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Spin and pseudospin symmetries and the equivalent spectra of relativistic spin-1/2 and spin-0 particles

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We show that the conditions which originate the spin and pseudospin symmetries in the Dirac equation are the same that produce equivalent energy spectra of relativistic spin-1/2 and spin-0 particles in the presence of vector and scalar potentials. The conclusions do not depend on the particular shapes of the potentials and can be important in different fields of physics. When both scalar and vector potentials are spherical, these conditions for isospectrality imply that the spin-orbit and Darwin terms of either the upper component or the lower component of the Dirac spinor vanish, making it equivalent, as far as energy is concerned, to a spin-0 state. In this case, besides energy, a scalar particle will also have the same orbital angular momentum as the (conserved) orbital angular momentum of either the upper or lower component of the corresponding spin-1/2 particle. We point out a few possible applications of this result.

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When describing some strong interacting systems it is often useful, because of simplicity, to approximate the behavior of relativistic spin-1/2 particles by scalar spin-0 particles obeying the Klein-Gordon equation. An example is the case of relativistic quark models used for studying quark-hadron duality because of the added complexity of structure functions of Dirac particles as compared to scalar ones. It turns out that some results (e.g., the onset of scaling in some structure functions) almost do not depend on the spin structure of the particle [1]. In this work we will give another example of an observable, the energy, whose value may not depend on the spinor structure of the particle, i.e., whether one has a spin-1/2 or a spin-0 particle. We will show that when a Dirac particle is subjected to scalar and vector potentials of equal magnitude, it will have exactly the same energy spectrum as a scalar particle of the same mass under the same potentials. As we will see, this happens because the spin-orbit and Darwin terms in the second-order equation for either the upper or lower spinor component vanish when the scalar and vector potentials have equal magnitude. It is not uncommon to find physical systems in which strong interacting relativistic particles are subject to Lorentz scalar potentials (or position-dependent effective masses) that are of the same order of magnitude of potentials which couple to the energy (time components of Lorentz four-vectors). For instance, the scalar and vector (hereafter meaning time-component of a four-vector potential) nuclear mean-field potentials have opposite signs but similar magnitudes, whereas relativistic models of mesons with a heavy and a light quark, like D- or B-mesons, explain the observed small spin-orbit splitting by having vector and scalar potentials with the same sign and similar strengths [2].

It is well-known that all the components of the free Dirac spinor, i.e., the solution of the free Dirac equation, satisfy the free Klein-Gordon equation. Indeed, from the free Dirac equation

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\Psi = 0 \tag{1}$$

one gets

$$(-i\hbar\gamma^{\nu}\partial_{\nu} - mc)(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\Psi \\ = (\hbar^2\partial^{\mu}\partial_{\mu} + m^2c^2)\Psi = 0, \tag{2}$$

where use has been made of the relation $\gamma^{\mu}\gamma^{\nu}\partial_{\mu}\partial_{\nu} = \partial_{\mu}\partial^{\mu}$. In a similar way, for the time-independent free Dirac equation we would have

$$(c\,\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2)\psi = (-i\hbar c\,\boldsymbol{\alpha}\cdot\nabla + \beta mc^2)\psi = E\psi, \tag{3}$$

where, as usual, $\psi(\boldsymbol{r}) = \Psi(\boldsymbol{r},t)\exp(i\,E\,t/\hbar)$, $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$ and $\beta = \gamma^0$. Then, by left multiplying Eq. (3) by $c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2$, one gets the time-independent free Klein-Gordon equation

$$(c^2\boldsymbol{p}^2 + m^2c^4)\psi = (-\hbar^2c^2\nabla^2 + m^2c^2)\psi = E^2\psi, \tag{4}$$

where the relation $\{\beta,\boldsymbol{\alpha}\} = 0$ was used. This all means that the free four-component Dirac spinor, and of course all of its components, satisfy the Klein-Gordon equation. This is not surprising, because, after all, both free spin-1/2 and spin-0 particles obey the same relativistic dispersion relation, $E^2 = \boldsymbol{p}^2c^2 + m^2c^4$, in spite of having different spinor structures and thus different wave functions. Since there is no spin-dependent interaction, one expects both to have the same energy spectrum.

We consider now the case of a spin-1/2 particle subject to a Lorentz scalar potential V_s plus a vector potential V_v. The time-independent Dirac equation is given by

$$[c\,\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta(mc^2 + V_s)]\psi = (E - V_v)\psi. \tag{5}$$

It is convenient to define the four-spinors $\psi_\pm = P_\pm\psi = [(I \pm \beta)/2]\,\psi$ such that

$$\psi_+ = \begin{pmatrix}\phi\\ 0\end{pmatrix} \qquad \psi_- = \begin{pmatrix}0\\ \chi\end{pmatrix}, \tag{6}$$

where ϕ and χ are, respectively, the upper and lower two-component spinors. Using the properties and anticommutation relations of the matrices β and $\boldsymbol{\alpha}$ we can apply the projectors $P_\pm$ to the Dirac equation (5) and decompose it into two coupled equations for ψ_+ and ψ_-:

$$c\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\psi_- + (mc^2 + V_s)\psi_+ = (E - V_v)\psi_+, \tag{7}$$

$$c\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\psi_+ - (mc^2 + V_s)\psi_- = (E - V_v)\psi_-. \tag{8}$$

Applying the operator $c\,\boldsymbol{\alpha}\cdot\boldsymbol{p}$ on the left of these equations and using them to write ψ_+ and ψ_- in terms of $\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\psi_-$ and $\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\psi_+$, respectively, we finally get second-order equations for ψ_+ and ψ_-:

$$c^2\boldsymbol{p}^2\,\psi_+ + c^2\,\frac{[\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\Delta]\boldsymbol{\alpha}\cdot\boldsymbol{p}\psi_+}{E - \Delta + mc^2} = (E - \Delta + mc^2)(E - \Sigma - mc^2)\psi_+, \tag{9}$$

$$c^2\boldsymbol{p}^2\,\psi_- + c^2\,\frac{[\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\Sigma]\boldsymbol{\alpha}\cdot\boldsymbol{p}\psi_-}{E - \Sigma - mc^2} = (E - \Delta + mc^2)(E - \Sigma - mc^2)\psi_-, \tag{10}$$

where the square brackets [ ] mean that the operator $\boldsymbol{\alpha}\cdot\boldsymbol{p}$ only acts on the potential in front of it and we defined $\Sigma = V_v + V_s$ and $\Delta = V_v - V_s$. The second term in these equations can be further elaborated noting that the Dirac α_i matrices satisfy the relation $\alpha_i\alpha_j = \delta_{ij} + \frac{2}{\hbar}\,i\epsilon_{ijk}S_k$ where S_k, $k = 1, 2, 3$, are the spin operator components. The second-order equations now read

$$c^2\,\boldsymbol{p}^2\,\psi_+ + c^2\,\frac{[\boldsymbol{p}\,\Delta]\cdot\boldsymbol{p}\psi_+ + \frac{2i}{\hbar}\,[\boldsymbol{p}\,\Delta]\times\boldsymbol{p}\cdot\boldsymbol{S}\psi_+}{E - \Delta + mc^2} = (E - \Delta + mc^2)(E - \Sigma - mc^2)\psi_+, \tag{11}$$

$$c^2\,\boldsymbol{p}^2\,\psi_- + c^2\,\frac{[\boldsymbol{p}\,\Sigma]\cdot\boldsymbol{p}\psi_- + \frac{2i}{\hbar}\,[\boldsymbol{p}\,\Sigma]\times\boldsymbol{p}\cdot\boldsymbol{S}\psi_-}{E - \Sigma - mc^2} = (E - \Delta + mc^2)(E - \Sigma - mc^2)\psi_-. \tag{12}$$

Now, if $\boldsymbol{p}\,\Delta = 0$, meaning that Δ is constant or zero (if Δ goes to zero at infinity, the two conditions are equivalent), then the second term in Eq. (11) disappears and we have

$$\begin{aligned} c^2\,\boldsymbol{p}^2\psi_+ &= (E - \Delta + mc^2)(E - \Sigma - mc^2)\psi_+ \\ &= [(E - V_v)^2 - (mc^2 + V_s)^2]\psi_+, \end{aligned} \tag{13}$$

which is precisely the time-independent Klein-Gordon equation for a scalar potential V_s plus a vector potential V_v[1]. Since the second-order equation determines the eigenvalues for the spin-1/2 particle, this means that when $\boldsymbol{p}\,\Delta = 0$, a spin-1/2 and a spin-0 particle with the same mass and subject to the same potentials V_s and V_v will have the same energy spectrum, including *both* bound and scattering states. This last sufficient condition for isospectrality can be relaxed to demand that just the combination $mc^2 + V_s$ be the same for both particles, *allowing them to have different masses*. This is so because this weaker condition does not change the gradient of Δ and Σ and therefore the condition $\boldsymbol{p}\,\Delta = 0$ will still hold. On the other hand, if the scalar and vector potentials are such that $\boldsymbol{p}\,\Sigma = 0$, we would obtain a Klein-Gordon equation for ψ_-, and again the spectrum for spin-0 and spin-1/2 particles would be the same, provided they are subjected to the same vector potential and $mc^2 + V_s$ is the same for both particles. If both V_s and V_v are central potentials, i.e., only depend on the radial coordinate, then the numerators of the second terms in Eqs. (11) and (12) read

$$\begin{aligned} &[\boldsymbol{p}\,\Delta]\cdot\boldsymbol{p}\psi_+ + \frac{2i}{\hbar}\,[\boldsymbol{p}\,\Delta]\times\boldsymbol{p}\cdot\boldsymbol{S}\,\psi_+ \\ &\quad = -\hbar^2\Delta'\,\frac{\partial\psi_+}{\partial r} + \frac{2}{r}\,\Delta'\boldsymbol{L}\cdot\boldsymbol{S}\,\psi_+, \end{aligned} \tag{14}$$

$$\begin{aligned} &[\boldsymbol{p}\Sigma]\cdot\boldsymbol{p}\psi_- + \frac{2i}{\hbar}\,[\boldsymbol{p}\,\Sigma]\times\boldsymbol{p}\cdot\boldsymbol{S}\psi_- \\ &\quad = -\hbar^2\Sigma'\,\frac{\partial\psi_-}{\partial r} + \frac{2}{r}\,\Sigma'\boldsymbol{L}\cdot\boldsymbol{S}\,\psi_-, \end{aligned} \tag{15}$$

where Δ' and Σ' are the derivatives with respect to r of the radial potentials $\Delta(r)$ and $\Sigma(r)$, and $\boldsymbol{L} = \boldsymbol{r}\times\boldsymbol{p}$ is the orbital angular momentum operator. From these equations ones sees that these terms, which set apart the Dirac second-order equations for the upper and lower components of the Dirac spinor from the Klein-Gordon equation and thus are the origin of the different spectra for spin-1/2 and spin-0 particles, are composed of a derivative term, related to the Darwin term which appears in the Foldy-Wouthuysen expansion, and a $\boldsymbol{L}\cdot\boldsymbol{S}$ spin-orbit term. If $\Delta' = 0$ ($\Sigma' = 0$), then there is no spin-orbit term for the upper (lower) component of the Dirac spinor. In turn, since the second-order equation determines the energy eigenvalues, this means that the orbital angular momentum of the respective component is a good quantum number of the Dirac spinor. This can be a bit surprising, since one knows that in general the orbital quantum number is not a good quantum number for a Dirac particle, since $\boldsymbol{L}^2$ does not commute with a Dirac Hamiltonian with radial potentials. The reason why this does not happen in these cases was reported in Refs. [3,4], and we now review it in a slight different fashion. Let us consider in more detail the case of spherical potentials such that $\Delta' = 0$. One knows that a spinor that is a solution of a Dirac equation

[1]There are some authors who introduce a scalar potential $\mathcal{V}_s$ in the Klein-Gordon equation by making the replacement $m^2c^4 \to m^2c^4 + \mathcal{V}_s^2$. Here we introduce it, as most authors do, as an effective mass $m^{*2} = (m + V_s/c^2)^2$, since it is the way that it is introduced in the Dirac equation. The two potentials are related by $\mathcal{V}_s^2 = (mc^2 + V_s)^2 - m^2c^4$.

with spherically symmetric potentials can be generally written as

$$\psi_{jm}(\boldsymbol{r}) = \begin{pmatrix} \mathrm{i}\dfrac{g_{j\,l}(r)}{r}\mathcal{Y}_{j\,l\,m}(\hat{\boldsymbol{r}}) \\ \dfrac{f_{j\,\tilde{l}}(r)}{r}\mathcal{Y}_{j\,\tilde{l}\,m}(\hat{\boldsymbol{r}}) \end{pmatrix}, \qquad (16)$$

where $\mathcal{Y}_{j\,l\,m}$ are the spinor spherical harmonics. These result from the coupling of spherical harmonics and two-dimensional Pauli spinors χ_{m_s}, $\mathcal{Y}_{j\,l\,m} = \Sigma_{m_s}\Sigma_{m_l}\langle lm_l; 1/2m_s|jm\rangle Y_{l\,m_l}\chi_{m_s}$, where $\langle lm_l; 1/2m_s|jm\rangle$ is a Clebsch-Gordan coefficient and $\tilde{l} = l \pm 1$, the plus and minus signs being related to whether one has aligned or anti-aligned spin, i.e., $j = l \pm 1/2$. The spinor spherical harmonics for the lower component satisfy the relation $\mathcal{Y}_{j\,\tilde{l}\,m} = -\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}\,\mathcal{Y}_{j\,l\,m}$. The fact that the upper and lower components have different orbital angular momenta is related to the fact, mentioned before, that $\boldsymbol{L}^2$ does not commute with the Dirac Hamiltonian

$$\begin{aligned} H &= c\,\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta(V_s + mc^2) + V_v \\ &= c\,\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta mc^2 + \Sigma P_+ + \Delta P_-, \end{aligned} \qquad (17)$$

where $P_\pm$ are the projectors defined above. However, when $\Delta' = 0$, there is an extra SU(2) symmetry of H (so-called "spin symmetry") as first shown by Bell and Ruegg [5]. When we have spherical potentials, Ginocchio showed that there is an additional SU(2) symmetry (for a recent review see [4]). The generators of this last symmetry are

$$\mathcal{L} = \boldsymbol{L}P_+ + \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,P_- = \begin{pmatrix} \boldsymbol{L} & 0 \\ 0 & U_p\,\boldsymbol{L}\,U_p \end{pmatrix}, \qquad (18)$$

where $U_p = \boldsymbol{\sigma}\cdot\boldsymbol{p}/(\sqrt{p^2})$ is the helicity operator. One can check that $\mathcal{L}$ commutes with the Dirac Hamiltonian,

$$\begin{aligned} [H,\mathcal{L}] &= \left[c\,\boldsymbol{\alpha}\cdot\boldsymbol{p}, \boldsymbol{L}P_+ + \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,P_-\right] \\ &\quad + \left[\Delta, \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\right] + [\Sigma, \boldsymbol{L}] \\ &= \left[\Delta, \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\right] = 0, \end{aligned} \qquad (19)$$

where the last equality comes from the fact that $\Delta' = 0$. The Casimir $\mathcal{L}^2$ operator is given by $\mathcal{L}^2 = \boldsymbol{L}^2P_+ + \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}^2\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,P_-$. Applying this operator to the spinor ψ_{jm} (16), we get

$$\begin{aligned} \mathcal{L}^2\psi_{jm} &= \boldsymbol{L}^2\psi_{jm}^+ + \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}^2\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\psi_{jm}^- \\ &= \hbar^2 l(l+1)\psi_{jm}^+ + \frac{\boldsymbol{\alpha}\cdot\boldsymbol{p}\,c\boldsymbol{L}^2\,\psi_{jm}^+}{E - \Delta + mc^2} \\ &= \hbar^2 l(l+1)\psi_{jm}^+ + \hbar^2 l(l+1)\psi_{jm}^- \\ &= \hbar^2 l(l+1)\psi_{jm}, \end{aligned} \qquad (20)$$

where $\psi_{jm}^\pm = P_\pm\psi_{jm}$ and we used the relation, valid when $\Delta' = 0$, $\psi_{jm}^+ = (E - \Delta + mc^2)\frac{\boldsymbol{\alpha}\cdot\boldsymbol{p}}{cp^2}\psi_{jm}^-$. From Eq. (20) we see that ψ_{jm} is indeed an eigenstate of $\mathcal{L}^2$. Thus the orbital quantum number of the upper component l is a good quantum number of the system when the spherical potentials $V_s(r)$ and $V_v(r)$ are such that $V_v(r) = V_s(r) + C_\Delta$, where C_Δ is an arbitrary constant. Also, according to we have said before, there is a state of a spin-0 particle subjected to these same spherical potentials (or, at least, with a scalar potential such that the sum $V_s + mc^2$ is the same) that has the same energy and the same orbital angular momentum as ψ_{jm}. In addition, the wave function of this scalar particle would be proportional to the spatial part of the wave function of the upper component.

Note that the generator of the "spin symmetry" $\mathcal{S}$ is given by a similar expression as Eq. (18) just replacing $\boldsymbol{L}$ by $\hbar/2\,\boldsymbol{\sigma}$ [4,5], meaning that $\mathcal{S}^2 \equiv \boldsymbol{S}^2 = 3/4\hbar^2 I$ so that spin is also a good quantum number, as would be expected. Actually, one can show that the total angular momentum operator $\boldsymbol{J}$ can be written as $\mathcal{L} + \mathcal{S}$, so that l, m_l (eigenvalue of $\mathcal{L}_z$), $s = 1/2$, m_s (eigenvalue of $\mathcal{S}_z$) are good quantum numbers. Then, of course, j and $m = m_l + m_s$ are also good quantum numbers, but only in a trivial way, because there is no longer spin-orbit coupling. Therefore, in the spinor (16) one could just replace the spinor spherical harmonic $\mathcal{Y}_{j\,l\,m}$ by $Y_{l\,m_l}\chi_{m_s}$ and $\mathcal{Y}_{j\,\tilde{l}\,m}$ by $-\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}Y_{l\,m_l}\chi_{m_s}$. Note that if Δ is a nonrelativistic potential, $\Delta \ll mc^2$ and $\Delta' \ll m^2c^4/(\hbar c)$, i.e., it is slowly varying over a Compton wavelength. In this case, the spin-orbit term will also get suppressed. In fact, the derivative of the Δ potential is the origin of the well-known relativistic spin-orbit effect which appears as a relativistic correction term in atomic physics or in the v/c Foldy-Wouthuysen expansion (only the derivative of V_v appears because usually no Lorentz scalar potential V_s is considered, and therefore $\Delta = V_v$).

When $\Sigma' = 0$, or $V_v(r) = -V_s(r) + C_\Sigma$, with C_Σ an arbitrary constant, there is again a SU(2) symmetry, usually called pseudospin symmetry ([5,6]) which is relevant for describing the single-particle level structure of several nuclei. This symmetry has a dynamical character and cannot be fully realized in nuclei because in relativistic mean-field theories the Σ potential is the only binding potential for nucleons [7,8]. For harmonic oscillator potentials this is no longer the case, since Δ, acting as an effective mass going to infinity, can bind Dirac particles [9,10], even when $\Sigma = 0$. As before, in the special case of spherical potentials, there is another SU(2) symmetry whose generators are

$$\tilde{\mathcal{L}} = \frac{1}{p^2}\boldsymbol{\alpha}\cdot\boldsymbol{p}\,\boldsymbol{L}\,\boldsymbol{\alpha}\cdot\boldsymbol{p}\,P_+ + \boldsymbol{L}P_- = \begin{pmatrix} U_p\,\boldsymbol{L}\,U_p & 0 \\ 0 & \boldsymbol{L} \end{pmatrix}. \qquad (21)$$

In the same way as before, applying $\tilde{\mathcal{L}}^2$ to ψ_{jm}, we would find that $\tilde{\mathcal{L}}^2\psi_{jm} = \hbar^2\tilde{l}(\tilde{l}+1)\psi_{jm}$, that is, this time it is the orbital quantum number of the lower component $\tilde{l}$ which is a good quantum number of the system and can be used to classify energy levels. Again, provided the vector and scalar potentials are adequately related, there would be a corresponding state of a spin-0 particle with the same energy and same orbital angular momentum $\tilde{l}$, and, furthermore, its wave function would be proportional to the spatial part of the wave function of the lower component. As before, the pseudospin symmetry generator $\tilde{\mathcal{S}}$ can be obtained from $\tilde{\mathcal{L}}$ by replacing $\boldsymbol{L}$ by $\hbar/2\,\boldsymbol{\sigma}$. The good quantum numbers of the system would be, besides

$\tilde{l}$, $m_{\tilde{l}}$, $\tilde{s} \equiv s = 1/2$ and $m_{\tilde{s}}$. Again, $\boldsymbol{J} = \tilde{\mathcal{L}} + \tilde{\mathcal{S}}$. It is interesting that, as has been noted by Ginocchio [9], the generators of spin and pseudospin symmetries are related through a γ^5 transformation since $\tilde{\mathcal{S}} = \gamma^5 \mathcal{S} \gamma^5$ and $\tilde{\mathcal{L}} = \gamma^5 \mathcal{L} \gamma^5$. This property was used in a recent work to relate spin symmetric and pseudospin symmetric spectra of harmonic oscillator potentials [11]. There it was shown that for massless particles (or ultrarelativistic particles) the spin- and pseudospin spectra of Dirac particles are the same. In addition, this means that spin-symmetric massless eigenstates of γ^5 would be also pseudospin symmetric and vice versa. Since in this case $\Delta = \Sigma = 0$, or $V_v = V_s = 0$, this is, of course, just another way of stating the well-known fact that free massless Dirac particles have good chirality.

Naturally, for free spin-1/2 particles described by spherical waves, *both* l and $\tilde{l}$ are good quantum numbers, which just reflects the fact that one can have free spherical waves with any orbital angular momentum for the upper or lower component and still have the same energy, as long as their linear momentum magnitude is the same, or, put in another way, the energy of a free spin-1/2 particle cannot depend on its direction of motion.

In summary, we showed that when a relativistic spin-1/2 particle is subject to vector and scalar potentials such that $V_v = \pm V_s + C_\pm$, where $C_\pm$ are constants, its energy spectrum does not depend on their spinorial structure, being identical to the spectrum of a spin-0 particle which has no spinorial structure. This amounts to say that if the potentials have these configurations there is no spin-orbit coupling and Darwin term. If the scalar and vector potentials are spherical, one can classify the energy levels according to the orbital angular momentum quantum number of either the upper or the lower component of the Dirac spinor. This would then correspond to having a spin-0 particle with orbital angular momentum l or $\tilde{l}$, respectively. This spectral identity can of course happen only with potentials which do not involve the spinorial structure of the Dirac equation in an intrinsic way. For instance, a tensor potential of the form $\mathrm{i}\beta\sigma^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)$ does not have an analog in the Klein-Gordon equation, so that one could not have a spin-0 particle with the same spectrum as a spin-1/2 particle with such a potential. This is the case of the so-called Dirac oscillator [12] (see [10] for a complete reference list), in which the Dirac equation contains a potential of the form $\mathrm{i}\beta\sigma^{0i} m\omega r_i = \mathrm{i}m\omega\beta\boldsymbol{\alpha}\cdot\boldsymbol{r}$. Another important potential, the electromagnetic vector potential $\boldsymbol{A}$, which is the spatial part of the electromagnetic four-vector potential, can be added via the minimal coupling scheme to both the Dirac and the Klein-Gordon equations. Since $\boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A})\boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) = (\boldsymbol{p} - e\boldsymbol{A})^2 + 2e\hbar\nabla\times\boldsymbol{A}\cdot\boldsymbol{S}$, the spectra of spin-0 and spin-1/2 particles cannot be identical as long as there is a magnetic field present, even though the condition $V_v = \pm V_s + C_\pm$ is fulfilled. It is important also to remark that, since for an electromagnetic interaction V_v is the time-component of the electromagnetic four-vector potential, this last condition is gauge invariant in the present case, in which we are dealing with stationary states, i.e., time-independent potentials. So, in the absence of a external magnetic field (allowing, for instance, an electromagnetic vector potential $\boldsymbol{A}$ which is constant or a gradient of a scalar function), a spin-0 and spin-1/2 particle subject to the same electromagnetic potential V_v and a Lorentz scalar potential fulfilling the above relation would have the same spectrum.

The remark made above about the similarity of spin-0 and spin-1/2 wave functions can be relevant for calculations in which the observables do not depend on the spin structure of the particle, like some structure functions. One such calculation was made by Paris [13] in a massless confined Dirac particle, in which $V_v = V_s$. It would be interesting to see how a Klein-Gordon particle would behave under the same potentials. More generally, this spectral identity can also have experimental implications in different fields of physics, since, should such an identity be found, it would signal the presence of a Lorentz scalar field having a similar magnitude as that of a time-component of a Lorentz vector field, or at least differing just by a constant.

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