

Universal scaling of p_T distribution of particles in relativistic nuclear collisions

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With the experimental data from the STAR, PHENIX, and BRAHMS programs on the centrality and rapidity dependence of the p_T spectrum in Au+Au and d +Au collisions, we show that a scaling distribution exists that is independent of the colliding system, centrality, and rapidity. The parameter for the average transverse momentum ($\langle p_T \rangle$) increases from peripheral to central d +Au collisions. This increase accounts for the enhancement of particle production in those collisions. A nonextensive entropy is used to derive the scaling function.

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The distributions of produced particles play a very important role in the study of new states of matter produced in relativistic nuclear collisions. From the relationship between particle distributions at different centralities and rapidities, one can see whether there has been a suppression or an enhancement, thus the dynamical mechanism for that particle production can be revealed. In earlier works [1], a scaling distribution for produced pions in Au+Au collisions in the midrapidity region at energies currently available at the BNL Relativistic Heavy Ion Collider (RHIC) was found that is independent of centrality and colliding energy. Similar scaling behaviors have been found in Refs. [2,3]. The existence of such scaling behaviors of the spectrum for a species of particle may be an indication of a universal underlying particle production mechanism in the collisions. Now, one may ask two questions: (1) Can the scaling be extended to the noncentral rapidity region? (2) Can a scaling behavior similar to that shown in Ref. [1] be found for d +Au collisions at different centralities and rapidities? If the answers to these two questions are yes, then one should investigate the difference between the scaling functions for different colliding systems. In this paper, we look for the scaling behavior of particle distributions in Au+Au collisions in the noncentral rapidity region and in d +Au collisions at different centralities and rapidities at RHIC energies. The same universal scaling behavior is found for both Au+Au and d +Au collisions at different centralities and rapidities. A possible origin for the universal scaling distribution is given from the principle of maximum nonextensive entropy.

Generally, the invariant particle distribution $E \frac{d^3N}{dp^3}$ for a given species of particles depends on the collision centrality β , (pseudo)rapidity y (or η), and the transverse momentum p_T . When we focus on the particle production in a collision with given centrality in a small rapidity region around y , we should distinguish two different factors. One is the number density dN/dy in rapidity, which tells us how many particles are produced in the region. The other is the probability for a produced particle having transverse momentum p_T . It is this probability that dictates the shape of the transverse momentum distributions. In this paper, we are interested only in such a probability which can be defined as

$$\Phi(p_T, y, \beta) \equiv 2\pi E \frac{d^3N}{dp^3} \bigg/ \frac{dN}{dy}, \quad (1)$$

where $\frac{dN}{dy} = 2\pi \int E \frac{d^3N}{dp^3} p_T dp_T$ is the multiplicity of particles produced in a unit rapidity region. So $\Phi(p_T, y, \beta)$ satisfies the normalization condition

$$\int \Phi(p_T, y, \beta) p_T dp_T = 1. \quad (2)$$

In the definition of Φ , the centrality and rapidity dependence is written explicitly. Experimental data show a similar shape of the distributions for different centralities and rapidities. Because of the similarity in the shape, those distributions of pions produced in Au+Au collisions in the midrapidity region at different centralities have been put together in Ref. [1] to the same curve by shrinking p_T by a factor K and shifting the distribution by another factor A . Then a scaling variable is defined as $u = p_T / \langle p_T \rangle$, and the normalized distribution is obtained as a function $\Psi(u) = \langle p_T \rangle^2 \Phi(\langle p_T \rangle u, y, \beta)$. Here the average transverse momentum is defined as

$$\langle p_T \rangle = \int \Phi(p_T, y, \beta) p_T^2 dp_T. \quad (3)$$

The scaling function reads

$$\Psi(u) = 2.1 \times 10^4 (u^2 + 11.65)^{-4.8} \times [1 + 25 \exp(-1.864u)]. \quad (4)$$

The obtained $\Psi(u)$ is universal in the sense that it is independent of the centrality β at midrapidity in Au+Au collisions.

Now we extend the discussion to the particle production in the noncentral rapidity region in Au+Au collisions at 200 GeV. For this purpose, we work on the experimental data [4] for negatively charged particles h^- at large η with exactly the same procedures as in Ref. [1] and try to put all the data points to the curve for the scaling function $\Psi(u)$. It turns out that the scaling function given by Eq. (4) can also describe the charged particle production at $\eta = 2.6$ and 3.2 for different centralities, as can be seen in Fig. 1. The shrinking factor for p_T is given in Table I.

The second observation in this paper is that the same scaling p_T distribution can describe the p_T distributions of charged particles produced in d +Au collisions [5] and π^0 data [6] in the central rapidity region at different centralities, as shown in

TABLE I. Fitted parameter $\langle p_T \rangle$ in the scaling distribution for Au+Au and d +Au collisions at $\sqrt{s_{NN}} = 200$ GeV, for different centralities and rapidities shown in Figs. 1–3.

Fig. 1		Fig. 2		Fig. 3 (π^-)		
$\beta(\%)$	$\eta = 2.6$	$\eta = 3.2$	$\beta(\%)$	PHENIX	$\beta(\%)$	$y = 3$
0–10	0.4340	0.3448	0–20	0.563	0–20	0.3228
10–20	0.4359	0.3452	20–40	0.572	30–50	0.3151
20–40	0.4363	0.3458	40–60	0.601	60–80	0.3108
40–60	0.4366	0.3462	60–88	0.601		

Fig. 2. The fitted values of parameter $\langle p_T \rangle$ for the PHENIX data are also tabulated in Table I.

To investigate the rapidity dependence of the distribution in d +Au collisions, we take the data for π^+ and π^- at $y = 3$ for different centrality cuts [7] and try to put the data points to the obtained scaling curve with exactly the same procedure as mentioned above. The agreement is quite good, as shown in Fig. 3. The fitted values of the parameter $\langle p_T \rangle$ are also given in Table I. Then we can conclude that the same scaling function can describe pion and charged particle production in both Au+Au and d +Au collisions at different centralities and rapidities. In this sense, we can say that the obtained scaling distribution is universal.

To show the degree of agreement between the experimental data and the fitted results, we define B as the ratio of the data to the fitted results. For illustration, B , calculated from results in Fig. 3, as a function of u is shown in Fig. 4. Obviously, values of B are very close to 1, indicating that the scaling of the produced pion spectrum is true with high accuracy.

Once we have found a scaling law for the distributions for Au+Au and d +Au collisions for different centralities and rapidities, we need to dig out the physical implications behind the scaling behavior. From the scaling function $\Psi(u)$, one can see that the only parameter characterizing the normalized distribution is the average transverse momentum $\langle p_T \rangle$ which depends on the centrality, rapidity, and colliding energy and system. Once $\langle p_T \rangle$ is known, both the soft part with low

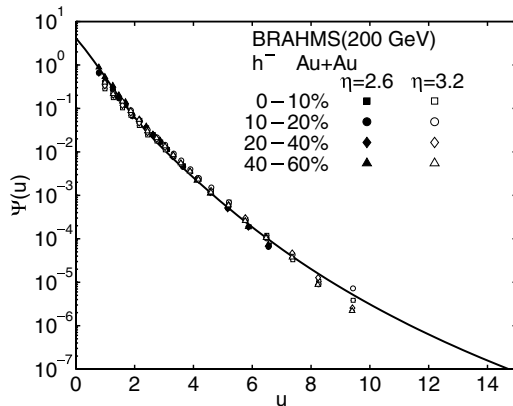


FIG. 1. Scaling distribution $\Psi(u)$ showing the coalescence of four rapidity bins in the BRAHMS data for h^- production at $\eta = 2.6$ and 3.2 in Au+Au collisions. The data points are taken from Fig. 1 in Ref. [4].

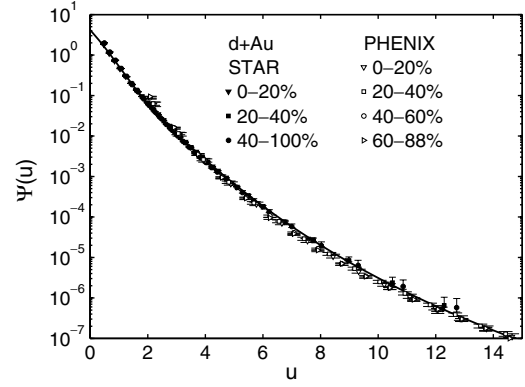


FIG. 2. Scaling distribution $\Psi(u)$ showing the coalescence spectra from the STAR data for charged particle and from PHENIX π^0 data production in d +Au collisions at midrapidity. The data points are taken from Refs. [5,6].

p_T and the hard part with high p_T are determined from $\Psi(u)$. Traditionally it was assumed that the bulk of particles is produced by soft processes which determine $\langle p_T \rangle$. If the soft and hard processes can be separated, as used in the two-component model [8], and each part has a different dynamical origin, it is extremely hard to imagine that they can be united into a single scaling function. Thus, we conclude that the hard process must contribute considerably to the average p_T and is a coherent part of particle production. In fact, we have known from the study of particle correlations in central Au+Au collisions at RHIC energies that the lost energy from hard partons must heat the medium and increase the mean transverse momentum.

On the other hand, we have known from experiments that particle production is suppressed, called jet quenching [9], in central Au+Au collisions because of the possible formation of dense hot medium; but it is enhanced in d +Au collisions because of the Cronin effect [10], possibly because of the p_T broadening from multiple soft collisions before a hard one [11] or from the recombination of soft partons with those from showers [12]. One can show that the enhanced production in d +Au collisions can be accounted for from our fitted results

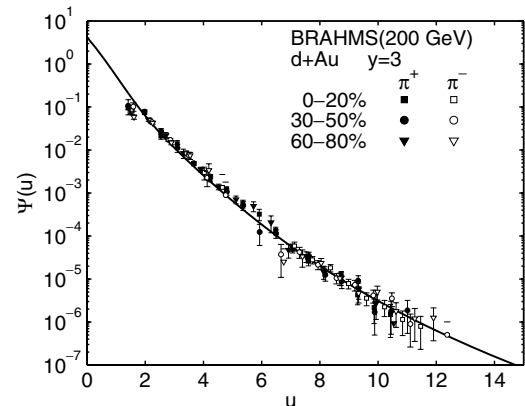


FIG. 3. Scaling distribution $\Psi(u)$ showing the coalescence of four centrality bins in the BRAHMS data for π^- production in d +Au collisions at forward direction. Data points are taken from Ref. [7].

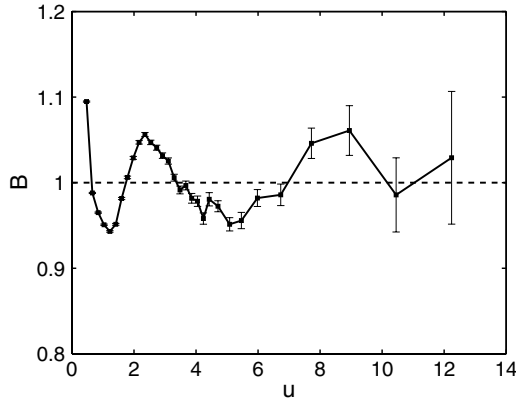


FIG. 4. Ratio B of the experimental data and fitted results, as shown in Fig. 3, for pion production as forward direction.

and that the mean transverse momentum $\langle p_T \rangle$ increases from peripheral to central collisions. We can define a new variable Q_{CP} to measure the suppression or enhancement from the scaling function $\Phi(p_T, y, \beta)$ as

$$Q_{CP} = \frac{\Phi(p_T, y, \beta)_C}{\Phi(p_T, y, \beta)_P} = \frac{(\Psi(p_T/\langle p_T \rangle)/\langle p_T \rangle^2)_C}{(\Psi(p_T/\langle p_T \rangle)/\langle p_T \rangle^2)_P}. \quad (5)$$

An advantage of this definition of Q_{CP} over the traditional R_{CP} is that it involves only the probability for a produced particle having certain transverse momentum and is independent of the total multiplicity. So the overall enhancement in the backward region ($y < 0$) in $d+Au$ collisions cannot be shown in Q_{CP} . In calculating the value of Q_{CP} , we do not need to use the number of binary hard collisions, which is model dependent. Q_{CP} is shown in Fig. 5(f) as a function of p_T for $d+Au$ collisions at midrapidity, and it increases slowly to about 1.33. Thus the production of high p_T pions is enhanced by about 30%. This is in agreement with experimental observations [13]. In this way, the enhancement or suppression behavior can be shown clearly in a model-independent way.

Finally, we investigate nucleon-nucleon collisions at $\sqrt{s} = 200$ GeV to see whether the particle spectrum can be described by the same scaling function. The experimental data on the p_T distribution in the central rapidity region in $p+p$ collisions

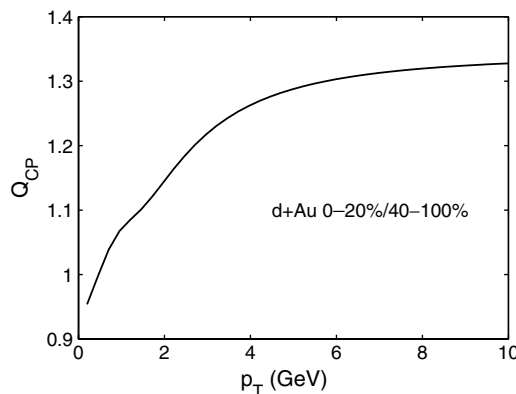


FIG. 5. Q_{CP} as a function of p_T for $d+Au$ collisions. Centrality cuts are 0–20% and 40–100%, respectively.

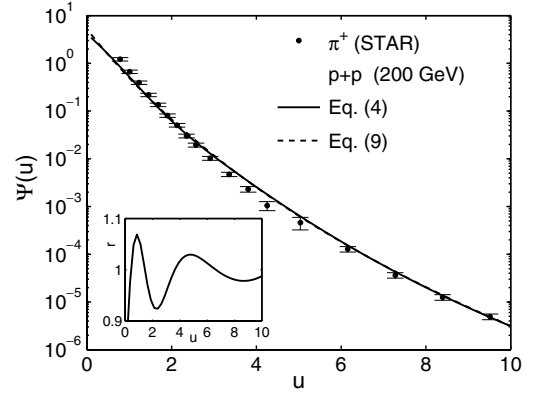


FIG. 6. Comparison between $\Psi(u)$ from fitting the data and from the maximum entropy method. The insert gives the ratio between them. Points are π^+ distribution in $p+p$ collisions from Ref. [15].

can be parametrized [14] as

$$\frac{dN}{p_T dp_T} = A(a + p_T)^{-n}, \quad (6)$$

where A , a , and n can be determined from fitting the data. It is astonishing that this parametrization for pion distribution in $p+p$ collisions, with the parameters suitably chosen, can also be put to the scaling curve for $\Psi(u)$ by shrinking horizontally and shifting vertically. As shown in Fig. 6, the π^+ spectrum in $p+p$ collisions can be fitted to the scaling function. Then, we come to our final conclusion that the same scaling function can be used—for different colliding systems at different centralities and rapidities—to give the probability for a produced particle having p_T once the parameter $\langle p_T \rangle$ for the average transverse momentum is known, thus the scaling function is universal.

Since $\Psi(u)$ is universal, the centrality and rapidity dependence in $\Phi(p_T, y, \beta)$ is totally encoded in $\langle p_T \rangle$. Straightforwardly, we can have

$$\begin{aligned} \langle p_T^n \rangle &\equiv \int p_T^n \Phi(p_T, y, \beta) p_T dp_T \\ &= \langle p_T \rangle^n \int_0^\infty u^n \Psi(u) u du. \end{aligned} \quad (7)$$

Therefore, the ratio $\langle p_T^n \rangle / \langle p_T \rangle^n$ is a constant independent of the colliding system, the centrality, and the rapidity. This consequence can be easily checked experimentally. For $n = 2, 3$, and 4 , the ratio equals to 1.65, 4.08, and 14.39.

We are now in a position to derive an expression for the particle distribution from some statistical consideration. One method is from the principle of maximum entropy. The scaling function is not an exponential form, therefore the well-known Shannon entropy cannot be used for our purpose. Instead, as suggested in Ref. [16], a nonextensive entropy may be used, considering the complex nonlinear interactions and the possible presence of fractal chaotic behavior in nuclear collisions, to derive an expression for the particle distribution. The entropy we will use here is the Tsallis entropy [17] which

reads for our case

$$S_q = \left(1 - \int \Phi^q(p_T) p_T dp_T \right) / (q - 1), \quad (8)$$

with parameter q as a measure of the nonextensiveness of the system. The maximum entropy under conditions demanded by Eqs. (2) and (3) gives, after the same shrinking and shifting as above,

$$\Psi(u) = A(a + u)^{-n}, \quad (9)$$

with $n = 1/(1 - q)$ and $a = (n - 3)/2$, $A = a^{n-2}(n - 1)(n - 2)$. So there is one parameter q to determine the shape of the distribution. The functional form of $\Psi(u)$ derived from the maximum entropy method is very different from that given by Eq. (4) but is the same as that used in parametrizing the $p+p$ data. Then one can compare the distributions $\Psi(u)$ from our fitting procedure and from the maximum entropy. As shown in Fig. 6, they agree with each other extremely well in a very wide range of u when $n = 13.2$. There, in log scale, one can hardly see any difference between them. A small difference can be seen if we show a ratio between them in a linear scale. As can be seen from the insert in Fig. 6, the largest difference

is less than 10%, though the distribution covers six orders of magnitude in the shown range of u . This agreement indicates that the produced particle system, including both the soft and hard particles, may be a nonextensive fractal system, and the fractal property is independent of the colliding system. The parameter characterizing the fractal property of the system is then $q = 1 - 1/n = 0.924$, not too far from 1.

In summary, a scaling distribution for pions is found for Au+Au, d+Au, and $p+p$ collisions at RHIC energy. The scaling distribution is universal, since it does not depend on the colliding system, centrality, or rapidity. What characterizes the dynamical difference in the particle production is the value of the average transverse momentum $\langle p_T \rangle$. A simple prediction which can be checked experimentally is that $\langle p_T^n \rangle / \langle p_T \rangle^n$ is independent of the colliding system, the colliding centrality, and the rapidity. A possible statistical origin for the scaling distribution of pions is discussed from a nonextensive entropy. Different particles may have different scaling properties.

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- [1] R. C. Hwa and C. B. Yang, Phys. Rev. Lett. **90**, 212301 (2003); Phys. Rev. C **67**, 034902 (2003); **67**, 064902 (2003).
 [2] I. Zborovsky and M. Tokarev, Preprint JINR E2-2006-34, Dubna, 2006, hep-ph/0603221, and references therein.
 [3] J. Schaffner-Bielich, D. Kharzeev, L. McLerran, and R. Venugopalan, Nucl. Phys. **A705**, 494 (2002).
 [4] C. Ristea (BRAHMS Collaboration), AIP Conf. Proc. **828**, 353 (2006).
 [5] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **70**, 064907 (2004).
 [6] S. S. Adler *et al.* (PHENIX Collaboration), nucl-th/0610036.
 [7] H. Yang (BRAHMS Collaboration), Rom. Rep. Phys. **58**, 5 (2006).
 [8] X. N. Wang and M. Gyulassy, Phys. Rev. Lett. **86**, 3496 (2001).
 [9] X. N. Wang, Phys. Rev. C **58**, 2321 (1998).
 [10] J. W. Cronin *et al.*, Phys. Rev. D **11**, 3105 (1975); D. Antreasyan, J. W. Cronin, H. J. Frisch, M. J. Shochet, L. Kluberg, P. A. Piroué, and R. L. Sumner, *ibid.* **19**, 764 (1979).
 [11] For a review, see A. Accardi, hep-ph/0212148 and references therein.
 [12] R. C. Hwa and C. B. Yang, Phys. Rev. Lett. **93**, 082302 (2004).
 [13] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. C **74**, 024904 (2006).
 [14] R. Hagedorn, Riv. Nuovo Cimento **6**, 1 (1983).
 [15] J. Adams *et al.* (STAR Collaboration), Phys. Lett. **B637**, 161 (2006).
 [16] A. S. Parvan and T. S. Biro, Phys. Lett. **A340**, 375 (2005).
 [17] C. Tsallis, J. Stat. Phys. **52**, 479 (1988); Braz. J. Phys. **29**, 1 (1999).