Application of density dependent parametrization models to asymmetric nuclear matter

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Density dependent parametrization models of the nucleon-meson effective couplings, including the isovector scalar δ-field, are applied to asymmetric nuclear matter. The nuclear equation of state and the neutron star properties are studied in an effective Lagrangian density approach, using the relativistic mean field hadron theory. It is known that the introduction of a δ-meson in the constant coupling scheme leads to an increase of the symmetry energy at high density and so to larger neutron star masses, in a pure nucleon-lepton scheme. We use here a more microscopic density dependent model of the nucleon-meson couplings to study the properties of neutron star matter and to reexamine the δ-field effects in asymmetric nuclear matter. Our calculations show that, due to the increase of the effective δ coupling at high density, with density dependent couplings the neutron star masses in fact can be even reduced.

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The nonlinear Walecka model (NLWM) [1,2] and derivative scalar couplings [3], based on the relativistic mean-field (RMF) approach, have been extensively used to study the properties of nuclear and neutron matter, β-stable nuclei, and then extended to the drip-line regions. In the last years some authors [4–8] have stressed the importance of including the isovector scalar virtual $\delta(a_0(980))$ field in hadronic effective field theories for asymmetric nuclear matter. The role of the δ meson in isospin channels appears relevant at high densities [4–8] and so of great interest in nuclear astrophysics.

In order to describe the medium dependence of nuclear interactions, a density dependent relativistic hadron field (DDRH) theory has been recently suggested [9–11]. The density dependent meson-nucleon couplings are based on microscopic Dirac-Brueckner (DB) calculations [10,12] and adjusted to reproduce some nuclear matter and finite nuclei properties [9–11]. Here we will see the predictions of the density dependent coupling models when applied to the neutron stars (NS). In fact it is known that the introduction of the δ-meson in the constant coupling model [8] leads to heavier neutron stars in a nucleon-lepton picture. This is not obvious for density dependent models.

The Lagrangian density with δ mesons, used in this work, is like the one of RMF approaches, with nucleons coupled to isoscalar (scalar, vector) σ, ω_μ and isovector (scalar, vector) δ, ρ_μ, effective meson fields. The most important difference to conventional RMF theories is the contribution from the rearrangement self-energies to the DDRH baryon field equation. The meson-nucleon couplings g_σ, g_ω, g_ρ, and g_δ are assumed to be vertex functions of Lorentz-scalar bilinear forms of the nucleon field operators. In most applications of DDRH theory, these couplings are chosen as functions of the vector density $\hat{\rho}^2 = \hat{j}_\mu \hat{j}^\mu$ with $\hat{j}_\mu = \bar{\psi}\gamma_\mu\psi$.

The equation of state (EOS) for nuclear matter at $T = 0$ is obtained from the energy-momentum tensor. In a mean field approximation the energy density has the form [11]

$$\epsilon = \sum_{i=n,p} 2\int \frac{d^3k}{(2\pi)^3} E_i^\star(k) + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho^2 + \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_3^2 + \frac{1}{2}\frac{g_\delta^2}{m_\delta^2}\rho_{s3}^2, \tag{1}$$

and the pressure is

$$p = \sum_{i=n,p} \frac{2}{3}\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{E_i^\star(k)} - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho^2 + \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_3^2 - \frac{1}{2}\frac{g_\delta^2}{m_\delta^2}\rho_{s3}^2 - \Sigma_o^R\rho, \tag{2}$$

with $E_i{}^\star = \sqrt{k^2 + M_i{}^{\star 2}}$ and nucleon effective masses given by $M_i{}^\star = M - g_\sigma\sigma \pm g_\delta\delta_3$, $i = n(+), p(-)$. The scalar fields, σ (isoscalar) and δ_3 (isovector), are expressed in terms of the corresponding local scalar densities. In the pressure a rearrangement term appears, in the density dependent case, as

$$\Sigma_0^R = \left(\frac{\partial g_\sigma}{\partial\rho}\right)\frac{g_\sigma}{m_\sigma^2}\rho_s^2 + \left(\frac{\partial g_\delta}{\partial\rho}\right)\frac{g_\delta}{m_\delta^2}\rho_{s3}^2 - \left(\frac{\partial g_\omega}{\partial\rho}\right)\frac{g_\omega}{m_\omega^2}\rho^2 - \left(\frac{\partial g_\rho}{\partial\rho}\right)\frac{g_\rho}{m_\rho^2}\rho_3^2, \tag{3}$$

where $\rho_3 = \rho_p - \rho_n$ and $\rho_{s3} = \rho_{sp} - \rho_{sn}$, with ρ, ρ_s being the baryon and the scalar densities.

The chemical potentials for protons and neutrons can be written as, respectively,

$$\mu_i = \sqrt{k_{F_i}^2 + M_i^{\star 2}} + \frac{g_\omega^2}{m_\omega^2}\rho \pm \frac{g_\rho^2}{m_\rho^2}\rho_3 - \Sigma_o^R, \tag{4}$$

for $i = n(-), p(+)$, with the nucleon Fermi momentum $k_{F_i} = (3\pi^2\rho_i)^{1/3}$.

Since we are interested in the effects of the nuclear EOS we will consider only pure nucleonic (+leptons) neutron star structure, i.e., without strangeness bearing baryons and even deconfined quarks, see [13,14]. The composition is determined by the requirements of β-equilibrium and charge neutrality, that for a (n, p, e^-) system can be written as

$$\begin{aligned} \mu_e &= \mu_n - \mu_p = 4E_{sym}(\rho)(1 - 2X_p), \\ \rho_e &= \frac{1}{3\pi^2}\mu_e^3 = \rho_p = X_p\rho, \end{aligned} \tag{5}$$

where X_p is the proton fraction ρ_p/ρ. The symmetry energy can be obtained from the energy per nucleon in the parabolic approximation:

$$E_{sym}(\rho) = [E/A(\rho, \alpha) - E/A(\rho, \alpha = 0)]/\alpha^2, \tag{6}$$

where α asymmetry parameter $\alpha \equiv (N - Z)/A = -\rho_3/\rho$. Then, for a given ρ, the X_p is related to the nuclear symmetry energy by

$$3\pi^2\rho X_p - [4E_{sym}(\rho)(1 - 2X_p)]^3 = 0. \tag{7}$$

In the case of the (n, p, e^-, μ^-) system, the conditions read $\mu_\mu = \mu_e = \mu_n - \mu_p$ and $\rho_p = \rho_e + \rho_\mu$, with the muon density ρ_μ expressed as a function of its chemical potential

$$\rho_\mu = \frac{1}{3\pi^2}\left(\mu_\mu^2 - m_\mu^2\right)^{3/2}\theta(\mu_e - m_\mu). \tag{8}$$

The proton fraction X_p for (*npe*) and (*npe*μ) systems are then deduced. The EOS for the β-stable (*npe*) and (*npe*μ) matter can be estimated by using the obtained values of X_p. The equilibrium properties of the neutron stars will be finally studied by solving Tolmann-Oppenheimer-Volkov (TOV) equations [15] inserting the derived nuclear EOS as input. We note that the presence of muons slightly increases the proton fraction for a fixed density, making the matter softer. We will see this effect in the final equilibrium properties.

The parameters of the model include nucleon, (M = 939 MeV), and meson ($m_\sigma, m_\omega, m_\rho, m_\delta$, see Table I) masses and the density dependent meson-nucleon couplings. The density dependence parametrization used here, inspired by DB calculations [10,12], was proposed [7,11,16] as $g_i(\rho) = g_i(\rho_0)f_i(x)$, for $i = \sigma, \omega, \rho, \delta$, where $x = \rho/\rho_0$, ρ_0 saturation density.

TABLE I. Parameters of the model.

Meson	TW [11]		DDRHρ	DDRH$\rho\delta$	
	σ	ω	ρ	ρ	δ
m_i(MeV)	550	783	770	770	980
$g_i(\rho_0)$	10.73	13.29	3.59	5.86	7.59

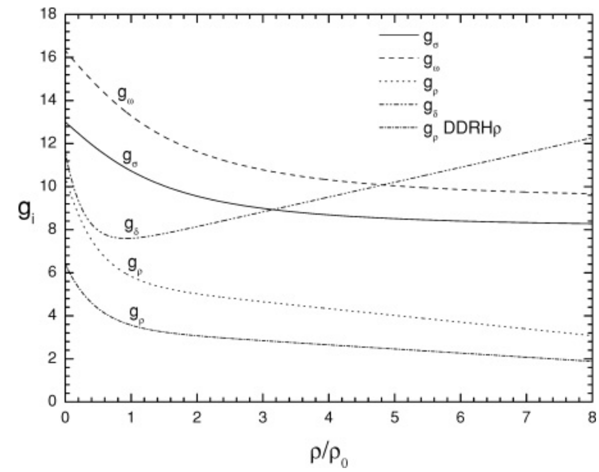

FIG. 1. Density dependence of the meson-nucleon couplings.

Parametrization form and parameters are taken from Ref. [11] for σ, ω mesons and from Refs. [7,16] for ρ, δ mesons, respectively. The density dependent couplings as a function of baryon density are displayed in Fig. 1.

For symmetric matter at saturation ($\rho_0 = 0.153$ fm^{-3}) we get a binding energy $E/A = \epsilon/\rho - M = -16.25$ MeV and a compressibility modulus $K = 240$ MeV. In order to note the effects of the coupling density dependence we will compare the results with a nonlinear (NL) relativistic mean field model *with constant couplings* which presents very similar saturation properties (Set A of Ref. [8]), including a symmetry energy $E_{sym} = 31.3$ MeV. Both effective models, NL and DDRH are rather soft for symmetric matter at high density, in agreement with relativistic collision data, [17], and Dirac-Brueckner expectations [18].

As shown in Refs. [4,8] when we include the δ coupling we have to increase the ρ coupling in order to keep the same symmetry term at saturation (see Table I). Since at higher densities the δ coupling is increasing while the ρ one is decreasing (see Fig. 1), as a result in the DDRH choice the symmetry term will be less repulsive than in the NL case.

The β-equilibrium nuclear matter is relevant for the composition of the neutron stars, as discussed previously. The EOS, pressure vs density, for (*npe*) matter in the density dependent DDRH vs NL-RMF models is reported in Fig. 2. We see that, at variance with the NL results, in the DDRH cases the EOS without δ-meson is stiffer than that with the δ-meson. This is partially due to the softening of the symmetry term in the DDRH$\rho\delta$ choice joined to a larger negative contribution to the pressure from the rearrangement term, see Eq. (2), as shown inside Fig. 2. We note that both effects are related to the density increase of the effective g_δ coupling (see Fig. 1) as expected from Dirac-Brueckner calculations [10,12].

We use the two effective nucleon-meson Lagrangians, with and without density dependent couplings, to calculate neutron star (NS) properties, with particular attention to the δ-field effects. The correlation between neutron star mass and radius

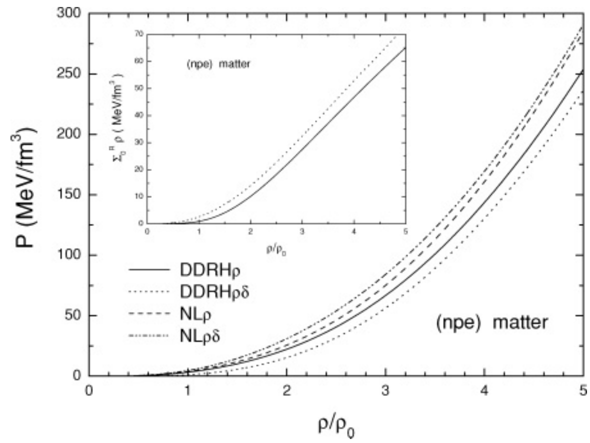

FIG. 2. Equation of state for (*npe*) matter in different models. Insert: density dependence of the rearrangement terms in the DDRH cases.

TABLE II. Maximum mass, corresponding radius, and central density of the star by the different models.

Neutron star	Model properties	Dens. dip		RMF	
		DDRHρ	DDRH$\rho\delta$	NLρ	NL$\rho\delta$
(npe) matter	$M_S/M_\odot$	2.108	2.01	2.14	2.21
	R (km)	11.00	10.29	11.02	11.55
	ρ_c/ρ_0	6.99	7.41	6.78	6.44
($npe\mu$) matter	$M_S/M_\odot$	2.106	1.98	2.12	2.18
	R (km)	10.91	10.27	10.91	11.30
	ρ_c/ρ_0	7.14	7.44	6.93	6.71

for the β-equilibrium (npe) and ($npe\mu$) matter obtained by the DDRH (density dependent) and NL-RMF (constant couplings) parametrizations are shown in Fig. 3. The obtained maximum mass, corresponding radius and central density for the (npe) and ($npe\mu$) neutron star matter are reported in Table II.

We first note that the NLρ and DDRHρ results are rather similar, with the DDRHρ interaction leading to a little softer matter, slightly smaller NS mass M_S and radius R and larger central density (see Table II). When we include the δ coupling we observe a clear effect in opposite directions: the DDRH case becomes much softer while the NL-RMF choice shows a much stiffer behavior. This can be seen from Table II, for the variations in M_S/R and central densities, but in fact it is quite impressive as it appears in Fig. 3: with reference to the close DDRHρ/NLρ curves we see a clear shift to the "left" of the DDRH$\rho\delta$ predictions and just the opposite to the "right" for the NL$\rho\delta$ expectations.

In general we also see, from Table II, that the (npe) star matter, for all models, has slightly larger masses and radii, and lower central densities, than the ($npe\mu$) star matter. This is due to the fact that the ($npe\mu$) star matter has some larger proton fraction in the regions above a critical baryon density where the muon appears, as already noted above.

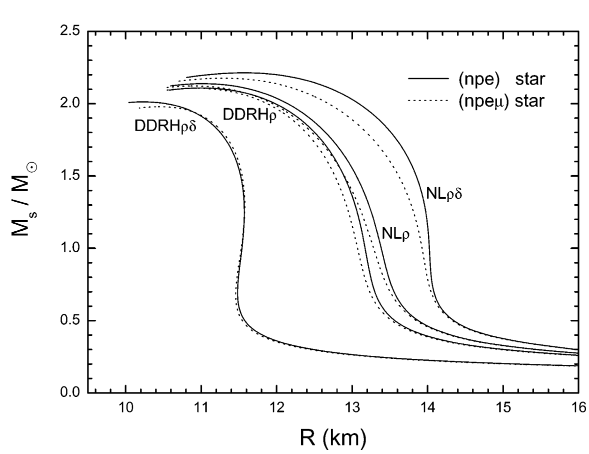

FIG. 3. Mass of the neutron star as a function of the radius of the neutron star in the two models.

All microscopic approaches of Dirac-Brueckner type to an effective meson-nucleon Lagrangian picture of the nuclear matter are predicting a density dependence of the couplings. We have studied the relative effects on the nuclear EOS at high baryon and isospin density, with application to nucleon-lepton neutron star properties. In particular we have focused our attention on the contribution of the isovector-scalar δ-meson. In fact in the "constant coupling" (NL-RMF) scheme the δ leads to very repulsive symmetry energy at high density. At variance in the "density dependence" case (DDRH) we can have a "softer" dense asymmetric matter due to combined mechanisms of a decrease of the isovector-vector g_ρ coupling and an increase of the g_δ (isovector scalar), which even lead to a larger pressure reduction from the rearrangement terms. The effect is clearly seen on equilibration properties of (npe) and/or ($npe\mu$) neutron stars, with a decrease of the NS mass in the DDRH case when the δ contribution is included. We note that in any case pure nucleon-lepton models cannot easily predict maximum NS masses below two solar units. In fact there is no observational evidence that prevents the existence of NSs with such large masses. In this respect of particular importance has been the recently reported compact object PSR J0751+1807 [19] with lower mass limit around two solar units at 68% C.L.

Our results seem to indicate that the large uncertainty of nucleon matter predictions, see the recent reviews [20,21], of relevance even for hybrid quark models, can be associated to the density dependence of the effective meson-nucleon couplings, in particular of the g_δ.

In conclusion we remark the interest of future work on two main directions:

(i) The importance of further DB confirmations of the high density behavior of the meson-nucleon effective couplings, in particular of some fundamental ground for the expected increase of the g_δ;

(ii) The study of dynamical effects of the isovector meson fields at the high baryon and isospin densities that can be reached in relativistic heavy ion collisions with exotic beams. Differential flows and particle productions appear to be rather promising observables, see the recent Refs. [22–25].

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