Gamow peak in thermonuclear reactions at high temperatures

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The Gamow peak represents one of the most important concepts in the study of thermonuclear reactions in stars. It is widely used in order to determine, at a given plasma temperature, the effective stellar energy region in which most charged-particle induced nuclear reactions occur. It is of importance in the design of nuclear astrophysics measurements, including those involving radioactive ion beams, and for the determination of stellar reaction rates. We demonstrate that the Gamow peak concept breaks down under certain conditions if a nuclear reaction proceeds through narrow resonances at elevated temperatures. It is shown that an effective stellar energy window does indeed exist in which most thermonuclear reactions take place at a given temperature, but that this energy window can differ significantly from the commonly used Gamow peak. We expect that these findings are especially important for thermonuclear reactions in the advanced burning stages of massive stars and in explosive stellar environments.

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The probability for a thermonuclear reaction to occur in a star depends mainly on two factors. The first factor is the velocity distribution of the nuclei in the plasma and is usually given by a Maxwell-Boltzmann distribution. The second factor is the nuclear reaction cross section and contains the tunneling probability through the Coulomb barrier. Most thermonuclear reactions in stars take place at neither very low energies where the reaction cross section is very small, nor at relatively high energies where the number of nuclei is very small. Nuclear reactions in a stellar plasma will occur near energies where the product of velocity distribution and cross section is at maximum.

The thermonuclear rate per particle pair for a reaction involving two nuclei is given by [1]

$$\langle \sigma v \rangle = \frac{\sqrt{8/(\pi m)}}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\,e^{-E/kT} dE, \tag{1}$$

where $\sigma(E)$ is the nuclear reaction cross section at energy E and the factor $\mathrm{E}e^{-E/kT}$ contains the Maxwell-Boltzmann distribution. All energies are given in the center-of-mass system. The quantities T, k, and m are the plasma temperature, the Boltzmann constant and the reduced mass of the interacting nuclei, respectively.

In this article we report on some interesting properties of Eq. (1) when a nuclear reaction proceeds through narrow resonances. For example, the total reaction rates of charged particle induced reactions on target nuclei in the $A = 20$–40 mass range are dominated by the contributions from narrow resonances. The effects that we describe occur at temperatures in excess of ≈ 1 GK and are thus important for advanced stellar burning stages in massive stars and in certain explosive stellar environments.

For a nonresonant reaction, the cross section depends only weakly on the properties of the nuclear interior. Hence, it can be factored into a part describing the strongly energy-dependent tunneling probability through the Coulomb barrier (the *Gamow factor* $e^{-2\pi\eta}$) and a part representing the weakly energy-dependent properties of the nuclear interior (the *astrophysical S-factor*). The nonresonant reaction rate is then given by

$$\langle \sigma v \rangle_{nr} = \frac{\sqrt{8/(\pi m)}}{(kT)^{3/2}} \int_0^\infty S(E) e^{-2\pi\eta - E/kT}\, dE, \tag{2}$$

where $\sigma(E) \equiv E^{-1} e^{-2\pi\eta} S(E)$. The quantity $\eta \propto E^{-1/2}$ is the Sommerfeld parameter and depends on the charges, masses and the energy of the interacting nuclei. Obviously, the reaction rate is dominated by the exponential factor in Eq. (2) which is referred to as the *Gamow peak*. Its maximum is located at

$$E_0 = 0.1220 \left(Z_p^2 Z_t^2 \frac{M_p M_t}{M_p + M_t} T_9^2 \right)^{1/3} \text{(MeV)}, \tag{3}$$

where M_i and Z_i are the masses (in amu) and charges of projectile and target, respectively; T_9 is the plasma temperature in GK. Approximating the Gamow peak by a Gaussian yields for the $1/e$ width

$$\Delta E_0 = 0.2368 \left(Z_p^2 Z_t^2 \frac{M_p M_t}{M_p + M_t} T_9^5 \right)^{1/6} \text{(MeV)}. \tag{4}$$

The region $E_0 \pm \Delta E_0/2$ represents the effective energy window for nonresonant thermonuclear reactions in stars. For increasing temperature, this window shifts towards higher energy and becomes broader according to Eqs. (3) and (4).

The Gamow peak strictly applies to nonresonant reactions. However, it is easily demonstrated that the concept of a Gamow peak also plays a crucial role when a nuclear reaction proceeds through narrow resonances. In this case, the reaction cross section does not only depend on the tunneling probability through the Coulomb barrier but, in addition, has a strong energy dependence caused by the influence of the nuclear interior. The reaction cross section for an isolated resonance, located at an energy E_r, is given by the well-known one-level Breit-Wigner formula

$$\sigma_r(E) = \frac{\lambda^2}{4\pi} \frac{(2J+1)}{(2j_p+1)(2j_t+1)} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4} \quad (5)$$

with λ the de Broglie wavelength of the projectile, J the resonance spin and j_p, j_t the projectile and target spins. The partial widths Γ_i represent the probability (in energy units) for the formation or decay of the compound nucleus via the entrance or exit channel, while Γ is the total resonance width.

The partial width for a charged particle a (usually either a proton or α-particle) is strongly energy dependent because the projectile must penetrate the Coulomb barrier. It is sufficient for our purposes to use the expression $\Gamma_a = c_1\ P_\ell(E) \propto e^{-2\pi\eta}$, where the penetration factor $P_\ell(E)$ for the Coulomb and centripetal barriers is approximated by the Gamow factor [4]. The quantity c_1 depends on the nuclear structure of the resonance. The penetration factor determines the energy-dependence of charged-particle partial widths: these vary by many orders of magnitude, from vanishingly small values at very low interaction energies to many keV at higher energies.

On the other hand, γ-ray partial widths depend only weakly on the interaction energy via $\Gamma_\gamma = c_2\ E_\gamma^{2L+1} \propto (E+Q)^{2L+1}$, where E_γ and L are the energy and multipolarity, respectively, of the γ-ray transition under consideration, and Q is the reaction Q-value. The quantity c_2 depends on the nuclear structure of the two levels involved in the interaction. Typically, γ-ray partial widths amount to $\approx$meV–eV.

If a number of narrow, radiative capture, resonances occur over a given stellar energy range, then which of these are expected to contribute significantly to the total thermonuclear rates? Suppose for the moment that the particle partial width of a given resonance is much smaller than the γ-ray partial width, $\Gamma_a \ll \Gamma_\gamma$, which is usually the case at low interaction energies. From Eqs. (1) and (5) one finds

$$\langle \sigma v \rangle_r \propto \int_0^\infty \frac{\Gamma_\gamma}{(E_r - E)^2 + \Gamma_\gamma^2/4} e^{-2\pi\eta} e^{-E/kT}\, dE. \quad (6)$$

The result is interesting because it shows that the integrand can be written as the product of two factors: (i) the Gamow peak $e^{-2\pi\eta - E/kT}$, and (ii) a resonant S-factor curve of Lorentzian shape. Note that the Lorentzian has a FWHM of Γ_γ and a maximum height of $4/\Gamma_\gamma$. Thus for a narrow resonance a variation of Γ_γ has no influence on the area under the Lorentzian curve: the reaction rate is independent of the γ-ray partial width. If a reaction cross section exhibits a number of narrow resonances, then it is obvious from Eq. (6) that those resonances located in the region of the Gamow peak (at energies of $E_0 \pm \Delta E_0/2$) will be the major contributors to the total stellar reaction rates. Of course, the quantity c_1 varies from resonance to resonance and its value also influences the reaction rates. Nevertheless, the Gamow factor contains the dominant energy dependence. If there are narrow resonances located in the Gamow peak, then other resonances located either below or above the Gamow peak will be of minor importance. The Gamow peak concept for narrow resonances has been described before [2]. It is widely used in nuclear astrophysics because it provides a simple and straightforward way for identifying the most important resonances in a given thermonuclear reaction.

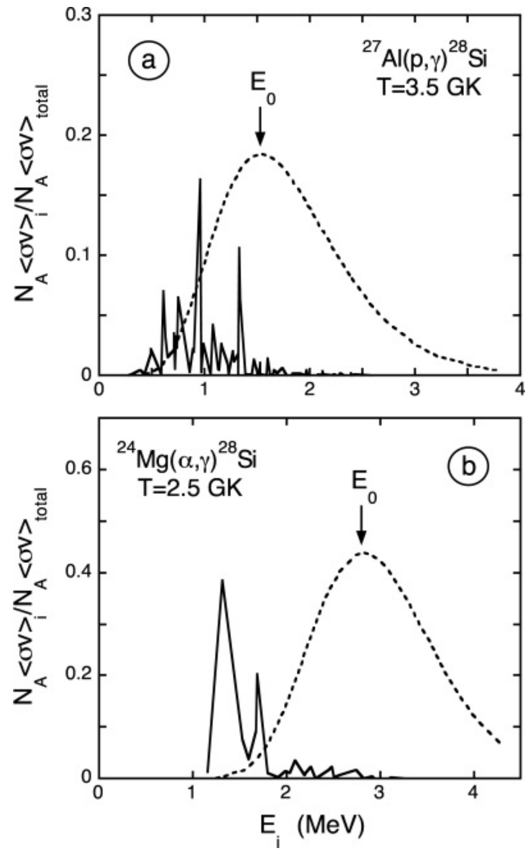

FIG. 1. Fractional contributions of narrow resonances to the total reaction rate versus resonance energy for (a) ^{27}Al(p,γ)^{28}Si at T = 3.5 GK and (b) ^{24}Mg(α,γ)^{28}Si at T = 2.5 GK. The Gamow peak is shown as dotted line. Resonance energies and strengths are adopted from Ref. [3].

Before continuing the discussion, it is worthwhile to visualize our expectations. If one would graph, at a given temperature, the fractional contribution of each narrow resonance to the total thermonuclear rates, then we expect a distribution with a shape that is strongly influenced by the Gamow peak: the largest contribution to the total rates would arise from energies near E_0, while about equal contributions are expected from the energy regions below and above E_0. Consider now Fig. 1, showing the fractional resonance contributions for the ^{27}Al(p,γ)^{28}Si reaction at a temperature of T = 3.5 GK (top panel) and for the ^{24}Mg(α,γ)^{28}Si reaction at T = 2.5 GK (bottom panel). The dotted lines show the Gamow peak. Contrary to expectations, it can be seen that for both reactions the largest contribution to the total thermonuclear rates arises from energies *below* the Gamow peak maximum. These results are not influenced by unobserved resonances, since all of the important resonances have been measured directly over the displayed energy ranges [3]. Neither are these results peculiar to the two reactions presented. Indeed, qualitatively similar results are obtained for all other reactions that we surveyed (see below).

How can these surprising results be explained? The answer is to be found in the curious interplay of particle and γ-ray partial widths. Suppose that there are a number of resonances located throughout the energy range of interest. At relatively low energies, the particle partial widths are much smaller than the γ-ray partial widths. For this condition, the Gamow peak represents the effective energy window according to Eq. (6). At higher energies, however, exactly the opposite situation prevails. While the γ-ray partial width is still on the order of meV–eV, the particle partial width may amount to several keV or more. For the condition $\Gamma_a \gg \Gamma_\gamma$, Eqs. (1) and (5) yield for a given resonance

$$\langle \sigma v \rangle_r \propto \int_0^\infty \frac{\Gamma_a}{(E_r - E)^2 + \Gamma_a^2/4} (E+Q)^{2L+1} e^{-E/kT} dE. \tag{7}$$

This result is also interesting because it shows that the integrand can be written as the product of three factors: (i) the Maxwell-Boltzmann term $e^{-E/kT}$, (ii) the weakly energy-dependent factor $(E+Q)^{2L+1}$, and (iii) a resonant S-factor curve of Lorentzian shape. The Lorentzian has a FWHM of Γ_a and a maximum height of $4/\Gamma_a$. Hence for a narrow resonance a variation of Γ_a has no influence on the area under the Lorentzian curve: the reaction rate is independent of the particle partial width. In other words, *a Gamow peak does not exist if* $\Gamma_a \gg \Gamma_\gamma$. Of course, the quantity c_2 varies from resonance to resonance and its value will also influence the reaction rates. Nevertheless, the factor $e^{-E/kT}$ contains the dominant energy dependence and, consequently, the most important resonances are generally located at the lowest energies (while still fulfilling the condition $\Gamma_a \gg \Gamma_\gamma$). Note, that this statement is independent of temperature: the overall magnitude of the reaction rate changes with T according to the factor $e^{-E/kT}$ in Eq. (7), but the most important resonances are still those located at the lowest energies.

For the following discussion, it is useful to introduce two quantities: E' is the expectation value of the distribution of fractional resonance contributions and $\Delta E'$ denotes the width of the stellar energy region that is centered about E' and contributes 68% of the total reaction rate. Based on the above ideas, the features seen in Fig. 1 have a simple explanation. In realistic situations, at any given stellar temperature, three different groups of resonances contribute to the total reaction rates of a given nuclear reaction. They can be distinguished based on the relative magnitude of their partial widths: (i) $\Gamma_a \ll \Gamma_\gamma$, (ii) $\Gamma_a \gg \Gamma_\gamma$, and (iii) $\Gamma_a \approx \Gamma_\gamma$. We may conclude:

First, the group of resonances with $\Gamma_a \ll \Gamma_\gamma$ has a maximum fractional contribution to the total reaction rates near an energy of E_0, while the group of resonances with $\Gamma_a \gg \Gamma_\gamma$ makes the largest fractional contribution at the smallest resonance energies. For sufficiently high temperatures, the latter group will eventually contribute to the total reaction rates and, therefore, the most effective energy E' for thermonuclear burning must in general be located *below* the Gamow peak energy E_0.

Second, for the group of resonances with $\Gamma_a \ll \Gamma_\gamma$, the Gamow peak will shift with increasing temperature according to Eqs. (3) and (4). However, no such energy shift will occur for the group of resonances with $\Gamma_a \gg \Gamma_\gamma$. In the latter case, the same resonances dominate the fractional contribution to the total stellar reaction rates, independent of temperature, and these resonances are located at the lowest energies (while still fulfilling the condition $\Gamma_a \gg \Gamma_\gamma$). Since in general both groups of resonances are contributing to the total reaction rates, the effective stellar energy window $E' \pm \Delta E'/2$ will shift towards higher energies for increasing temperature, but this energy shift should be much smaller than what is predicted from Eqs. (3) and (4).

Third, at sufficiently high temperatures, the Gamow peak will be located at energies where the condition $\Gamma_a \gg \Gamma_\gamma$ is fulfilled for the majority of resonances. The group of resonances with $\Gamma_a \ll \Gamma_\gamma$ makes then a negligible contribution to the total rates and the location of the effective energy window $E' \pm \Delta E'/2$ becomes independent of temperature. This maximum and temperature-independent value of E' is specific to the nuclear reaction and should be close to an energy where, on average, the γ-ray partial width is approximately equal to the particle partial width, $\Gamma_a \approx \Gamma_\gamma$.

A few selected results supporting these conclusions will now be discussed. Figure 2 displays the energy regions $E_0 \pm \Delta E_0/2$ (black lines) and $E' \pm \Delta E'/2$ (red lines) versus temperature for the ^{35}Cl$(p,\gamma)^{36}$Ar reaction. Only at the lowest temperature ($T \approx 0.5$ GK) does the Gamow peak roughly correspond to the effective thermonuclear energy range. The difference between the energy regions $E' \pm \Delta E'/2$ and $E_0 \pm \Delta E_0/2$ becomes more pronounced with increasing temperature. At the highest temperature shown ($T \approx 2.5$ GK), the entire energy region $E' \pm \Delta E'/2$ is located below the Gamow peak energy E_0. Furthermore, the effective stellar energy window has a weaker temperature dependence and is also much narrower compared to the Gamow peak at all temperatures.

Effective thermonuclear energy windows and Gamow peaks are compared in Fig. 3 for all proton-capture reactions that we surveyed. At a relatively low temperature of $T = 0.6$ GK (top panel), the effective stellar energy windows (red bars) are located within the Gamow peaks (black bars). The

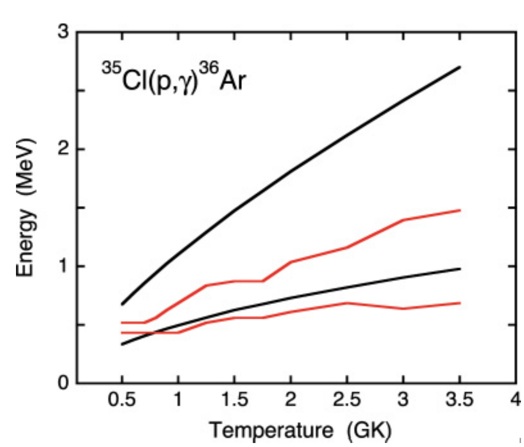

FIG. 2. (Color) Gamow peak (region between black lines) and effective thermonuclear energy range (region between red lines) versus stellar temperature for the ^{35}Cl(p,γ)^{36}Ar reaction. Resonance energies and strengths are adopted from Ref. [3].

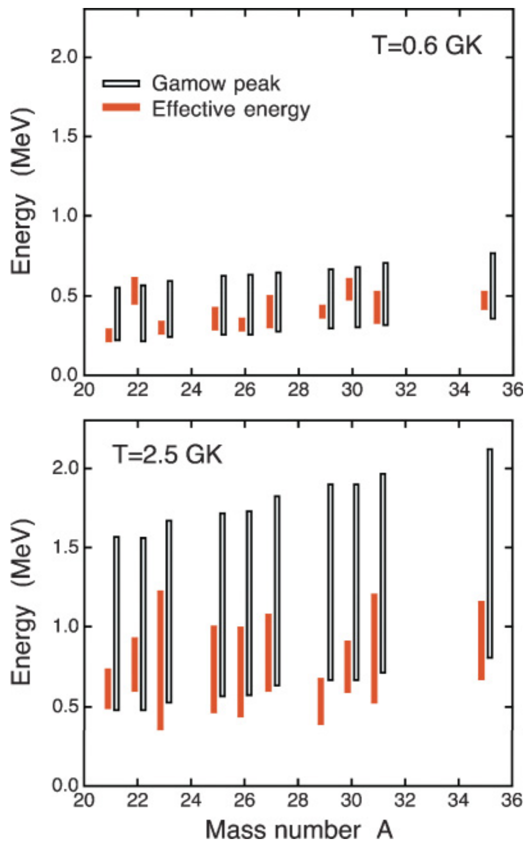

FIG. 3. (Color) Gamow peak (black bars) and effective thermonuclear energy range (red bars) versus target mass number for the proton-capture reactions on ^{21,22}Ne, ^{23}Na, ^{25,26}Mg, ^{27}Al, ^{29,30}Si, ^{31}P and ^{35}Cl at two different temperatures: $T = 0.6$ GK (top panel) and $T = 2.5$ GK (bottom panel). Resonance energies and strengths are adopted from Ref. [3].

former energy regions scatter significantly for different reactions. This is caused by the fact that at this lower temperature the reaction rates are dominated by relatively few resonances. Therefore, the contributions from single, particularly strong, resonances are more likely to dominate the total reaction rates. At a higher temperature of $T = 2.5$ GK (bottom panel), it is again obvious that for each reaction the energy region $E' \pm \Delta E'/2$ is located below E_0 and that the effective energy window is much narrower than the Gamow peak. More resonances contribute to the reaction rates at this higher temperature and thus their individual properties are not as important. Consequently, the effective stellar energy windows have a similar location for all proton-capture reactions shown ($E' \approx 0.8$ MeV).

We demonstrated that the Gamow peak may differ significantly from the effective stellar energy window of nuclear burning, especially if a reaction proceeds through narrow resonances at elevated temperatures. This is indeed the case for the majority of thermonuclear reactions in the advanced burning stages of massive stars and in explosive stellar burning environments.

It is shown that it is frequently inappropriate to estimate thermonuclear reaction rates by considering only those resonances that are located in the region of the Gamow peak. Similarly, laboratory measurements of astrophysically important nuclear reactions, including those involving radioactive ion beams, should be performed over an energy range of $E' \pm \Delta E'/2$ instead of $E_0 \pm \Delta E_0/2$. Our attention is focused here on proton-capture reactions, but qualitatively similar results are obtained for (α, γ) reactions.

The present results have also far-reaching implications for the applicability of nuclear reaction models to the calculation of stellar reaction rates. A detailed account of this work will be presented elsewhere.

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[1] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (University of Chicago Press, Chicago, 1983).
[2] W. A. Fowler and F. Hoyle, Astrophys. J. Suppl. **9**, 201 (1964) app. C.
[3] P. M. Endt, Nucl. Phys. **A521**, 1 (1990); **A633**, 1 (1998).
[4] The penetration factor is not equal to the Gamow factor. The former quantity depends on the orbital angular momentum quantum number ℓ, while the latter quantity represents the transmission probability for s-wave particles at low energies. However, we are only interested here in the *energy-dependence* of the particle partial width. It can be approximated by the Gamow factor, as long as the bombarding energy is well below the Coulomb barrier [1]. Furthermore, the *energy-dependence* of the particle partial width is relatively insensitive to the orbital angular momentum, as can be seen from Fig. 2 of Iliadis *et al.*, Phys. Rev. C **53**, 475 (1996).