Evidence for strong refraction of ^{3}He in an α-particle condensate

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We have analyzed ^{3}He scattering from ^{12}C at 34.7 and 72 MeV in a coupled channel method with a double folding potential derived from the precise wave functions for the ground 0^+ state and 0_2^+ (7.65 MeV) Hoyle state, which has been suggested to be an α particle condensate. It is found that strong refraction of ^{3}He in the Hoyle state can be clearly seen in the experimental angular distribution at a *low* incident energy region as an Airy minimum of the *prerainbow oscillations*.

Bose-Einstein condensation (BEC) has been well established in a dilute gas [1]. The remarkable properties of superconductivity and superfluidity in both ^{3}He and ^{4}He are related to BEC. In hadronic systems of strong interaction BEC has been paid much attention in pion condensation and kaon condensation. To the best of our knowledge, the refractive effect of BEC states in the scattering has been rarely investigated. In this paper we show that the strong refractive effect is observed in the Hoyle state of ^{12}C, which has been suggested to be an α particle condensate in nuclei.

The pioneering work of Hokkaido group by Uegaki *et al.* [2] and Kamimura and Fukushima [3] showed in the microscopic cluster model that the 0_2^+ state of ^{12}C, the Hoyle state, has a loosely coupled three α cluster structure with an $\alpha \otimes {}^8$Be configuration. Recently it has been shown that the wave functions of Uegaki *et al.* [2] and Kamimura and Fukushima [3] are almost completely equivalent to the wave function that the three α particles are sitting in the lowest $0s$ state like a dilute gas and speculated that the Hoyle state is a Bose-Einstein condensate of three α particles [4–6].

In a BEC of atomic gas the system is magnetically trapped. On the other hand, as the nucleus is a self-binding finite system due to strong nuclear interaction, there is no external field to trap the system. Although the BEC of atomic gas appears at near *zero temperature*, the gaseous state of weakly interacting α particles like the Hoyle state appears at a *highly excited energy* near the threshold. If such a dilute state due to BEC exists, typical macroscopic physical quantities peculiar to it would exist. The huge radius of its state may be one of them. To measure such a huge radius of the excited state is very challenging. However, no such experiment has been reported. Recently Kokalova *et al.* [7] proposed a new experimental way of testing the BEC of α particles in nuclei by directly observing the enhancement of α particle emission and the multiplicity partition of the possible emitted α particles.

In this paper we show that the strong refractive effect of incident ^{3}He in the Hoyle state of ^{12}C can be seen in the *prerainbow oscillations* in the *low* incident energy region where there is a pocket in the effective potential (nuclear plus Coulomb plus centrifugal). Usually the refractive effect has been discussed as nuclear rainbow scattering in the *high* incident energy region where the analogy between the meteorological rainbow and the nuclear rainbow can be

discussed based on the classical deflection function [8,9]. We discussed the relation between the nuclear rainbow and BEC in α particle scattering from ^{12}C(0_2^+) at *high* incident energies in a previous paper [10] because experimental data were only available above $E_\alpha = 139$ MeV. However, it is expected that the refractive effect becomes much larger and can be seen clearly at *low* incident energies. In fact, in optics [11], refractive index n is related to the optical potential V as follows:

$$n(r) = \sqrt{1 - \frac{V(r)}{E_{\text{c.m.}}}}. \tag{1}$$

Also for α particle scattering from ^{12}C there is a well-known long-standing difficulty that a global potential for the $\alpha + {}^{12}$C system has not been seen in the *low* incident energy region [12].

The prerainbow structure has been mostly studied in elastic scattering [13,14]. There had been no systematic theoretical and experimental studies of the prerainbow oscillations in *inelastic* scattering. However, recently it has been shown that inelastic prerainbow Airy structure can also be understood in a way similar to elastic scattering [15]. This suggests that inelastic prerainbow oscillation may also be useful for the study of the nuclear properties of the excited states of the target nucleus because the internal region of the interaction potential can be well determined even for inelastic channels.

Fortunately the prerainbow Airy structure in the scattering from the Hoyle state was measured in the low incident energy region $E_L = 34.7$ MeV many years ago in ^{3}He scattering [16] as well as the high energy scattering at 72 MeV [17] where the falloff of the cross sections is seen in the experimental data. However, unfortunately these inelastic scattering data from the Hoyle state have been forgotten and have never been studied from the theoretical point of view. Also elastic ^{3}He scattering from ^{12}C has not been studied systematically compared with $\alpha + {}^{12}$C scattering. Most of the analyses of elastic ^{3}He scattering from nuclei have been done with a conventional Woods-Saxon potential [18]. Khallaf *et al.* [19,20] have analyzed ^{3}He + ^{12}C elastic scattering with a folding potential.

We study elastic and inelastic ^{3}He + ^{12}C scattering in the microscopic coupled channel method by taking into account simultaneously the 0_1^+ (0.0 MeV), 2^+ (4.44 MeV),
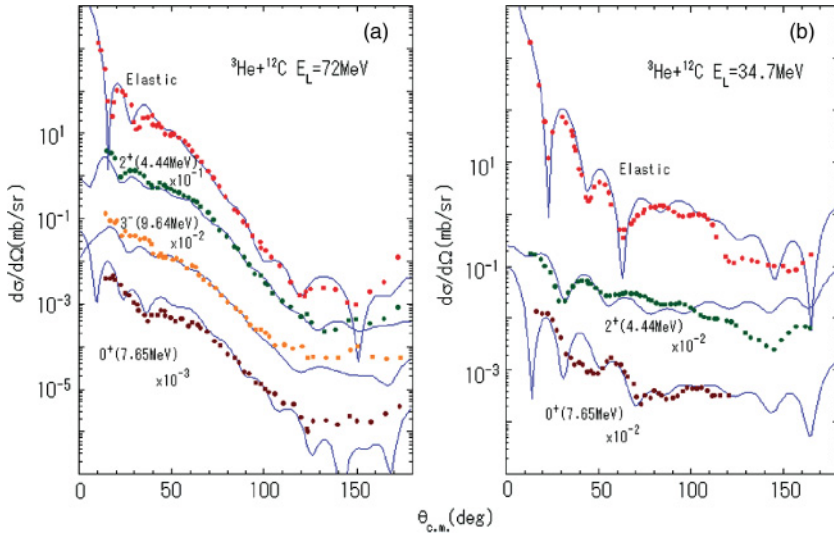
FIG. 1. (Color online) The calculated elastic and inelastic angular distributions (solid lines) for the 2^+ (4.44 MeV), 0_2^+ (7.65 MeV), and 3^- (9.63 MeV) states of ^{12}C in ^{3}He+^{12}C scattering at $E_L = 72$ and 34.7 MeV are compared with the experimental data (points) [16,17].

0_2^+ (7.65 MeV), and 3^- (9.63 MeV) states of ^{12}C. The diagonal and coupling potentials for the ^{3}He+^{12}C system are calculated by the double folding model:

$$V_{ij}(\mathbf{R}) = \int \rho_{00}^{(^3\text{He})}(\mathbf{r}_1)\rho_{ij}^{(^{12}\text{C})}(\mathbf{r}_2)$$
$$\times v_{NN}(E, \rho, \mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2, \qquad (2)$$

where $\rho_{00}^{(^3\text{He})}(\mathbf{r})$ is the ground state density of ^{3}He taken from Ref. [21], while v_{NN} denotes the density-dependent M3Y effective interaction (DDM3Y) [22]. $\rho_{ij}^{(^{12}\text{C})}(\mathbf{r})$ represents the diagonal $(i = j)$ or transition $(i \neq j)$ nucleon density of ^{12}C calculated in the resonating group method by Kamimura *et al.* [3]. The folding potential is very sensitive to the wave functions used, which serves as a good test of the validity of the wave function [23]. This wave function for the Hoyle state is almost completely equivalent to the Bose-Einstein condensate wave function [5]. In the analysis we introduce the normalization factor N_R for the real part of the potential and phenomenological imaginary potentials with a Woods-Saxon form factor (volume absorption) and a derivative of the Woods-Saxon form factor (surface absorption) for each channel.

In Fig. 1(a) calculated angular distributions are shown in comparison with the experimental data of elastic and inelastic ^{3}He scattering from ^{12}C at $E_L = 72$ MeV. The characteristic features of the falloff of the cross sections beyond the rainbow angle in the experimental angular distributions for the shell-like ground, 2^+, 3^- states, and the 0_2^+ state with the well-developed α-cluster structure are simultaneously well reproduced. It is noted that for the ground and the 2^+ states the agreement of the calculations with the data is fairy good up to large angles. In Fig. 1(b) the same comparison is shown for the experimental data at 34.7 MeV [16]. In this energy region the falloff of the cross sections in the angular distributions characteristic to rainbow scattering is no longer seen in the experimental data. The broad bumps in the intermediate angular region of the angular distributions in elastic and inelastic scattering are reproduced by the calculations.

The discrepancy between the experimental data and the calculation seen only for the 0_2^+ state at forward angles where the nearside contribution increases [Fig. 2(b)] may be mostly due to the truncation of the explicit coupling to the higher excited states. In fact, for example, most of the imaginary potentials for the 0_2^+ come from the coupling to the 2_2^+ state, which has a well-developed α-cluster structure with almost the same configuration as the 0_2^+ state. We confirmed that the present calculation also reproduces the experimental angular distribution at $E_L = 119$ MeV of Hyakutake *et al.* [24] very well.

To see the refractive effect in Fig. 2 the calculated angular distributions are decomposed into farside and nearside components following Fuller's prescription [25]. At $E_L = 72$ MeV the first Airy minimum $A1$ appears at $30°$ for elastic scattering and $35°$ for the 0_2^+ state. For elastic scattering a clear minimum is not seen in the angular distribution of the farside cross sections and the Airy minimum in the experimental data at $30°$ is obscured by the interference between the farside and nearside amplitudes. On the other hand, the $A1$ minimum for the 0_2^+ state is clearly seen in the farside cross sections because the minimum is shifted to a larger angle where the nearside contribution is much smaller. The situation is more clearly seen in the Airy structure at a *low* incident energy region where there is a pocket in the effective potential and no typical rainbow falloff of the dark side appears. At $E_L = 34.7$ MeV in Fig. 2(b) the Airy minimum $A1$ appears at $60°$ for elastic scattering and $75°$ for the 0_2^+ state. The latter is further shifted to a larger angle and the Airy minimum is not at all obscured by the nearside contributions, which decrease rapidly as the scattering angle increases. For the 0_2^+ state, in the wider range of angles the nearside contributions are much smaller than the farside contributions compared with the elastic scattering case. Thus the difference of the refraction between the ground state and the 0_2^+ state is much more clearly seen at 34.7 MeV than at 72 MeV. The Airy minimum for the 0_2^+ state is three times more shifted to a larger angle than the case of $E_L = 72$ MeV. The absorption is incomplete for the 0_2^+ state and a more precise prerainbow Airy oscillation is seen than at elastic scattering. Thus we can say that the refractive effect of the 0_2^+ state is
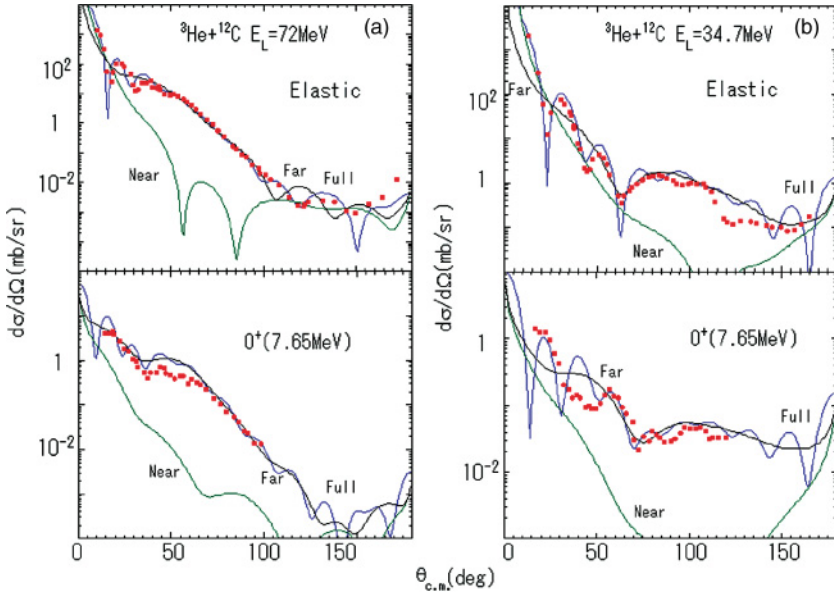
FIG. 2. (Color online) The angular distributions for the ground and 0_2^+ (7.65 MeV) states of ^{12}C in ^{3}He+^{12}C scattering at $E_L = 72$ and 34.7 MeV are decomposed into farside (dashed lines) and nearside (dotted lines) contributions.

more clearly seen in the prerainbow structure at a *low* incident energy region than at a high incident energy region where a typical nuclear rainbow appears.

In Table I the properties of the real folding potential and imaginary potential parameters used are given. The obtained volume integral per nucleon pair of the potential $J_V = 365$ MeV fm^3 for elastic scattering at $E_L = 119$ MeV is consistent with 352 MeV fm^3 of the unique potential of Hyakutake *et al.* [24]. The obtained $J_V = 408$ MeV fm^3 at $E_L = 72$ MeV for elastic scattering is closer to a deeper potential set $A4$ ($J_V = 437$ MeV fm^3) than a shallower set set A2 ($J_V = 275$ MeV fm^3) of Dem'yanova *et al.* [17]. The volume integral for elastic scattering increases as energy decreases and becomes 445 MeV fm^3 at $E_L = 34.7$ MeV. The energy dependence of the present double folding potential for the ^{3}He+^{12}C system is consistent with a systematic study in Ref. [18]. The volume integral for ^{3}He scattering is deeper than that for α-particle scattering although its energy evolution is similar. We compare the refractive effect for the ground state and the 0_2^+ state at $E_L = 34.7$ MeV. The obtained $J_V = 583$ MeV fm^3 for the 0_2^+ state is much larger than $J_V =$

445 MeV fm^3 for the ground state. The calculated root mean square radius of the real potential is 4.385 fm for the 0_2^+ state and 3.574 fm for the ground state. This shows that the refractive effect is extremely stronger for the 0_2^+ state than the ground state, and that the potential, that is the lens, for the 0_2^+ state is much more extended than that for the ground state in agreement with the dilute distribution of the density of the 0_2^+ state.

From the experimental data itself of the Airy minimum of the 0_2^+ state we can see that the density distribution of the 0_2^+ state is far more extended and dilute than the ground state. For elastic scattering the rainbow angle θ_N can be given analytically if the Woods-Saxon form factor is assumed for the real part of the optical potential [26]:

$$\theta_N \approx |V_C - 0.56\, V(\frac{R}{a})^{\frac{1}{2}}|/E_{\text{c.m.}}, \qquad (3)$$

where $V_C = Z_1 Z_2 e^2/R$ and V, R, and a are the strength, radius, and diffuseness parameters of the potential, respectively. This means that the larger θ_N is, the deeper the potential strength is (or the larger the radius is). This also means that the

TABLE I. The volume integral per nucleon pair J_V, root mean square radius $\langle R^2 \rangle^{1/2}$, of the folding potential, and the parameters of the imaginary potentials in the conventional notation. The normalization factor N_R is fixed to 1.28.

E_L (MeV)	J^π	J_V (MeV fm^3)	$\langle R^2 \rangle^{1/2}$ (fm)	W_V (MeV)	R_V (fm)	a_V (fm)	W_S (MeV)	R_S (fm)	a_S (fm)
72	0_1^+	408	3.576	5.0	5.6	0.60	4.0	2.6	0.20
	2^+	403	3.562	8.0	5.0	0.10	7.0	2.7	0.30
	3^-	461	3.827	9.0	4.9	0.10	9.0	2.4	0.10
	0_2^+	529	4.385	18.0	4.5	0.20	12.0	2.6	0.50
34.7	0_1^+	445	3.574	6.0	4.8	0.50	6.0	2.6	0.50
	2^+	440	3.559	4.0	4.3	0.30	6.0	2.4	0.40
	3^-	506	3.829	9.0	4.9	0.40	9.0	2.6	0.50
	0_2^+	586	4.385	19.0	5.4	0.60	14.0	2.8	0.60

more the first Airy minimum is shifted to a larger angle, the deeper the potential becomes (or the larger the radius of the potential becomes). Based on the similarity of the systematic energy evolution in a wide range of incident energies of the Airy structure between elastic scattering and inelastic scattering [15], the above discussion between the position of the first Airy minimum of the prerainbow oscillations and the depth (and the radius) of the potential will hold qualitatively in the present case. Because the Airy minimum of the prerainbow oscillations for the 0_2^+ state appears at a larger angle as seen in Fig. 2(b), the radius parameter R of the corresponding potential is far larger than that for the ground state considering that the depth V of the potential for the 0_2^+ is smaller than that for the ground state. Therefore the shift of the angular position of the Airy minimum of the prerainbow oscillations for the 0_2^+ state from that for the ground may be used to measure the size of the lens.

The present approach to know the size of the lens of the excited state qualitatively, namely, how dilute the excited state is, can be applied to the $n\alpha$-particle states of $4N$-nuclei near the threshold such as the four α-particle state in ^{16}O and ten α-particle state in ^{40}Ca by using α particle, ^{3}He and ^{16}O as a projectile, for which absorption is incomplete. For non-$4N$ nuclei the $\frac{3}{2}^-$ state at 8.56 MeV in ^{11}B analog to the 0_2^+ state of ^{12}C is considered to have a dilute density distribution [27].

Also it has been suggested that analog states appear in neutron rich nuclei, for example, the $\frac{1}{2}^-$ (8.86 MeV) state in ^{13}C [28], the 0^+(9.746 MeV) state in ^{14}C [29] and the 0^+($\sim$29 MeV) state in ^{16}C [30], which are considered to have one, two, and four additional neutrons to the 0_2^+ state of ^{12}C. It is interesting to observe systematically how the prerainbow Airy minimum in inelastic scattering at low incident energy region is shifted as additional neutrons are added (removed) to (from) ^{12}C.

To summarize, it is found that a strong refraction of ^{3}He in the 0_2^+ (7.65 MeV) Hoyle state of ^{12}C, which has been suggested to be an α particle condensate, can be clearly seen in the experimental angular distribution at *low* incident energy region where there is a pocket in the effective potential as an Airy minimum of the *prerainbow oscillations*. Because of this strong refraction, the Airy minimum is shifted to a considerably larger angle compared with that of the normal ground state, it is clearly observed, having not been obscured by the nearside contributions. The present finding may also hold for an α particle condensate in heavier nuclei like ^{40}Ca and excited states with dilute density distribution.

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