (C) or UCOM (D) short-range corrections.

Table I is very interesting in the sense that there we can access the magnitudes of the various corrections to the bare matrix element. The magnitudes of the corrections coming from the finite nucleon size and the higher-order terms of the nucleonic current together amount to 25–30%. In fact, the magnitude of our bare matrix element is roughly the one reported in Refs. [21,36]. Even after correcting by the higher-order terms and the nucleon finite-size effect the matrix elements agree roughly, as shown in Table II. From the table we also see that our computed Jastrow corrections are much less than the ones obtained in Refs. [21,36]. The reason for this is not clear. The net effect is that our final matrix elements, especially the UCOM-corrected ones, are much larger than those of Refs. [21,36]. On the other hand, our shell model computed Jastrow correlation corrections for ^{48}Ca [33] agree with the ones of Ref. [41].

TABLE II. Matrix element $|M^{(0\nu)}|$ of Eq. (3) for ^{76}Ge obtained in the present calculation and by Šimkovic *et al.* [36]. Shown are the results without and with the short-range correlations (s.r.c.) for $g_A = 1.254$.

without s.r.c.		with s.r.c.		
Present	[36]	Jastrow	UCOM	[36]
6.36	5.16	4.72	6.08	2.80

TABLE III. Beta decay log ft values for transitions from the 2_1^- states of ^{76}As and ^{82}Br to one- and two-phonon states in the indicated final nuclei.

J_f	^{76}Se		^{82}Kr	
	Exp.	Th.	Exp.	Th.
$0^+_{g.s.}$	9.7	9.0	8.9	9.3
2_1^+	8.1	7.7	7.9	7.7
0_2^+	10.3	9.2	$\geqslant 9.6$	9.4
2_2^+	8.2	8.7	8.0	9.0
4_1^+	11.1	10.9	?	11.1

To check the consistency of our calculations we also computed the single β^- decay rates from the lowest 2^- states of the intermediate nuclei ^{76}As and ^{82}Br to the lowest 2^+ collective state, 2_1^+, and its $0_2^+, 2_2^+$ and 4_1^+ two-phonon excitations (see, e.g., Ref. [38]) in ^{76}Se and ^{82}Kr. The wave function of the 2_1^+ state was calculated by the use of the quasiparticle random-phase approximation (QRPA) [18] and its energy was fixed to the experimental one [38]. Beta decays to the mentioned final states were computed by the method of the multiple commutator model (MCM) of Ref. [38].

The 2^- wave function was calculated by using the central value of g_{pp} in the intervals of Eq. (4). In the case of a 2^- initial state this choice works fine since the calculated beta decay rates depend only weakly on g_{pp} within the range relevant for the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays. We compare the computed log ft values with the available data in Table III. From this table it is seen that the computed numbers nicely reproduce the trends of the measured ones, although the assumption that the states $0_2^+, 2_2^+$ and 4_1^+ are pure two-phonon excitations is an idealized one. It has to be noted that there is no experimental data on beta decay of the lowest 1^+ state in ^{76}As and ^{82}Br. For some double-beta-decaying systems the 1^+ data exists and matching of beta and double beta decay could be more problematic due to the stronger g_{pp} dependence of the beta decay rates from a 1^+ intermediate state [25].

Our final results for the $0\nu\beta\beta$ nuclear matrix elements have been collected in Table IV. These matrix elements were calculated with the UCOM short-range corrections by also taking into account the finite size of the nucleons and the

TABLE IV. Nuclear $0\nu\beta\beta$ matrix elements of Eq. (3) for the decays of ^{76}Ge and ^{82}Se. The UCOM and other corrections are included. The used g_{pp} values are also indicated.

g_{pp}	^{76}Ge		^{82}Se	
	1.02	1.06	0.96	1.0
$M_F^{(0\nu)}$	1.923	1.803	1.304	1.214
$M_{GT}^{(0\nu)}$	−4.632	−4.208	−3.293	−2.950
$M^{(0\nu)}$	−6.555	−5.355	−4.597	−3.722

TABLE V. Values of the matrix element $|M^{(0\nu)}|$ of Eq. (3) obtained in several recent calculations.

Nucleus	Present	[21]	[36]	[26]
^{76}Ge	5.36–6.56	2.26–2.74	2.80	4.03–5.92
^{82}Se	3.72–4.60	1.86–2.45	2.64	2.82–4.14

higher-order terms in the nucleonic weak current. The two different values of the matrix elements correspond to the $g_{\rm pp}$ and $g_{\rm A}$ parameter combinations indicated earlier in the text. As can be seen, the values of the final matrix elements vary between

$$\begin{aligned} 5.355 \leqslant |M^{(0\nu)}| \leqslant 6.555 \quad \text{for} \quad {}^{76}\text{Ge}, \\ 3.722 \leqslant |M^{(0\nu)}| \leqslant 4.597 \quad \text{for} \quad {}^{82}\text{Se}. \end{aligned} \tag{5}$$

We compare these matrix elements with other recent calculations in Table V. There the values $1.0 \leqslant g_{\rm A} \leqslant 1.254$ are used for the axial-vector coupling constant, except that Šimkovic *et al.* [36] use $g_{\rm A} = 1.254$. The results of Civitarese *et al.* [26] are based on the formalism introduced in Ref. [42] where the finite size of the nucleon and the nucleonic weak current were obtained from a relativistic quark-confinement model. In Ref. [42] the generated nucleonic weak current is incomplete as compared to the present formalism, adopted from Ref. [36]. Also the short-range correlations were not taken into account. In this sense the last column of Table V should be compared to the fourth column "+A+B" of Table I. As already said, the differences between the two results can be explained by the different treatment of the weak-interaction current and the nucleon form factor. We then conclude that our present results are more complete that the ones of Ref. [26] and thus should be more reliable. It is worth mentioning that in Ref. [33] the quoted matrix elements were calculated for the "default" value $g_{\rm pp} = 1.00$ without taking into account the higher-order terms in the nucleonic current and the nucleon finite-size effect.

In the matrix element calculations there may be other uncertainties than the ones induced by the uncertainty in the value of $g_{\rm pp}$. Such uncertainties could occur from sources such as deformation, the mean-field single-particle energies, and the adopted two-body interaction. In Ref. [21] it was shown that the effect of the adopted two-body interaction is very small as long as the interaction is microscopic. Our adopted Bonn-A interaction is of this type and included in the survey of Ref. [21]. In Ref. [21] it was furthermore demonstrated that the size of the single-particle space does not produce sizable effects as long as the value of $g_{\rm pp}$ is determined from the $2\nu\beta\beta$ data, as is done in the present calculations. By the same argument, only small effects are expected from different parametrizations of the Woods-Saxon mean-field potential and the resulting slightly different single-particle energies. These two sources of uncertainty produce effects that can be expected to be smaller than the one coming from the short-range correlations.

The role of deformation is the most uncertain one. The presently discussed nuclei of the two double beta decay chains are pf-shell nuclei and most likely they have no or very small static deformation. Instead, they are most likely soft anharmonic vibrators. The deformation allows of a new suppression mechanism of $2\nu\beta\beta$ decay, namely through the overlap factor used to take into account the nonorthogonality of the intermediate states generated by using the initial and final ground states as starting points in two separate pnQRPA calculations. This suppression mechanism is enhanced when the deformations of the initial and final nuclei of double beta decay are different [43]. However, in Ref. [44] it was deduced experimentally that ^{76}Ge and ^{76}Se exhibit quantitatively very similar neutron pair correlations. This would indicate similarity of their ground states and no suppression would occur through different ground state deformations. For ^{82}Se and ^{82}Kr this question is still open. In any case, the role of deformation in $0\nu\beta\beta$ decay is still largely unexplored and no definitive conclusion about the importance of deformation can be drawn for the present.

Our final matrix elements can be converted to $0\nu\beta\beta$ half-lives by choosing a value for the effective neutrino mass in Eq. (2). Expressing the effective mass in units of eV and using the phase-space integrals tabulated in Ref. [14], we obtain for the predicted half-lives

$$\begin{aligned} t_{1/2}^{(0\nu)} &= (0.96 - 1.44) \times 10^{24}\ \text{yr}/(\langle m_\nu \rangle[\text{eV}])^2 \quad \text{for} \quad {}^{76}\text{Ge}, \\ t_{1/2}^{(0\nu)} &= (4.53 - 6.90) \times 10^{23}\ \text{yr}/(\langle m_\nu \rangle[\text{eV}])^2 \quad \text{for} \quad {}^{82}\text{Se}. \end{aligned} \tag{6}$$

In summary, we have calculated the nuclear matrix elements for the $0\nu\beta\beta$ decays of ^{76}Ge and ^{82}Se by using the proton-neutron quasiparticle random-phase approximation with a realistic two-body interaction and a realistic single-particle space. The numerical calculations were done by including the higher-order terms of the nucleonic weak currents, the nucleon's finite-size corrections and the nucleon-nucleon short-range correlation effects. The short-range correlations have been calculated by using the unitary correlation operator formalism that is superior to the traditionally adopted rudimentary Jastrow procedure. The UCOM reduces the bare values of the computed matrix elements less than the Jastrow procedure, leading to larger matrix elements than the ones quoted in the recent literature. This reduces the predicted theoretical $0\nu\beta\beta$ half-lives of ^{76}Ge and ^{82}Se by a significant amount and thus directly influences the neutrino-mass sensitivities of the running and future double beta experiments.

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