PHYSICAL REVIEW C **75**, 044301 (2007)

α **transitions to coexisting 0⁺ states in Pb and Po isotopes**

Chang Xu^1 and Zhongzhou Ren^{1,2,3}

¹*Department of Physics, Nanjing University, Nanjing 210008, People's Republic of China*

²*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, People's Republic of China*

³*CPNPC, Nanjing University, Nanjing 210008, People's Republic of China*

(Received 21 January 2007; published 6 April 2007)

The α -transitions ($\Delta \ell = 0$) to ground and first excited 0⁺ states in neutron deficient Pb and Po isotopes are systematically analyzed by the density-dependent cluster model. The magnitude of nuclear deformation of the coexisting 0_1^+ and 0_2^+ states is extracted directly from the experimental α -decay energies and half-lives. The phenomenon of shape coexistence around the $Z = 82$ shell closure is clearly demonstrated in our present analysis. The obtained deformation values from $\text{Rn} \rightarrow \text{Po} \rightarrow \text{Pb}$ decay chains are generally consistent with both the available experimental and theoretical studies.

DOI: [10.1103/PhysRevC.75.044301](http://dx.doi.org/10.1103/PhysRevC.75.044301) PACS number(s): 23*.*60*.*+e, 21*.*10*.*Tg, 21*.*60*.*Gx, 27*.*80*.*+w

The phenomenon of shape coexistence in the vicinity of $Z = 82$ proton shell closure has received particular attention in recent years [\[1,2\]](#page-4-0). Extensive experimental studies have been carried out to identify low-lying excited 0^+ states in even-mass nuclei around the shell closure by means of *α*-decay, *β*-decay, in-beam measurements, and so on [\[3–14\]](#page-4-0). The occurrence of these low-lying intruder states is mainly due to the multiparticle multihole excitations across the proton shell gap in a shell model picture. Considerable theoretical researches have been devoted to pursuing a quantitative description of shape-coexisting structure in Pb and Po region, such as the Nilsson-Strutinsky approach, the Hartree-Fock-Bogoliubov calculation, the interacting boson model, etc. [\[15–21\]](#page-4-0). The spherical, oblate, and prolate minima of the potential energy surfaces calculated by various theoretical approaches are generally consistent with the experimental facts, while the calculated excitation energies of the low-lying 0^+ states reasonably agree with the experimental data.

The *α*-decay fine structure of Pb and Po nuclei has also been studied extensively from the theoretical side [\[22–25\]](#page-4-0). This is because α -decay is a very effective method to provide internal structure information on nuclei close to the proton drip line. For many even-mass nuclei around the $Z = 82$ region, the low-lying excited 0^+ states in their energy spectra, corresponding to different configurations, respectively, were populated by α -decay fine structure in experiments [\[9\]](#page-4-0). Halflives ratio (HR), which is defined as the ratio between measured and calculated α -decay half-lives, is a sensitive probe to see whether an α -transition occurring between two states with the same spin and parity is hindered or not [\[26–31\]](#page-4-0). The significant deviation of this quantity from the value 1.0 often implies a change of configuration between two states. For this reason, the α -decay is well suited to analyzing shape-coexisting phenomena in the ground and excited states of a given nucleus, especially for the light Pb and Po isotopes. Previous theoretical studies basically concerned with reproducing the experimental *α*-decay hindrance factors by using deformation parameters from the potential energy surface calculations [\[25\]](#page-4-0).

The main purpose of this paper is to study shape-coexisting structure in even-mass neutron deficient Pb and Po isotopes by combining the ideas of *α*-decay analysis and potential energy

surface calculation together. The α -transitions ($\Delta \ell = 0$) to ground and excited 0⁺ states in ¹⁸⁶*,*188*,*190*,*192*,*194Pb and 190*,*192*,*194*,*196*,*198Po are systematically studied by the densitydependent cluster model. The variation of *α*-decay width, i.e., the half-lives ratio, as a function of nuclear deformation is analyzed for each $\Delta \ell = 0$ *α*-transition. The magnitude of deformation of the ground and excited 0^+ states in these nuclei is extracted directly from their *α*-decay energies and half-lives. Our present analysis on shape-coexisting nuclei is totally based on the measured α -decay fine structure data, which provides a novel way to deduce deformation parameters because the collective bands built on the excited 0^+ states of these nuclei are rather difficult to identify in experiments.

Firstly we briefly introduce the framework of the densitydependent cluster model (DDCM). In DDCM, the *α*-cluster is considered to penetrate the deformed Coulomb barrier after its formation in the parent nucleus. The *α*-decay width is mainly determined by the product of *α*-cluster preformation factor and penetration probability. The later one is very sensitive to the details of the α -core interaction, which is the sum of the nuclear potential, the Coulomb potential, and the centrifugal potential [\[32\]](#page-4-0)

$$
V_{\text{Total}}(\mathbf{R}, \theta) = V_{\text{N}}(\mathbf{R}, \theta) + V_{\text{C}}(\mathbf{R}, \theta) + \frac{\hbar^2}{2\mu} \frac{\left(\ell + \frac{1}{2}\right)^2}{\mathbf{R}^2}, \quad (1)
$$

where R is the distance between the mass centers of the *α*-particle and the core. *θ* is the orientation angle of the α -particle with respect to the symmetry axis of the daughter nucleus. ℓ is the angular momentum carried by the α -particle and μ is the reduced mass of the α -core system. The nuclear potential is obtained from the double-folding integral of the renormalized M3Y nucleon-nucleon potential with the matter density distributions of the *α*-particle and the daughter nucleus [\[33\]](#page-4-0). The Coulomb potential is also obtained from the well established double-folding model by including the effect of finite size of the *α*-cluster. We assume a spherical *α*-particle interacts with an axially-symmetric deformed daughter nucleus. The mass density distribution of the spherical *α*-particle is taken as the widely-used Gaussian form [\[33\]](#page-4-0). The mass density distribution of the daughter nucleus is a deformed Fermi distribution with standard parameters [\[32\]](#page-4-0)

$$
\rho_2(r_2, \theta) = \rho_0 \Bigg/ \Bigg\{ 1 + \exp \Bigg[\frac{r_2 - R_0 [1 + \beta Y_{20}(\theta)]}{a} \Bigg] \Bigg\}, \tag{2}
$$

where the parameters $R_0 = 1.07 A_d^{1/3}$ fm and $a = 0.54$ fm. β is the deformation parameter of the daughter nucleus. In DDCM, the double-folding potential can be evaluated by a sum of different multipole components [\[32\]](#page-4-0)

$$
V_{\text{NorC}}(\mathbf{R}, \theta) = \sum_{l=0,2,4...} V_{\text{NorC}}^{l}(\mathbf{R}, \theta), \tag{3}
$$

and the multipole component of the double-folding potential is written as

$$
V_{\text{NorC}}^{l}(\mathbf{R}, \theta) = \frac{2}{\pi} [(2l+1)/4\pi]^{1/2}
$$

$$
\times \int_{0}^{\infty} dk k^{2} j_{l}(k\mathbf{R}) \tilde{\rho}_{1}(k) \tilde{\rho}_{2}^{(l)}(k) \tilde{v}(k) P_{l}(\cos \theta),
$$
 (4)

where $\tilde{\rho}_1(k)$ is the Fourier transformation of the density distribution of the *α*-particle and $\tilde{\rho}_2^{(l)}(k)$ is the intrinsic form factor corresponding to the daughter nucleus. $\tilde{v}(k)$ is the Fourier transformation of the effective M3Y interaction or the proton-proton Coulomb interaction. $P_l(\cos \theta)$ is the Legendre function of degree *l*. The M3Y nucleon-nucleon interaction is given by two direct terms with different ranges, and by an exchange term with a delta interaction [\[33\]](#page-4-0)

$$
v(\mathbf{s}) = 7999 \frac{\exp(-4\mathbf{s})}{4\mathbf{s}} - 2134 \frac{\exp(-2.5\mathbf{s})}{2.5\mathbf{s}} + J_{00}\delta(\mathbf{s}),
$$

\n
$$
J_{00} = -276(1 - 0.005 \, \text{E}_{\alpha}/\text{A}_{\alpha}),
$$
\n(5)

where the quantity |**s**| is the distance between a nucleon in the core and a nucleon in the α -particle ($\mathbf{s} = \mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1$). E_α is the *α*-decay energy and A_α is the mass number of the *α*-particle. The depth of the nuclear potential is determined separately for each decay in order to generate a quasibound state by employing the Bohr-Sommerfeld condition. Once the α -core potential has been determined, the polar-angle dependent penetration probability of *α*-decay in the deformed version of DDCM can be given by [\[32\]](#page-4-0)

$$
\mathcal{P}_{\theta} = \exp\bigg[-2\int_{R_2(\theta)}^{R_3(\theta)} \sqrt{\frac{2\mu}{\hbar^2} |Q_{\alpha} - V_{\text{Total}}(R, \theta)|} \, dR\bigg], \tag{6}
$$

where $R_2(\theta)$ and $R_3(\theta)$ are the second and third classical turning points of a certain orientation angle *θ*. The total penetration probability P_{Total} is obtained by averaging P_{θ} in all directions [\[32\]](#page-4-0)

$$
\mathcal{P}_{\text{Total}} = \frac{1}{2} \int_0^{\pi} \mathcal{P}_{\theta} \sin(\theta) \, \mathrm{d}\theta. \tag{7}
$$

Finally, the α -decay width in the deformed version of DDCM is given by [\[32\]](#page-4-0)

$$
\Gamma = P_{\alpha} \mathcal{F} \frac{\hbar^2}{4\mu} \frac{1}{2} \int_0^{\pi} \mathcal{P}_{\theta} \sin(\theta) d\theta, \qquad (8)
$$

where F is the normalization factor and P_α is the α -cluster preformation factor in the parent nucleus. The microscopic description of the preformation amplitude is still an open problem in physics. Experiments have shown that the preformation factor varies smoothly in the open-shell region and has a value smaller than 1.0. We have recently performed a global calculation on favored *α*-decay half-lives where a constant preformation factor $P_\alpha = 0.38$ is found for all eveneven α -emitters [\[32\]](#page-4-0). This value is consistent with both the experimental facts and the microscopic calculations [\[34,35\]](#page-4-0). In present calculations, we still adopt the constant value $(P_\alpha = 0.38)$ for α -transitions to ground 0^+_1 states in the neutron deficient Pb and Po isotopes. As compared with the ground state transitions, the situation of α -transitions to the excited $0₂⁺$ states in the daughter nucleus is much more complex. According to the *n*P-*n*H picture, such transitions often involve the removal of a pair of protons below the proton shell closure to form an *α*-cluster in the parent nucleus. So the *α*-cluster preformation factor in these cases should be smaller than that of the ground-state transitions where only the valence nucleons are involved. This is also evidenced by our previous analysis of *α*-decay branching ratios where the excitation probabilities of the daughter nucleus after disintegration were found to obey the Boltzmann distribution approximately [\[36\]](#page-4-0)

$$
w_{\ell}(\mathbf{E}_{\ell}^*) = \exp[-c\mathbf{E}_{\ell}^*],\tag{9}
$$

where E_{ℓ}^{*} is the excitation energy of ℓ state in the daughter nucleus and *c* is a free parameter. The excitation probability is directly related to the *α*-cluster preformation factor in the parent nucleus. We stress that our assumption of the Boltzmann distribution is reasonable in physics and it leads to good agreement between experimental and calculated α -decay branching ratios [\[36\]](#page-4-0). In present study, we describe the *α*-cluster preformation probabilities for different *α*-transitions by the product of the constant formation factor and the Boltzmann distribution function $[P_\alpha \times w_\ell(E^*_{\ell})]$. It is easy to find that the preformation factor of the *α*-particle is fixed to a constant value of 0.38 for the ground-state transitions ($0^+ \rightarrow$ $0₁⁺$). For *α*-transitions to the excited $0⁺$ states, a smaller value of the formation factor is automatically determined from the experimental excitation energy E_{ℓ}^* . The value $c = 1.5$ is taken directly from previous analysis [\[36\]](#page-4-0) and we do not modify any parameter in our calculations.

We perform a systematic study of even-mass neutron deficient $^{194-202}$ Rn → $^{190-198}$ Po → $^{186-194}$ Pb decay chains by the deformed version of DDCM. Let us take the *α*-emitter ¹⁹⁴Rn $\left[\alpha \otimes^{190}$ Po] as an example to illustrate the details of our calculations. The α -decay of ¹⁹⁴Rn with a decay energy of 7.862 MeV and a half-life of 0.78 ms was reported by Andreyev *et. al* very recently [\[13\]](#page-4-0). It is the lightest even-mass Rn isotope identified in experiment so far [\[13\]](#page-4-0). In Fig. [1,](#page-2-0) we plot the variation of the theoretical *α*-decay half-life of 194 Rn as a function of the deformation parameter β (corresponding to the daughter nucleus 190 Po). The solid line represents the experimental *α*-decay half-life $[T_{1/2}(Exp)]$ and the dotted line stands for the calculated one $[T_{1/2}(Cal)]$. It is seen from Fig. [1](#page-2-0) that the calculated half-life of 194 Rn is much higher than the experimental one in the spherical case $\beta = 0$. By taking

FIG. 1. The variation of the theoretical α -decay half-life of ¹⁹⁴Rn as a function of parameter *β*.

the deformation freedom into account, the theoretical *α*-decay half-life decreases smoothly with increasing nuclear deformation. The deformation parameter for $190P$ o can be extracted at the intersection point of two lines where the calculated half-life by DDCM is exactly equal to the experimental one, i.e., the half-lives ratio HR = $T_{1/2}$ (Exp)/ $T_{1/2}$ (Cal) = 1.0. Through a careful analysis of the experimental *α*-decay half-life, we obtain a deformation value of $\beta = 0.364$ for the ground 0⁺ state of 190Po. This is in accord with the potential energy surface calculations where $190P$ o was also predicted to be deformed in its ground state [\[18\]](#page-4-0).

In Fig. 2, the α -transitions from the ground state of ¹⁹⁰Po to the ground 0^+_1 and excited 0^+_2 states of ¹⁸⁶Pb are plotted in the same way. The experimental partial half-lives of α -transitions to these two states are 2.5 ms and 74 ms, respectively [\[8\]](#page-4-0). Obviously, the $0^+ \rightarrow 0^+$ α -transition is hindered as compared with the ground state transition. Thus the residual daughter nucleus after disintegration has the highest probability to stay

in its ground state. It is shown in Fig. 2 that the ground 0^+_1 state in 186Pb has a nearly-spherical shape and we deduce a deformation value of $\beta = 0.097$ from the experimental *α*-decay half-life $[T_{1/2}(Exp) = 2.5$ ms]. This spherical ground state in ¹⁸⁶Pb is caused by the $Z = 82$ proton shell effect. In contrast, the excited low-lying 0_2^+ state in ¹⁸⁶Pb is mainly the deformed configuration with a large extracted *β* value of 0.328. The occurrence of the deformed excited 0^+_2 states in 186Pb is strongly associated with proton excitations across the $Z = 82$ shell closure. Although it is difficult to distinguish the prolate or oblate shape for the deformed state due to the overall effect of nuclear deformation on the *α*-decay half-lives, the phenomenon of coexisting spherical and deformed shapes in 186Pb is clearly demonstrated in Fig. 2.

The numerical results of other neutron-deficient Pb and Po isotopes are given in Table [I.](#page-3-0) In Table [I,](#page-3-0) the parent and daughter nuclei are listed in the first column. The second column denotes the spin and parity of the parent nucleus ground state. The third column denotes the spin and parity of the ground and excited states in the daughter nucleus. The experimental α -decay energies and half-lives are given in columns 4 and 5, respectively [\[9,10,13\]](#page-4-0). The extracted deformation parameters are listed in the last column of Table [I.](#page-3-0) It is seen from Table [I](#page-3-0) that the variation of the experimental α -decay half-lives is as large as 10^{10} times $[7.8 \times 10^{-4}$ s $\sim 1.4 \times 10^{+7}$ s]. As a consequence, it is extremely difficult to reproduce the experimental data at the level of $HR = T_{1/2}(Exp)/T_{1/2}(Cal) = 1.0$. In fact, the experimental half-lives of the neutron-deficient Rn and Po isotopes are more or less underestimated in the spherical calculations. However, the agreement between the experimental data and the spherical results can be greatly improved by taking nuclear deformation freedom into account. This is because the overall effect of nuclear deformation will result in a higher *α*-cluster penetration probability through the deformed Coulomb barrier. We note that the penetration factor is more sensitive to nuclear deformation than other terms of the decay width and thus the nuclear deformation mainly affects the barrier penetration probability. Fortunately, the *α*-core interaction in DDCM is calculated from the microscopic double-folding model [\[32\]](#page-4-0). Based on the popular M3Y nucleon-nucleon interaction and

FIG. 2. The variation of the theoretical α -decay half-life of ¹⁹⁰Po as a function of parameter *β*.

Decay	I^{π} (p)	I^{π} (d)	Q (MeV)	T_{α} (Expt.)	β (Theo.)
194 Rn \rightarrow 190 Po + α	0^{+}	0^{+}_{1}	7.862	7.8×10^{-4}	0.364
196 Rn \rightarrow 192 Po + α	0^{+}	0^{+}_{1}	7.616	4.4×10^{-3}	0.360
198 Rn \rightarrow 194 Po + α	0^{+}	0^{+}_{1}	7.354	6.4×10^{-2}	0.267
200 Rn \rightarrow 196 Po + α	0^+	0^{+}_{1}	7.043	1.1×10^{0}	0.213
\rightarrow 196 Po ^m + α	0^{+}	0^{+}_{2}	6.485	1.3×10^{2}	0.347
202 Rn \rightarrow 198 Po + α	0^{+}	0^{+}_{1}	6.775	9.9×10^{0}	0.221
\rightarrow ¹⁹⁸ Po ^m + α	0^{+}	0^{+}_{2}	5.959	5.5×10^{3}	0.302
$^{190}Po \rightarrow ^{186}Pb + \alpha$	0^{+}	0^{+}_{1}	7.695	2.5×10^{-3}	0.097
$^{192}Po \rightarrow ^{188}Pb + \alpha$	0^{+}	0^{+}_{1}	7.319	3.2×10^{-2}	0.141
$^{194}Po \rightarrow ^{190}Pb + \alpha$	0^+	0^{+}_{1}	6.986	4.2×10^{-1}	0.147
$^{196}Po \rightarrow ^{192}Pb + \alpha$	0^{+}	0^{+}_{1}	6.657	6.2×10^{0}	0.162
$^{198}Po \rightarrow ^{194}Pb + \alpha$	0^{+}	0^{+}_{1}	6.307	1.8×10^{2}	0.131
$^{190}Po \rightarrow ^{186}Pb^m + \alpha$	0^{+}	0^{+}_{2}	7.162	7.4×10^{-2}	0.328
$^{192}Po \rightarrow ^{188}Pb^m + \alpha$	0^+	0^{+}_{2}	6.731	2.3×10^{0}	0.344
$^{194}Po \rightarrow ^{190}Pb^m + \alpha$	0^+	0^{+}_{2}	6.328	1.8×10^{2}	0.273
$^{196}Po \rightarrow ^{192}Pb^m + \alpha$	$^{0+}$	0^{+}_{2}	5.888	2.8×10^{4}	0.212
¹⁹⁸ Po \rightarrow ¹⁹⁴ Pb ^m + α	0^{+}	0^{+}_{2}	5.377	1.4×10^{7}	0.200

TABLE I. The theoretical deformations extracted from the experimental *α*-decay energies and half-lives. The unit of the α -decay half-lives is in seconds.

the standard proton-proton Coulomb interaction, the deformed Coulomb barrier in DDCM are well defined in physics. The total penetration probability of the *α*-particle through the deformed Coulomb barrier is obtained by a careful averaging procedure along different orientation angles [\[32\]](#page-4-0). Therefore the influence of the core deformation on the *α*-decay half-lives is properly included in the deformed version of DDCM [\[32\]](#page-4-0). For the *α*-emitters in Table I, we have also performed systematic calculations on their partial *α*-decay half-lives. Similarly, the deformation parameters of the ground and first excited 0^+ states in these nuclei are deduced by the deformed version of DDCM. In principle, the extracted *β* value represents an average deformation after configuration-mixing for the even-mass Pb and Po isotopes. To illustrate the variation of these extracted β values clearly, we plot in Fig. 3 the deformation parameters of the coexisting $0₁⁺$ and $0₂⁺$ states of Pb and Po isotopes by full and hollow circles respectively.

It is seen from Fig. 3 that the ground state deformation parameters in ^{188*,*190*,*^{192*,*194}Pb are very small [$\beta \sim 0.10$],} while the deformation parameters of the low-lying excited $0⁺$ states in these nuclei are relatively much larger [*β* ∼ 0.25]. This closely resembles the case of ¹⁸⁶Pb and agrees with the experimental facts. It is also seen from Fig. 3 that the even-mass Po isotopes are deformed in their ground states and the β values range from 0.213 to 0.364. Experimentally the low-lying excited 0^+_2 states in ^{196,198}Po have also been identified by α - and β -studies. DDCM yields slightly larger deformation parameters for these two states as compared with the ground state ones. The magnitude of these extracted deformation parameters of Pb and Po isotopes is generally consistent with the available experimental and theoretical studies [\[14–20\]](#page-4-0). More importantly, several interesting features of these shape-coexisting nuclei are clearly shown in Fig. 3: (i) Pb isotopes: spherical in their ground 0^+_1 states and

deformed in the excited 0^+_2 states; (ii) Po isotopes: deformed in both the ground and excited 0^+ states; (iii) decreasing of nuclear deformation with increasing neutron number for the ground states of ¹⁹⁰*,*192*,*194*,*196*,*198Po. Notice that our present analysis is totally based on the measured *α*-decay energies and half-lives. The values of deformation parameters are obtained without adjusting any parameter in DDCM. The overall agreement with the experiment is satisfactory and our present analysis provides an alternative way to study nuclear deformation in these shape-coexisting nuclei.

In conclusion, the α -transitions to ground and first excited 0^+ states in the neutron-deficient Pb and Po isotopes are systematically calculated by the density-dependent cluster model. The influence of nuclear deformation on the *α*-decay half-lives is properly taken into account by averaging the

FIG. 3. The deformation parameters of the ground $0₁⁺$ and excited $0₂⁺$ states in even-mass Pb and Po isotopes.

α-cluster penetration probability along different orientation angles. The α -cluster preformation factor is approximately characterized by the Boltzmann distribution for *α*-transitions to the coexisting 0^+_1 and 0^+_2 states. The deformation parameters, which are extracted directly from the experimental *α*-decay data, are in satisfactory agreement with both the experimental facts and theoretical calculations. The shapecoexisting phenomenon in the neutron-deficient Pb and Po nuclei is successfully described by the deformed version of DDCM. The present analysis confirms that α -decay is a

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powerful tool to investigate the shape-coexisting nuclei around the $Z = 82$ shell closure.

ACKNOWLEDGMENTS

Zhongzhou Ren thanks Professor S. Hofmann and Professor A. N. Andreyev for sending us their manuscript before publication. This work is supported by National Natural Science Foundation of China (Nos. 10125521, 10535010) and by 973 National Major State Basic Research and Development of China (No. G2000077400).

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