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# Polarized neutron matter: A lowest order constrained variational approach

G. H. Bordbar* and M. Bigdeli

*Department of Physics, Shiraz University, Shiraz 71454, Iran, and*

*Research Institute for Astronomy and Astrophysics of Maragha, Post Office Box 55134-441, Maragha, Iran*

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In this paper, we calculate some properties of polarized neutron matter using the lowest order constrained variational method with the $AV_{18}$ potential and employing a microscopic point of view. A comparison is also made between our results and those of other many-body techniques.

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## I. INTRODUCTION

Pulsars are rapidly rotating neutron stars with strong surface magnetic fields in the range of $10^{12}$–$10^{13}$ G [1–3]. The physical origin of this magnetic field remains an open problem and there is still no general consensus regarding the mechanism to generate such strong magnetic fields in a neutron star. There exist several possible ways for generating a magnetic field in a neutron star, from the nuclear physics point of view; however, one of the most interesting and stimulating mechanisms that has been suggested is the possible existence of a phase transition to a ferromagnetic state at densities corresponding to theoretically stable neutron stars and, therefore, of a ferromagnetic core in the liquid interior of such compact objects. Such a possibility has been studied by several authors using different theoretical approaches [4–25], but the results are still contradictory. Whereas some calculations, such as for instance the ones based on Skyrme-like interactions, predict the transition to occur at densities in the range (1–4)$\rho_0$ ($\rho_0 = 0.16\ \text{fm}^{-3}$), others, such as recent Monte Carlo [20] and Brueckner-Hartree-Fock calculations [21–23] using modern two- and three-body realistic interactions, exclude such a transition, at least up to densities around $5\rho_0$. This transition could have important consequences for the evolution of proto-neutron stars, in particular for the spin correlations in the medium, which strongly affect the neutrino cross section and the neutrino mean free path inside the star [26].

In recent years, we have computed the equation of state of symmetrical and asymmetrical nuclear matter along with some of their properties, such as symmetry energy and pressure [27–30], and properties of spin-polarized liquid $^3$He [31] using the lowest order constrained variational (LOCV) approach. The LOCV method, which was developed several years ago, is a useful tool for the determination of the properties of neutron, nuclear, and asymmetric nuclear matter at zero and finite temperature [27–39]. The LOCV method is a fully self-consistent formalism and it does not bring any free parameters into the calculation. It employs a normalization constraint to keep the higher order term as small as possible [27,34]. The functional minimization procedure represents an enormous computational simplification over unconstrained methods that attempt to go beyond lowest order.

In the present work, we compute the properties of polarized neutron matter using the LOCV method with the $AV_{18}$ potential [40] and employing microscopic calculations where we treat explicitly the spin projection in the many-body wave functions.

## II. BASIC THEORY

### A. LOCV formalism

We consider a trial many-body wave function of the form

$$\psi = F\phi, \tag{1}$$

where $\phi$ is the uncorrelated ground-state wave function (simply the Slater determinant of plane waves) of $N$ independent neutrons and $F = F(1\ldots N)$ is an appropriate $N$-body correlation operator, which can be replaced by a Jastrow form, that is,

$$F = S\prod_{i>j} f(ij), \tag{2}$$

in which $S$ is a symmetrizing operator. We consider a cluster expansion of the energy functional up to the two-body term,

$$E([f]) = \frac{1}{N}\frac{\langle\psi|H\psi\rangle}{\langle\psi|\psi\rangle} = E_1 + E_2. \tag{3}$$

The one-body term $E_1$ for polarized neutron matter can be written as a Fermi momentum functional $[k_F{}^{(i)} = (6\pi^2\rho^{(i)})^{\frac{1}{3}}]$:

$$E_1 = \sum_{i=1,2}\frac{3}{5}\frac{\hbar^2 k_F^{(i)^2}}{2m}\frac{\rho^{(i)}}{\rho}. \tag{4}$$

Labels 1 and 2 are used instead of spin up and spin down neutrons, respectively, and $\rho = \rho^{(1)} + \rho^{(2)}$ is the total neutron matter density. The two-body energy $E_2$ is

$$E_2 = \frac{1}{2A}\sum_{ij}\langle ij|\nu(12)|ij - ji\rangle, \tag{5}$$

where $\nu(12) = -\frac{\hbar^2}{2m}[f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12)$ and $f(12)$ and $V(12)$ are the two-body correlation and

*Corresponding author. Email address: bordbar@physics.susc.ac.ir

 

potential, respectively. For the two-body correlation function, $f(12)$, we consider the following form [27,28]:

$$f(12) = \sum_{k=1}^{3} f^{(k)}(12) O^{(k)}(12), \qquad (6)$$

where the operators $O^{(k)}(12)$ are given by

$$O^{(k=1-3)}(12) = 1, \quad \left(\tfrac{2}{3} + \tfrac{1}{6} S_{12}\right), \quad \left(\tfrac{1}{3} - \tfrac{1}{6} S_{12}\right) \qquad (7)$$

and $S_{12}$ is the tensor operator.

After doing some algebra we find the following equation for the two-body energy:

$$\begin{aligned} E_2 = {} & \frac{2}{\pi^4 \rho} \left(\frac{h^2}{2m}\right) \sum_{JLSS_z} \frac{(2J+1)}{2(2S+1)} [1 - (-1)^{L+S+1}] \\ & \times \left| \left\langle \frac{1}{2}\sigma_{z1} \frac{1}{2}\sigma_{z2} \middle| SS_z \right\rangle \right|^2 \int dr \left\{ \left[ f_\alpha^{(1)'^2} a_\alpha^{(1)^2}(k_f r) \right.\right. \\ & + \frac{2m}{h^2} \big(\{V_c - 3V_\sigma + V_\tau - 3V_{\sigma\tau} + 2(V_T - 3V_{\sigma\tau}) \\ & + 2V_{\tau z}\} a_\alpha^{(1)\,2}(k_f r) + [V_{l2} - 3V_{l2\sigma} + V_{l2\tau} \\ & \left. - 3V_{l2\sigma\tau}] c_\alpha^{(1)^2}(k_f r)\big) \big(f_\alpha^{(1)}\big)^2 \right] \\ & + \sum_{k=2,3} \left[ f_\alpha^{(k)'^2} a_\alpha^{(k)^2}(k_f r) + \frac{2m}{h^2} \big(\{V_c + V_\sigma + V_\tau \right. \\ & + V_{\sigma\tau} + (-6k + 14)(V_{tz} + V_t) \\ & - (k-1)(V_{ls\tau} + V_{ls}) + [V_T + V_{\sigma\tau} \\ & + (-6k + 14)V_{tT}][2 + 2V_{\tau z}]\} a_\alpha^{(k)^2}(k_f r) \\ & + [V_{l2} + V_{l2\sigma} + V_{l2\tau} + V_{l2\sigma\tau}] c_\alpha^{(k)^2}(k_f r) \\ & \left. + [V_{ls2} + V_{ls2\tau}] d_\alpha^{(k)^2}(k_f r)\big) f_\alpha^{(k)^2} \right] \\ & + \frac{2m}{h^2} \{V_{ls} + V_{ls\tau} - 2(V_{l2} + V_{l2\sigma} + V_{l2\sigma\tau} + V_{l2\tau}) \\ & - 3(V_{ls2} + V_{ls2\tau})\} b_\alpha^2(k_f r) f_\alpha^{(2)} f_\alpha^{(3)} \\ & \left. + \frac{1}{r^2} \big(f_\alpha^{(2)} - f_\alpha^{(3)}\big)^2 b_\alpha^2(k_f r) \right\}, \end{aligned} \qquad (8)$$

where $\alpha = \{J, L, S, S_z\}$ and the coefficients $a_\alpha^{(1)^2}$, etc. are defined as

$$a_\alpha^{(1)^2}(x) = x^2 I_{L,S_z}(x), \qquad (9)$$

$$a_\alpha^{(2)^2}(x) = x^2 \big[\beta I_{J-1,S_z}(x) + \gamma I_{J+1,S_z}(x)\big], \qquad (10)$$

$$a_\alpha^{(3)^2}(x) = x^2 \big[\gamma I_{J-1,S_z}(x) + \beta I_{J+1,S_z}(x)\big], \qquad (11)$$

$$b_\alpha^{(2)}(x) = x^2 \big[\beta_{23} I_{J-1,S_z}(x) - \beta_{23} I_{J+1,S_z}(x)\big], \qquad (12)$$

$$c_\alpha^{(1)^2}(x) = x^2 \nu_1 I_{L,S_z}(x), \qquad (13)$$

$$c_\alpha^{(2)^2}(x) = x^2 \big[\eta_2 I_{J-1,S_z}(x) + \nu_2 I_{J+1,S_z}(x)\big], \qquad (14)$$

$$c_\alpha^{(3)^2}(x) = x^2 \big[\eta_3 I_{J-1,S_z}(x) + \nu_3 I_{J+1,S_z}(x)\big], \qquad (15)$$

$$d_\alpha^{(2)^2}(x) = x^2 \big[\xi_2 I_{J-1,S_z}(x) + \lambda_2 I_{J+1,S_z}(x)\big], \qquad (16)$$

$$d_\alpha^{(3)^2}(x) = x^2 \big[\xi_3 I_{J-1,S_z}(x) + \lambda_3 I_{J+1,S_z}(x)\big], \qquad (17)$$

with

$$\beta = \frac{J+1}{2J+1}, \quad \gamma = \frac{J}{2J+1}, \quad \beta_{23} = \frac{2J(J+1)}{2J+1}, \qquad (18)$$

$$\begin{aligned} \nu_1 &= L(L+1), \quad \nu_2 = \frac{J^2(J+1)}{2J+1}, \\ \nu_3 &= \frac{J^3 + 2J^2 + 3J + 2}{2J+1}, \end{aligned} \qquad (19)$$

$$\eta_2 = \frac{J(J^2 + 2J + 1)}{2J+1}, \quad \eta_3 = \frac{J(J^2 + J + 2)}{2J+1}, \qquad (20)$$

$$\xi_2 = \frac{J^3 + 2J^2 + 2J + 1}{2J+1}, \quad \xi_3 = \frac{J(J^2 + J + 4)}{2J+1}, \qquad (21)$$

$$\lambda_2 = \frac{J(J^2 + J + 1)}{2J+1}, \quad \lambda_3 = \frac{J^3 + 2J^2 + 5J + 4}{2J+1}, \qquad (22)$$

and

$$I_{J,S_z}(x) = \int dq P_{S_z}(q) J_J^2(xq). \qquad (23)$$

In this last equation $J_J(x)$ is the Bessel's function and $P_{S_z}(q)$ is defined as

$$\begin{aligned} P_{S_z}(q) = {} & \tfrac{2}{3}\pi \big[\big(k_F^{\sigma_{z1}}\big)^3 + \big(k_F^{\sigma_{z2}}\big)^3 - \tfrac{3}{2}\big(\big(k_F^{\sigma_{z1}}\big)^2 + \big(k_F^{\sigma_{z2}}\big)^2\big) q \\ & - \tfrac{3}{16}\big(\big(k_F^{\sigma_{z1}}\big)^2 - \big(k_F^{\sigma_{z2}}\big)^2\big)^2 q^{-1} + q^3\big] \end{aligned} \qquad (24)$$

for $\frac{1}{2}\left|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}\right| < q < \frac{1}{2}\left|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}\right|$,

$$P_{S_z}(q) = \tfrac{4}{3}\pi \min\left(k_F^{\sigma_{z1}}, k_F^{\sigma_{z2}}\right) \qquad (25)$$

for $q < \frac{1}{2}\left|k_F^{\sigma_{z1}} - k_F^{\sigma_{z2}}\right|$, and

$$P_{S_z}(q) = 0 \qquad (26)$$

for $q > \frac{1}{2}|k_F^{\sigma_{z1}} + k_F^{\sigma_{z2}}|$, where $\sigma_{z1}$ or $\sigma_{z2} = \frac{1}{2}, -\frac{1}{2}$ for spin up and spin down, respectively.

Now, we can minimize the two-body energy [Eq. (8)] with respect to the variations in the function $f_\alpha{}^{(i)}$ but subject to the normalization constraint [28],

$$\frac{1}{A} \sum_{ij} \langle ij | h_{S_z}^2 - f^2(12) | ij \rangle_a = 0, \qquad (27)$$

where in the case of spin-polarized neutron matter the function $h_{S_z}(r)$ is defined as

$$\begin{aligned} h_{S_z}(r) &= \left[1 - 9\left(\frac{J_J^2(k_F^i)}{k_F^i}\right)^2\right]^{-1/2}, \quad S_z = \pm 1, \\ &= 1, \quad S_z = 0. \end{aligned} \qquad (28)$$

From the minimization of the two-body cluster energy, we get a set of coupled and uncoupled differential equations that are the same as presented in Ref. [28].

### B. Magnetic susceptibility

The magnetic susceptibility $\chi$, which characterizes the response of a system to the magnetic field, is defined by

$$\chi = \left(\frac{\partial M}{\partial H}\right)_{H=0}, \tag{29}$$

where $M$ is the magnetization of the system per unit volume and $H$ is the magnetic field. By some simplification, the magnetic susceptibility can be written as

$$\chi = \frac{\mu^2 \rho}{\left(\frac{\partial^2 E}{\partial \delta^2}\right)_{\delta=0}}, \tag{30}$$

where $\mu$ is the magnetic moment of the neutron and $\delta$ is the spin-polarization parameter, which is defined as

$$\delta = \frac{\rho^{(1)} - \rho^{(2)}}{\rho}. \tag{31}$$

Usually, one is interested in calculating the ratio of $\chi$ to the magnetic susceptibility for a degenerate free Fermi gas ($\chi_F$). The susceptibility $\chi_F$ can be straightforwardly obtained from Eq. (30) by using the total energy per particle of a free Fermi gas,

$$\chi_F = \frac{\mu^2 m}{\hbar^2 \pi^2} k_F, \tag{32}$$

where $k_F = (3\pi^2\rho)^{1/3}$ is the Fermi momentum. After a little algebra one finds

$$\frac{\chi}{\chi_F} = \frac{2}{3} \frac{E_F}{\left(\frac{\partial^2 E}{\partial \delta^2}\right)_{\delta=0}}, \tag{33}$$

where $E_F = \hbar^2 k_F^2 / 2m$ is the Fermi energy.

## III. RESULTS

In Fig. 1, we have shown the energy per particle for various values of spin polarization of neutron matter as a function of density. As can be seen from this figure, the energy of neutron matter becomes repulsive by increasing the polarization for all relevant densities. According to this result, the spontaneous phase transition to a ferromagnetic state in neutron matter does not occur. If such a transition existed a crossing of the energies of different polarizations would be observed at some density, indicating that the ground state of the system would be ferromagnetic from that density on. As is shown in Fig. 1, there is no crossing point. The existence of a crossing point becomes less favorable as density increases. For the energy of neutron matter, we have also made a comparison between our results and the results of other many-body methods with the $AV_{18}$ potential, as shown in Fig. 2. The BGLS calculations are based on the Brueckner-Hartree-Fock approximation both for continuous choice (BHFC) and standard choice (BHFG) [41]. The APR results have been obtained by using the variational chain summation (VCS) method [42] and the EHMMP calculations have been carried out by using the lowest order Brueckner (LOB) technique [43]. We see that our results

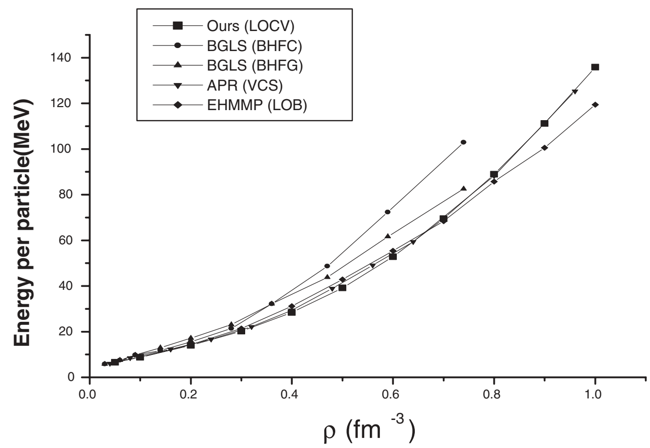


FIG. 2. Comparison between our results for the energy per particle of neutron matter and those of BGLS [41], APR [42], and EHMMP [43] calculations with the $AV_{18}$ potential.

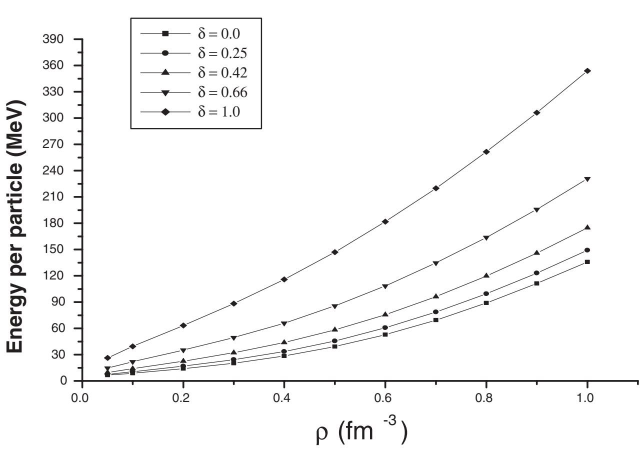


FIG. 1. The energy per particle vs density ($\rho$) for different values of the spin polarization ($\delta$) of the neutron matter.

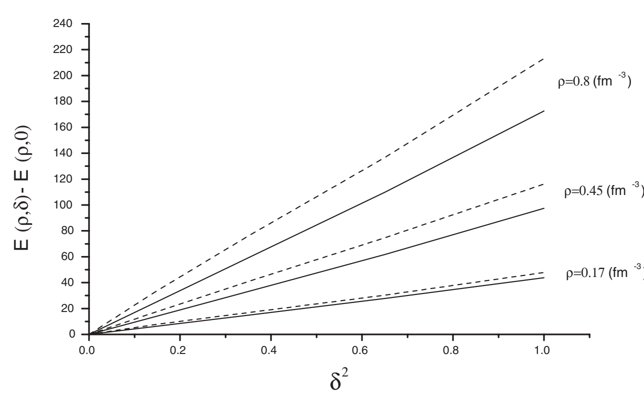


FIG. 3. Our results (full curves) for the energy difference of polarized and unpolarized cases vs quadratic spin polarization ($\delta$) for different values of the density ($\rho$) of the neutron matter. The results of ZLS [25] (dashed curves) are also presented for comparison.

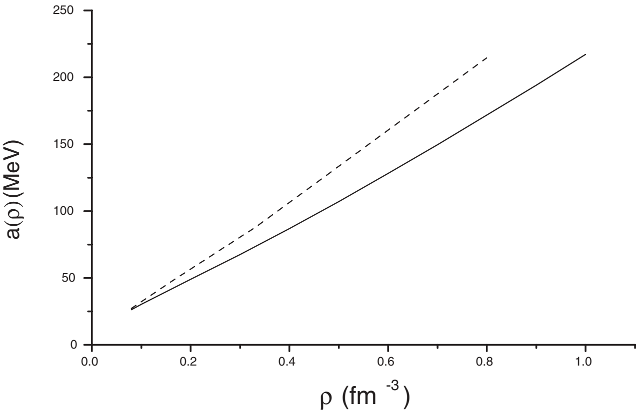


FIG. 4. Our results (full curve) for the parameter $a(\rho)$ as a function of the density ($\rho$). The results of ZLS [25] (dashed curve) are also given for comparison.

are in agreement with those of others, especially with the APR and EHMMP calculations.

For neutron matter, we have also considered the dependence of energy on the spin polarization $\delta$. Let us examine this dependency in quadratic spin-polarization form for different densities as shown in Fig. 3. As can be seen from this figure, the energy per particle increases as the polarization increases and the minimum value of energy occurs at $\delta = 0$ for all densities. This indicates that the ground state of neutron matter is paramagnetic. In Fig. 3, the results of ZLS calculations using the Brueckner-Hartree-Fock theory with the $AV_{18}$ potential [25] are also given for comparison. There is an agreement between our results and those of ZLS, especially at low densities. From Fig. 3, it is also seen that the variation of the energy of neutron matter versus $\delta^2$ is nearly linear. Therefore, one can characterize this dependency in the following analytical form:

$$E(\rho, \delta) = E(\rho, 0) + a(\rho)\delta^2. \tag{34}$$

The density-dependent parameter $a(\rho)$ can be interpreted as a measure of the energy required to produce a net spin

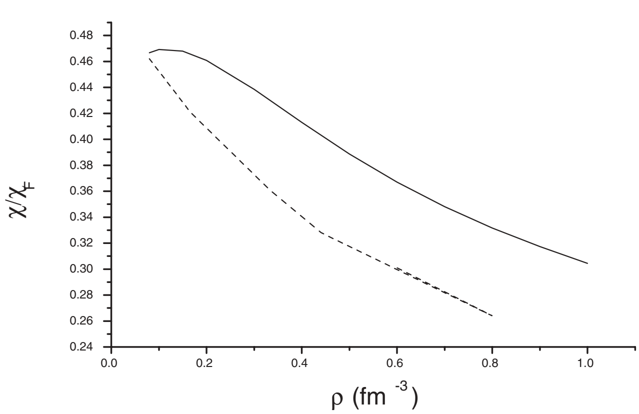


FIG. 5. Same as Fig. 4, but for the magnetic susceptibility ($\chi/\chi_F$).

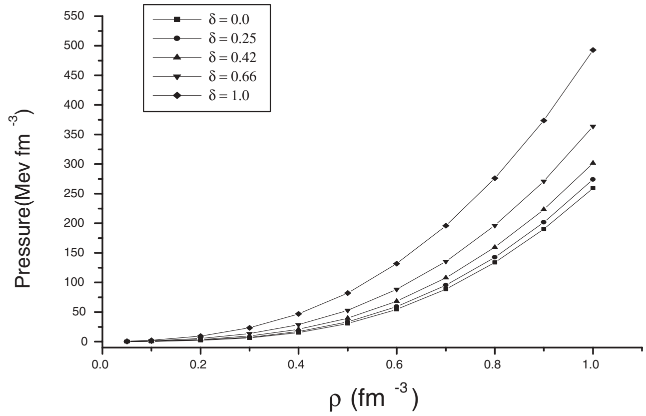


FIG. 6. The equation of state of neutron matter for different values of the spin polarization ($\delta$).

alignment in the direction of the magnetic field, and its value can be determined as the slope of each line in Fig. 3, for the corresponding density,

$$a(\rho) = \frac{\partial E(\rho, \delta)}{\partial \delta^2}. \tag{35}$$

In Fig. 4, the parameter $a(\rho)$ is shown as a function of the density and as can be seen the value of this parameter increases by increasing the density. In turn this indicates that the energy required to align spins in the same direction increases. One can infer again from this result that a phase transition to a ferromagnetic state is not to be expected from our calculation. The parameter $a(\rho)$ obtained by ZLS [25] is also shown in Fig. 4 for comparison.

In Fig. 5, we have plotted the ratio $\chi/\chi_F$ versus density. As can be seen from Fig. 5, this ratio changes continuously for all densities. Therefore, the ferromagnetic phase transition does not occur. For comparison, we have also shown the results of ZLS [25] in this figure.

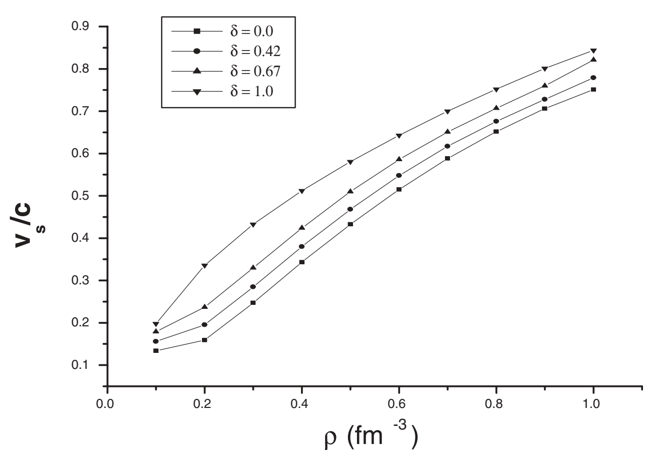


FIG. 7. The velocity of sound in units of $c$ vs density ($\rho$) for different values of the spin polarization ($\delta$).

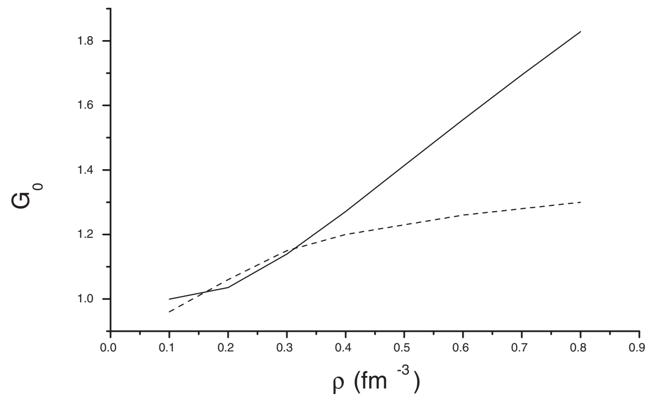


FIG. 8. Same as Fig. 4, but for the Landau parameter $G_0$.

The equation of state of polarized neutron matter, $P(\rho, \delta)$, can be simply obtained by using

$$P(\rho, \delta) = \rho^2 \frac{\partial E(\rho, \delta)}{\partial \rho}. \tag{36}$$

In Fig. 6, we have presented the pressure of neutron matter as a function of density $\rho$ at different polarizations. We see that the equation of state becomes stiffer by increasing the polarization. We also see that, with increasing density, the difference between the equations of state at different polarization becomes more appreciable. To check the causality condition for our equations of state, we have calculated the velocity of sound, $v_s$, as shown in Fig. 7. It is seen that the velocity of sound increases with both increasing polarization and density, but it is always less than the velocity of light in vacuum ($c$). Therefore, all calculated equations of state obey the causality condition.

As is known, the Landau parameter $G_0$ describes the spin-density fluctuation in the effective interaction. It is simply related to the magnetic susceptibility by the relation

$$\frac{\chi}{\chi_F} = \frac{m^*}{1 + G_0}, \tag{37}$$

where $m^*$ is the effective mass. A magnetic instability would require $G_0 < -1$. Our results for the Landau parameter have been presented in Fig. 8. It is seen that the value of $G_0$ is always positive and monotonically increasing up to highest density and does not show any magnetic instability for the neutron matter. In Fig. 8, the results of ZLS calculations [25] are also given for comparison.

## IV. SUMMARY AND CONCLUSIONS

The properties of neutron matter are of primary importance in the study of neutron stars and, in particular, strongly magnetized ones (i.e., pulsars). It is therefore important to calculate the properties of polarized neutron matter by using an efficient and sufficiently accurate method. We have recently computed various properties of neutron matter using the lowest order constrained variational scheme. To make our results more general, we used this method for polarized neutron matter. The energy per particle for various values of spin polarization of neutron matter was computed as a function of density and was shown to become repulsive as a result of increasing the polarization. In addition, we considered the dependence of the energy of neutron matter on the spin polarization and found it to increase with the spin polarization for all densities. This dependence was represented by a quadratic formula with the coefficient of the quadratic term $a(\rho)$ determined as a function of the density. This parameter, too, was shown to increase monotonically with density. Magnetic susceptibility, which characterizes the response of the system to the magnetic field, was calculated for the system under consideration. We have also computed the equation of state of neutron matter at different polarizations. Our results for higher values of polarization show a stiff equation of state. The velocity of sound was computed to check the causality condition of the equation of state and it was shown that it is always lower than the velocity of light in vacuum. We have also investigated the Landau parameter $G_0$, showing that the value of $G_0$ is always positive and monotonically increasing up to high densities. Finally, our results showed no phase transition to the ferromagnetic state. We have also compared the results of our calculations for the properties of neutron matter with other calculations.

## ACKNOWLEDGMENTS

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