

Isospin dependence of 1S_0 proton and neutron superfluidity in asymmetric nuclear matter

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We investigate the 1S_0 neutron and proton superfluidity in isospin-asymmetric nuclear matter. We have concentrated on the isospin dependence of the pairing gaps and the effect of a microscopic three-body force. It is found that as the isospin asymmetry goes higher, the neutron 1S_0 superfluid phase shrinks gradually to a smaller density domain, whereas the proton one extends rapidly to a much wider density domain. The three-body force turns out to weaken the neutron 1S_0 superfluidity slightly, but it suppresses strongly the proton 1S_0 superfluidity at high densities in nuclear matter with large isospin asymmetry.

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I. INTRODUCTION

Superfluidity plays an important role in understanding a number of astrophysical phenomena in neutron stars [1–9]. In the inner crust of a neutron star where the total baryon density ρ is low, neutrons are thought to be superfluid in the 1S_0 channel. In the nuclear core part (i.e, the outer core part), protons may form a superfluid in the 1S_0 partial wave states, and neutron superfluidity is expected in the 3P_2 - 3F_2 partial wave channel. It is generally expected that the cooling processes via neutrino emission [3–5], the properties of rotating dynamics, the postglitch timing observations [6,7], and the possible vertex pinning [8] of a neutron star are rather sensitive to the presence of neutron and proton superfluid phases as well as to their pairing strength. Recently it has been shown [9] that a weak proton 1S_0 superfluidity is consistent with the cooling data if the effect of the possible presence of accreted envelopes is taken into account in the cooling scenario.

Nucleon superfluidity in symmetric nuclear matter and pure neutron matter has been investigated extensively by many authors using various theoretical approaches [10–24]. All these investigations have predicted the occurrence of the 1S_0 nucleon superfluid phase in the low-density region although the obtained strengths of the superfluidity are remarkably sensitive to the different approaches and the different approximations adopted. There also exist a few investigations on the nucleon superfluidity in β -stable neutron star matter [25]. It is shown that, owing to the small proton fraction in β -stable matter, the proton superfluid phase in the 1S_0 channel may extend to much higher baryon densities with a much smaller maximum of the pairing gap as compared to the case of symmetric nuclear matter. Therefore it is of interest to investigate the variation of nucleon superfluidity versus isospin asymmetry in asymmetric nuclear matter, which is expected to be helpful for understanding the properties of nucleon superfluid phases in neutron stars.

The aim of this work is devoted to the isospin dependence of the 1S_0 neutron and proton superfluid phases in asymmetric nuclear matter and to investigating the influence of

three-body forces, which turn out to be crucial for reproducing the empirical saturation properties of nuclear matter in a nonrelativistic microscopic approach [26–28].

II. THEORETICAL APPROACHES

For our present purpose, we shall not go beyond the BCS framework. In this case, the pairing gap that characterizes the superfluidity in a homogeneous Fermi system is determined by the standard BCS gap equation [29], that is,

$$\Delta_{\vec{k}} = - \sum_{\vec{k}'} v(\vec{k}, \vec{k}') \frac{1}{2E_{\vec{k}'}} \Delta_{\vec{k}'}, \quad (1)$$

where $v(\vec{k}, \vec{k}')$ is the realistic NN interaction in momentum space and $E_{\vec{k}} = \sqrt{(\epsilon_{\vec{k}} - \epsilon_F)^2 + \Delta_{\vec{k}}^2}$, with $\epsilon_{\vec{k}}$ and ϵ_F being the single-particle (s.p.) energy and its value at the Fermi surface, respectively. In the BCS gap equation, the most important ingredients are the realistic NN interaction $v(\vec{k}, \vec{k}')$ and the neutron and proton s.p. energies $\epsilon_{\vec{k}}$ in asymmetric nuclear matter. For the NN interaction, we adopt the Argonne V18 (AV_{18}) two-body interaction [30] plus a microscopic three-body force (TBF). The TBF adopted in the present calculation was originally proposed in Ref. [26] based on the meson-exchange current approach. The parameters of the TBF, that is, the coupling constants and the form factors, were determined [27] from the one-boson-exchange potential model to meet the self-consistent requirement with the adopted AV_{18} two-body force. A more detailed description of the TBF model and the related approximations can be found in Ref. [26].

The proton and neutron s.p. energies in asymmetric nuclear matter are calculated by using the BHF approach for isospin-asymmetric nuclear matter [31]. The starting point of the BHF approach is the Brueckner G matrix, which satisfies the Bethe-Goldstone (BG) equation [32]:

$$G(\rho, \beta; \omega) = v + v \sum_{k_1 k_2} \frac{|k_1 k_2\rangle Q(k_1, k_2) \langle k_1 k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho, \beta; \omega), \quad (2)$$

where $k_i \equiv (\vec{k}_i, \sigma_i, \tau_i)$ denotes the momentum and the z -components of spin and isospin of a nucleon, respectively, ω is the starting energy, and $Q(k_1, k_2)$ is the Pauli operator. The

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isospin-asymmetry parameter is defined as $\beta = (\rho_n - \rho_p)/\rho$, where ρ_n , ρ_p , and ρ are the neutron, proton, and total nucleon number densities, respectively. The s.p. energy is given by $\epsilon(k) = \hbar^2 k^2/(2m) + U(k)$. In solving the BG equation, we adopt the continuous choice [32] for the s.p. potential $U(k)$ since it has been proven to converge much faster than the gap choice [33]. Under the continuous choice, the s.p. potential describes physically the nuclear mean field felt by a nucleon in the nuclear medium [34] and is calculated from the real part of the on-shell G matrix.

The neutron and proton superfluidity in nuclear matter can be described by their pairing energy gaps at their respective Fermi surfaces. To solve the gap equation, we follow the scheme given in Ref. [10]. The proton and neutron s.p. energies in the gap equation are calculated from the BHF approach. In our calculations, the TBF contribution has been included by reducing the TBF to an equivalently effective two-body interaction according to the standard scheme as described in Ref. [26]. A detailed description and justification of the method are discussed in Refs. [26,27].

III. RESULTS AND DISCUSSIONS

In Fig. 1 is shown the neutron energy gap in the 1S_0 partial wave channel $\Delta_F^n = \Delta(k_F^n)$ as a function of the total baryon density ρ . The curves along the direction of the arrow from the bottom correspond to $\beta = 0.2, 0.4, 0.6$, and 0.8 , respectively. We see that the neutron 1S_0 superfluid phase exists only at low densities ($\rho \leq 0.13 \text{ fm}^{-3}$) and the peaks of the pairing gaps are located around $\rho = 0.02 \text{ fm}^{-3}$, which is compatible with the previous predictions for pure neutron matter and symmetric nuclear matter [10,24]. As the isospin asymmetry β increases, the neutron fraction increases for a given total density and as a consequence the density domain for the neutron 1S_0 superfluidity shrinks gradually. In the case without including the TBF, the density region for the presence of the neutron superfluidity reduces from $\rho \leq 0.13 \text{ fm}^{-3}$ to $\rho \leq 0.1 \text{ fm}^{-3}$ as the isospin asymmetry rises from $\beta = 0.2$ to $\beta = 0.8$. In Ref. [24], we calculated the 1S_0 pairing gaps in symmetric nuclear matter ($\beta = 0$) and pure neutron matter ($\beta = 1$). We found that the neutron 1S_0

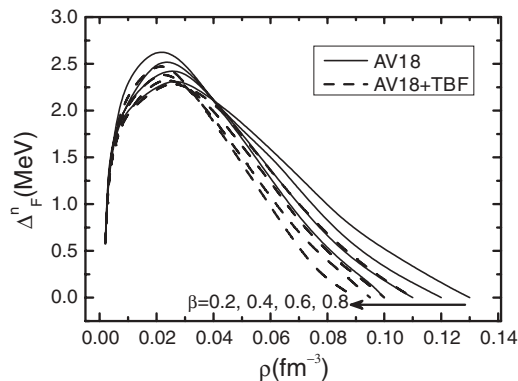


FIG. 1. Neutron 1S_0 energy gap as a function of density in asymmetric nuclear matter at various asymmetries. The solid curves are predicted by adopting purely the AV_{18} two-body interaction; the dashed ones are predicted by using the AV_{18} plus the TBF.

superfluid phase is expected to appear in a density range of $\rho_B \leq 0.135 \text{ fm}^{-3}$ for symmetric nuclear matter and $\rho_B \leq 0.09 \text{ fm}^{-3}$ for pure neutron matter, consistent with our present results. The isospin dependence of the pairing gap obtained here is readily understood as follows. In asymmetric nuclear matter, the strength of the neutron pairing gap is related directly to the neutron number density. As the asymmetry rises, the neutron excess and the neutron number density for a given total nucleon density increase. As a consequence, the density domain for the existence of the neutron superfluid phase shrinks gradually going from symmetric matter to pure neutron matter. We notice that, in cases both with and without inclusion of the TBF, the peak values of the gaps increase and their locations shift slightly to lower densities as the matter becomes more isospin asymmetric. This is mainly attributed to the isospin dependence of the neutron s.p. energy spectrum in asymmetric nuclear matter. As is known, the neutron s.p. potential in asymmetric matter becomes shallower and the neutron effective mass gets larger for a larger isospin asymmetry according to the BHF and DBHF calculations [31,35]. We may also see from Fig. 1 that within the region of the neutron superfluid phase, as the isospin asymmetry decreases, the pairing gap becomes smaller at low densities below and in the vicinity of the peak position, whereas it gets larger at relatively high densities. This isospin behavior stems from the competition between two different mechanisms. On the one hand, at a fixed total density on the right side of the peak position, a higher asymmetry corresponds to a larger value of neutron density and a weaker neutron superfluidity. On the other hand, the isospin dependence of the neutron s.p. spectrum tends to enhance the neutron pairing gap as the isospin asymmetry increases. At low densities around and below the peak position, the latter effect is dominant, whereas as the density increases, the former mechanism becomes more and more effective. Comparing the solid curves with the corresponding dashed ones, we may see that the TBF affects mainly the pairing gap at high densities. Inclusion of the TBF weakens the neutron superfluidity at high densities and shrinks the predicted density range for the existence of the neutron superfluid phase.

In Fig. 2 is reported the proton 1S_0 pairing gap in asymmetric nuclear matter for four different asymmetries, $\beta = 0.2, 0.4, 0.6$, and 0.8 , respectively. The solid and dashed curves are

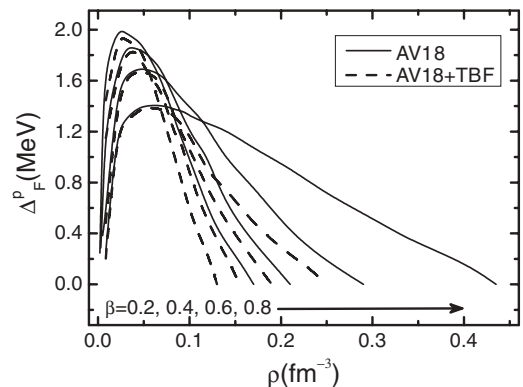


FIG. 2. The same as Fig. 1 but for proton 1S_0 pairing gap.

obtained, respectively, without and with inclusion of the TBF. The isospin dependence of the proton 1S_0 pairing gaps turns out to be completely different from that of the corresponding neutron ones in both cases with and without the TBF. It is seen that, as the isospin asymmetry increases, the density domains for the existence of the proton superfluid phases enlarge rapidly and the peaks of the pairing gaps become lower appreciably and shift to higher densities gradually. As the asymmetry rises from $\beta = 0.2$ to $\beta = 0.8$, the density domain extends from $\rho \leq 0.17 \text{ fm}^{-3}$ to $\rho \leq 0.435 \text{ fm}^{-3}$ in the case not including the TBF and from $\rho \leq 0.13 \text{ fm}^{-3}$ to $\rho \leq 0.25 \text{ fm}^{-3}$ in the case including the TBF; the peak values lower from about 2.0 MeV to about 1.4 MeV in both cases. The difference between the proton and neutron superfluid phases is especially pronounced at high isospin asymmetries. As compared to the neutron 1S_0 superfluidity, the proton 1S_0 superfluid phase in highly asymmetric matter extends to much higher densities but with a smaller peak value of the pairing gap. For example, in the case without inclusion of the TBF, at $\beta = 0.8$, the proton 1S_0 superfluidity is predicted to exist in the density region of $\rho \leq 0.435 \text{ fm}^{-3}$, which is much wider than that of $\rho \leq 0.1 \text{ fm}^{-3}$ for the neutron 1S_0 superfluidity. This behavior of the proton superfluidity versus isospin asymmetry can be readily explained in terms of the isospin dependence of the proton density and the proton s.p. spectrum in asymmetric nuclear matter. First, at a fixed total density, a higher asymmetry corresponds to a smaller proton concentration and a lower proton density. Such an isospin dependence of the proton density is directly responsible for the widening of the density domain as a function of asymmetry. Second, the proton s.p. potential becomes deeper going from symmetric matter to pure neutron matter [31], and this results in the lowering of the proton pairing peak versus asymmetry. At relatively low densities below and around the peak position, the variation of the proton s.p. spectrum as a function of asymmetry plays a major role in determining the isospin variation of the proton pairing gap and thus the proton superfluidity becomes weaker at a higher asymmetry. However, as the density increases, the effect resulting from the decreasing of the proton fraction versus asymmetry gets stronger. At high enough densities, it becomes predominant and leads to a turnover of the isospin behavior of the proton superfluidity (i.e., a stronger superfluidity at a higher asymmetry). The turnover is clearly seen in Fig. 2 and stems from the competition between these two isospin effects.

We notice that the TBF effect is negligibly small at low densities below and around the peak position. However, it gets stronger rapidly as the density goes up. The TBF turns out to induce a significant reduction of the proton pairing gaps in the high-density superfluidity domain. Consequently, it leads to a remarkable shrinking of the density domain for the existence of the proton superfluid phase. We notice that the TBF suppression of the 1S_0 proton superfluidity predicted here is particularly pronounced for highly asymmetric matter. For example, at $\beta = 0.8$, the density domain of the proton superfluid phase is reduced by about 50% (i.e., from $\rho \leq 0.435 \text{ fm}^{-3}$ to $\rho \leq 0.25 \text{ fm}^{-3}$) by inclusion of the TBF. In nuclear matter, proton pairs are embedded in the medium of neutrons and protons and both the surrounded protons and

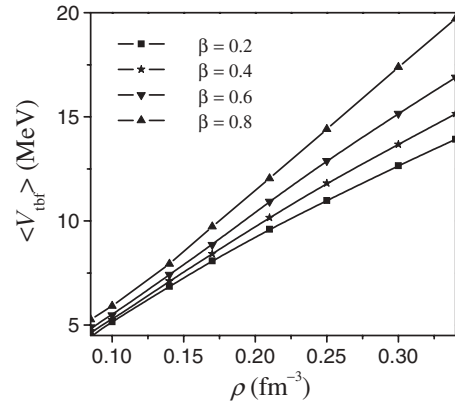


FIG. 3. The TBF contribution in the 1S_0 channel to the proton 1S_0 pairing interaction at the Fermi momentum vs the total density for several isospin asymmetries.

neutrons contribute to the TBF renormalization of the proton-proton interaction; therefore the TBF effect on the proton pairing correlations is determined by the *total* nucleon number density instead of the *proton* number density. Accordingly, in spite of the small proton fractions and the low proton densities at high asymmetries, the TBF modifies strongly the proton-proton pairing interactions in the high-density superfluidity domain and weakens considerably the corresponding proton pairing gaps. One can verify readily from Fig. 2 that the TBF suppression of the proton superfluidity is mainly in the high-density region and the reduction of the gap increases rapidly with increasing total density. We also notice from Fig. 2 that the TBF effect gets stronger as the asymmetry increases since the proton superfluid phase extends to larger densities for asymmetric matter at higher isospin asymmetries. Inclusion of the TBF weakens considerably the isospin dependence of the predicted proton superfluidity as compared to the results obtained by adopting purely the AV_{18} two-body force.

To clarify the TBF effect on the isospin dependence of the proton 1S_0 pairing gap in asymmetric nuclear matter more clearly, we plot in Fig. 3 the contribution of the averaged TBF in the 1S_0 channel to the proton pairing interaction at the Fermi momentum versus the total density for several isospin asymmetries. It is seen from the figure that the TBF generates a repulsive contribution to the proton pairing interaction in the 1S_0 channel. The repulsive contribution of the TBF increases continuously as the total density and the isospin asymmetry increase. It becomes especially strong at large densities and high asymmetries and may destroy the proton pairing remarkably.

IV. SUMMARY

In summary, we have calculated the neutron and proton 1S_0 pairing gaps in asymmetric nuclear matter based on the BHF approach and the BCS theory. We have especially investigated the isospin dependence of the pairing gaps in the 1S_0 channel and the influence of the TBF. It is shown that the isospin dependence of the proton 1S_0 superfluidity in asymmetric nuclear matter is completely different from that of the neutron one. The neutron 1S_0 superfluid phase exists only in the

low-density region for all isospin asymmetries. As the matter goes from symmetric nuclear matter to pure neutron matter, the peak value of the neutron 1S_0 pairing gap becomes larger and the density domain for the existence of neutron superfluidity shrinks gradually. The density domain for the proton superfluid phase enlarges rapidly as the isospin asymmetry rises and may extend to very high densities for highly asymmetric nuclear matter. The peak value of the proton pairing gap turns out to be smaller in asymmetric nuclear matter at a higher isospin asymmetry.

The TBF affects only weakly the neutron 1S_0 superfluid phase in asymmetric nuclear matter (i.e., it reduces slightly the pairing gap) because of the low-density region for this kind of superfluidity. However, it suppresses strongly the proton superfluidity in the 1S_0 channel at high densities, especially at high asymmetries. The density domain for the existence of the proton 1S_0 superfluid phase is reduced by about 50% from $\rho \leq 0.435 \text{ fm}^{-3}$ to $\rho \leq 0.25 \text{ fm}^{-3}$ by inclusion of the TBF.

In the present paper, we have concentrated on investigating the isospin dependence of the 1S_0 neutron and proton pairing gaps in asymmetric nuclear matter and the TBF effect within the BHF+BCS framework. We have not included the medium polarization effects in our present treatment. In the BHF+BCS framework, the irreducible pairing interactions are approximated by the bare nucleon-nucleon interactions and the BHF single-particle energy spectra are adopted in the

gap equation. For a strongly interacting Fermi system such as nuclear matter, on the one hand, the exact pairing interaction consists of the full RPA bubble series. On the other hand, the self-energy (the s.p. energy spectrum) is generally energy dependent (i.e., has a dispersive effect) [22], and the BHF s.p. energy spectrum is modified essentially by the ground-state many-body correlations, which may cause the depletion of the neutron and proton s.p. distributions [36]. The medium effects discussed here are expected to affect considerably the nucleon pairing in nuclear matter and their details have not yet been completely settled [13,21]. These medium polarization effects are especially unclear in the case of asymmetric nuclear matter [36]. Our present paper may serve as an intermediate step to the exploration of the isospin dependence of the nucleon superfluidity in asymmetric matter. A more complete and realistic understanding of the problem demands further efforts on the isospin dependence of the medium polarization effects.

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