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Jet shapes in opaque media

Antonio D. Polosa
INFN, Sezione di Roma, Rome, Italy

Carlos A. Salgado*
Dip. di Fisica, Università di Roma "La Sapienza" and INFN, Rome, Italy
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We present general arguments, based on the medium-induced radiative energy loss, to explain the presence of nontrivial angular structures within this perturbative mechanism. Simple analytical estimates of this effect are provided which are relevant to the current discussion on the interpretation of the data on two- and three-particle correlations of semihard particles at BNL's Relativistic Heavy Ion Collider.

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Jet quenching, which is the strong modification of the spectrum of particles produced at high transverse momenta in nucleus-nucleus collisions, is a solid experimental result of BNL's Relativistic Heavy Ion Collider (RHIC) program [1]. At present, its main observable is the strong suppression of the inclusive particle yield in the whole available range of $p_\perp \lesssim 20\,\mathrm{GeV}/c$ [1,2]. This suppression can be understood, in general terms, as being caused by the energy loss of highly energetic partons traversing the medium created in the collision. To further unravel the dynamical mechanism underlying this effect, the questions of where and how the energy is "lost" are of special relevance. The most promising experimental probes to investigate these issues are clearly related to the modification of the jet structures [3]. In the most successful approach to explaining jet quenching, the degradation of the energy of the parent parton produced in an elementary hard process is due to the medium-induced radiation of soft gluons [4–9], and the energy transfer to the medium (recoil) is neglected. As usual, the medium-modified parton shower will eventually convert into a hadron jet. In the opposite limit, one could assume that a large fraction of the original parton energy is transferred to the medium, with a fast local equilibration, and diffused through sound and/or dispersive modes. The latter possibility has been advocated [10] as the origin of the striking non-Gaussian shape of the azimuthal distributions in the opposite direction to the trigger particle [11].

In this paper, we show that the same perturbative mechanism able to describe the inclusive suppression data, namely, the radiative energy loss, produces two-peak shape structures in the azimuthal correlations. This happens under some special conditions which could be realized in the present data. The central point of our argument is the need for a more exclusive treatment of the distributions to describe experimental situations with restrictive kinematical constraints. To this end, we supplement the medium-modified jet formation formalism with the Sudakov suppression form factor following the well-known in-vacuum approach. Furthermore, we assume that the final hadronic distributions follow the parton level ones (parton-hadron duality).

The medium-induced gluon radiation. Let us suppose that a very energetic parton propagating through a medium of length L emits a gluon with energy ω and transverse momentum $k_\perp$ with respect to the fast parton. The formation time of the gluon is $t_{\mathrm{form}} \sim \frac{2\omega}{k_\perp^2}$. Its typical transverse momentum is

$$k_\perp^2 \sim \frac{\langle q_\perp^2\rangle_{\mathrm{med}}}{\lambda} t_{\mathrm{form}} \sim \sqrt{2\omega\hat{q}}. \qquad (1)$$

$\hat{q}$ is the transport coefficient [4] which characterizes the medium-induced transverse momentum squared $\langle q_\perp^2\rangle_{\mathrm{med}}$ transferred to the projectile per unit path length λ. In this picture, the gluon acquires transverse momentum because of the Brownian motion in the transverse plane due to multiple soft scatterings during t_{form}. The typical emission angle,

$$\sin\theta \equiv \frac{k_\perp}{\omega} \sim \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4}, \qquad (2)$$

defines a minimum emission energy $\hat{\omega} \sim (2\hat{q})^{1/3}$ below which the radiation is suppressed by formation time effects [9]. The latter is a crucial observation for the discussion to follow. Notice that for energies smaller than $\hat{\omega}$, the angular distribution of the medium-induced emitted gluons peaks at large values. In these conditions, the medium-induced spectrum [6,9] can be approximated by (taking $k_\perp^2 < \sqrt{2\omega\hat{q}}$, $\omega \ll \omega_c \equiv \frac{1}{2}\hat{q}L^2$)

$$\frac{dI^{\mathrm{med}}}{d\omega\, dk_\perp^2} \simeq \frac{\alpha_s C_R}{16\pi} L \frac{1}{\omega^2}. \qquad (3)$$

Letting $\hat{q} \sim 5$–$15\,\mathrm{GeV}^2/\mathrm{fm}$ for the most central Au-Au collisions at RHIC [12], Eq. (3) is valid for $\omega < \hat{\omega} \sim 3\,\mathrm{GeV}$ [we have taken $\hat{\omega} \sim 2(2\hat{q})^{1/3}$ as indicated by numerical results]. We have checked that this approximation is in agreement with the numerical results in Refs. [6,9] up to logarithmic corrections at small ω, which will be neglected in the following.

The parton shower evolution. Equation (3) gives the inclusive spectrum of gluons emitted by a high-energy parton traversing a medium. For practical applications, however, more exclusive distributions, giving the probability of one, two, ... emissions are needed. How to construct such probabilities, using Sudakov form factors, is a well-known procedure in

*Permanent address: Departamento de Física de Partículas, Universidade de Santiago de Compostela, Spain.

the vacuum. In the medium, a first attempt to deal with the strong trigger bias effects in inclusive particle production was to use an independent gluon emission approximation with corresponding Poissonian probabilities [13]. Here we propose to improve this assumption by including the virtuality through medium-modified Sudakov form factors.

Since we are interested in angular distributions, the parton shower description proposed in Ref. [14] is particularly convenient. Let us introduce the evolution variable

$$\xi = \frac{q_1 \cdot q_2}{\omega_1 \omega_2} \tag{4}$$

to define the branching of a particle (gluon in general) with virtuality q and energy E in two particles of virtuality $q_1, q_2 < q$ and energies $\omega_1 \equiv zE$, $\omega_2 \equiv (1-z)E$. If we assume that $\omega_1, \omega_2 \gg m_1, m_2$, then $\xi \simeq 1-\cos\theta_{12}$, with $\theta_{12} = \theta_1+\theta_2$, where θ_1, θ_2 are the angles formed by the daughter partons with the parent parton. With these definitions, the evolution in virtuality can be converted into an evolution in the variable ξ. The corresponding probability distribution for one branching is

$$d\mathcal{P}(\xi, z) = \frac{d\xi}{\xi}\, dz\, \frac{\alpha_s}{2\pi} P(z)\, \Delta(\xi_{\max}, \xi)\theta(\xi_{\max} - \xi) \times \theta(\xi - \xi_{\min}), \tag{5}$$

where $P(z)$ is the splitting function. The Sudakov form factor controlling the evolution is [15]

$$\Delta(\xi, E) = \exp\left\{ - \int_{\xi}^{\xi_{\max}} \frac{d\xi'}{\xi'} \int_{\epsilon}^{1-\epsilon} dz\, \frac{\alpha_s}{2\pi}\, P(z) \right\}, \tag{6}$$

and $\epsilon = Q_0/E\sqrt{\xi}$ with Q_0 a cutoff. $\Delta(\xi, E)$ can be interpreted as the probability of no branching between the scales ξ and $\xi_{\max}$.

The generalization to the medium case. The splitting probability is modified in the case that the shower develops in the presence of a medium. A modified splitting function can be defined by noticing that the total spectrum of emitted gluons [4–9] is the sum of a medium plus a vacuum contribution, with this last given by the usual splitting function

$$\frac{dI^{\rm tot}}{dzdk_\perp^2} = \frac{dI^{\rm vac}}{dzdk_\perp^2} + \frac{dI^{\rm med}}{dzdk_\perp^2}; \qquad \frac{dI^{\rm vac}}{dzdk_\perp^2} = \frac{\alpha_s}{2\pi}\frac{P(z)}{k_\perp^2}. \tag{7}$$

This suggests that we make the change

$$P(z) \to P(z) + \Delta P(z) \tag{8}$$

in Eqs. (5) and (6), where $\Delta P(z)$ is given by the medium spectrum, Eq. (3). This assumption is in agreement with the findings in Ref. [8], in which the in-medium evolution of fragmentation functions has been found to follow normal DGLAP (Dokshitzer-Gribor-Lipatov-Altarelli-Parisi) equations—equivalent at LO (leading order) to the above Sudakov formulation—with a modified splitting of the general form (8). A similar prescription has also been used in Ref. [16]. Notice also that the usual formulation in terms of *quenching weights* [13] is recovered once the virtuality is neglected.

The general splitting probability contains, in this way, a medium (ΔP) and a vacuum (P) part. The first one is enhanced by the medium length L and will be dominant for very opaque and long media. In the following, the vacuum contribution will be neglected. We have checked that its relevance is numerically negligible for all the cases studied, except for the most peripheral collisions. So, including the vacuum contribution does not change the main observations drawn in this paper. We therefore present simple analytical results using only the medium part of the spectrum.

Results. The general case of arbitrary ω_1 and ω_2 is difficult to study analytically. Let us study, instead, two extreme cases: (i) one of the particles takes most of the incoming energy $\omega_1 \gg \omega_2$ and, hence, $\theta_1 \simeq 0$—we call this the J configuration; (ii) the two particles share equally the available energy, $\omega_1 \simeq \omega_2$ and $\theta_1 \simeq \theta_2$—this is the Y configuration.

First let us consider case (i), where $\xi = 1 - \cos\theta_{12} \equiv 1 - \cos\theta$ with $\sin\theta = k_\perp/\omega$. From Eq. (3) we get

$$\frac{dI}{dz\, d\xi} = \frac{\alpha_s C_R}{8\pi} E\, L\, (1 - \xi). \tag{9}$$

Therefore, the corresponding Sudakov form factor reads

$$\Delta^{\rm med}(\xi_{\max}, \xi) = \exp\left\{ -\frac{\alpha_s C_R}{8\pi}\, L\, E \int_{\xi}^{\xi_{\max}} d\xi'\, (1 - \xi') \right\}, \tag{10}$$

where the small contribution from the integration in z has been neglected (we checked that the results do not vary as long as $\omega \gg Q_0$); $\alpha_s = 1/3$ will be assumed in the following. Taking $\xi = 1 - \cos\theta$, $\xi_{\max} = 1$ and inserting into the probability of splitting, Eq. (5), we get

$$\frac{d\mathcal{P}(\theta, z)}{dz\, d\theta} = \frac{\alpha_s C_R}{8\pi}\, E\, L \sin\theta \times \cos\theta \exp\left\{ -E\, L\, \frac{\alpha_s C_R}{16\pi} \cos^2\theta \right\} \tag{11}$$

as the probability distribution for a parton to split just once, emitting a gluon at angle θ with a fraction z of the incoming momentum.

Equation (11) is written in spherical coordinates with respect to the direction of the parent parton, with θ the polar angle. For symmetry, the spectrum is independent of the azimuthal angle β, which was integrated into the previous expressions but not in the following. Assuming that this parent parton was produced at 90° in the center-of-mass frame of the collision, we can transform the coordinates of the emitted gluon to the laboratory azimuthal Φ and polar $\theta_{\rm lab}$ angles with respect to the beam direction $\hat{z}$. Usually, one uses the pseudorapidity $\eta = -\log\tan(\theta_{\rm lab}/2)$. The Jacobian is

$$d\theta d\beta = \frac{d\eta\, d\Phi}{\cosh\eta\sqrt{\cosh^2\eta - \cos^2\Phi}}. \tag{12}$$

Taking, for simplicity, $\eta = 0$ in the most favorable detection region, the answer is simply

$$\left.\frac{d\mathcal{P}(\Phi, z)}{dz\, d\Phi}\right|_{\eta=0} = \frac{\alpha_s C_R}{16\pi^2}\, E\, L \cos\Phi \times \exp\left\{ -E\, L\, \frac{\alpha_s C_R}{16\pi} \cos^2\Phi \right\}, \tag{13}$$

giving the probability of one splitting as a function of Φ. Thus we reach our objective: the possibility of describing

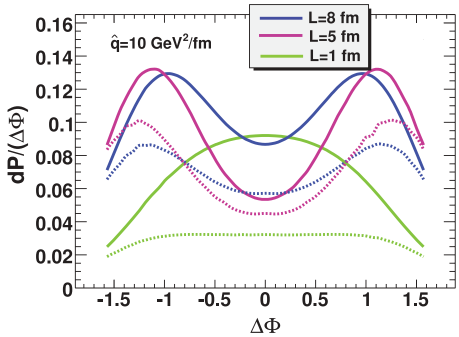

FIG. 1. (Color online) Probability of just one splitting, Eq. (11), as a function of laboratory azimuthal angle $\Delta\Phi$ for a gluon jet of $E = 7$ GeV. Different medium lengths are plotted for the J and Y configurations (solid and dotted lines, respectively).

nontrivial angular dependences, as shown in Eqs. (11) and (13) with a perturbative mechanism. The distribution found has two maxima whose positions are determined by

$$\Phi_{\max} = \pm \arccos\sqrt{\frac{8\pi}{E\,L\,\alpha_s C_R}}. \qquad (14)$$

The angular shape found in Eq. (13) is very similar to the one found experimentally. We do not intend to perform a detailed calculation of the experimental situation. We just introduce a simple model to take into account the additional smearing of the jet shape introduced by the triggering conditions. Setting $\eta = 0$ for the trigger particle, we take into account (i) the uncertainty due to the boosted center-of-mass frame of the partonic collision by integrating the probability (13) in $2\Delta\eta$ with $\Delta\eta = 1$,[1] and (ii) an additional uncertainty in azimuthal angle $\Delta\Phi$ given by a Gaussian with $\sigma = 0.4$ [18]. Specifically, we take

$$\frac{dP}{d\Delta\Phi\, dz} = \frac{1}{N}\int_{-\Delta\eta}^{\Delta\eta} d\eta \int d\Phi' \frac{dP}{d\Phi' dz d\eta} \times \exp\left\{-\frac{(\Delta\Phi - \Phi')^2}{2\sigma^2}\right\}, \qquad (15)$$

$N = 2\Delta\eta\sqrt{2\pi\sigma^2}$ being a normalization factor. The results are plotted in Fig. 2 for three different medium lengths and $E = 7$ GeV (the quoted value $\hat{q} = 10\,\mathrm{GeV}^2/\mathrm{fm}$ ensures that the results hold for gluon energies $\omega < \hat{\omega} \simeq 3$ GeV). To estimate the centrality dependence of the position of the maxima, we simply take $L = N_{\mathrm{part}}^{1/3}$. Although this geometric procedure is too simplistic, it provides us with a sense of the N_{part}-dependence of this effect; this is plotted in Fig. 2. We must also point out that the centrality dependence of the transport coefficient is not taken into account in these figures. The reduction of $\hat{q}$ with centrality ($\hat{q} \sim dN/dy \sim N_{\mathrm{part}}$) will make the radiation more collinear when $\hat{q}^{1/3} < \omega$. The addition of the vacuum contribution—explicitly neglected—would also change the shapes of the more peripheral cases. A more precise answer would need, however, a numerical analysis.

[1]We have checked that a convolution with a Gaussian as given in Ref. [17] does not affect our results.

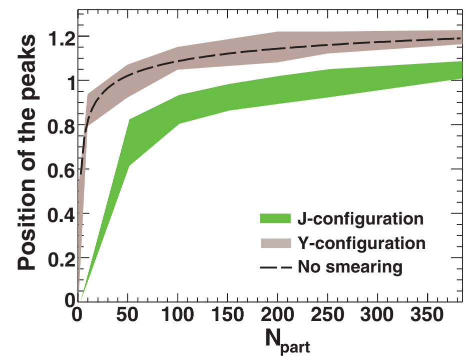

FIG. 2. (Color online) Position of $\Delta\Phi$ distribution peaks for J and Y configurations. We also show the analytical estimate (14) without smearing.

Let us now consider case (ii), the Y configuration, where $\omega_1 \simeq \omega_2$ and the two angles are similar, $\theta_1 \simeq \theta_2 \equiv \theta$. Now the variable $\xi = 1 - \cos\theta_{12} = 1 - \cos 2\theta$, and $k_\perp^2/\omega^2 = \sin^2\theta = \xi/2$. Repeating the same procedure followed before, we obtain

$$\frac{d\mathcal{P}(\Phi, z)}{dz\, d\Phi} = \frac{C_R\alpha_s}{64\pi^2}\, E\, L \exp\left\{-\frac{C_R\alpha_s}{32\pi}\, E\, L\cos\Phi\right\}. \qquad (16)$$

In this case, the maxima are outside the borders of the physical phase space, but a minimum still occurs at $\Phi = 0$. Anyway, the smeared distribution does present maxima. The corresponding curves and positions of the maxima are plotted in Figs. 1 and 2.

The picture that emerges from our analysis is the following: (i) the splitting probability of a highly energetic parton produced inside the medium by a hard process presents well-defined maxima in the laboratory azimuthal angle when the emitted gluons have energies $\omega < \hat{\omega}$, with $\hat{\omega} \sim 3$ GeV for the most central collisions at RHIC, and (ii) when the experimental triggering conditions are such that only a small number of splittings are possible (by, e.g., restrictive kinematical constraints), these structures should be observed. To relate this finding to present RHIC data on two-particle correlations at high $p_\perp$, we first observe that the steeply falling perturbative spectrum biases the observed trigger particle to small in-medium and in-vacuum energy losses. This allows the energy of the parent parton to be estimated as $E \sim p_\perp^{\mathrm{trigg}}/0.6$ [19] and the production point to be surface dominated [12]. In Ref. [20], three possibilities, which we take as representative examples, have been studied experimentally:

(i) $2.5 < p_\perp^{\mathrm{trigg}} < 4$ GeV and $1 < p_\perp^{\mathrm{assoc}} < 2.5$ GeV. Under these conditions, the available energy estimated from $p_\perp^{\mathrm{trigg}}$ allows only the production of a small number of particles in the $p_\perp^{\mathrm{assoc}}$ range, limiting the number of possible splittings, and the two-peak structure would appear.

(ii) $6 < p_\perp^{\mathrm{trigg}} < 10$ GeV and $1 < p_\perp^{\mathrm{assoc}} < 2.5$ GeV. Now the kinematic constraint is less restrictive, more splittings are possible, and the dip can be filled.

(iii) $p_\perp^{\mathrm{trigg}} > 8$ GeV and $p_\perp^{\mathrm{assoc}} > 6$ GeV. Under these conditions, the possible large angle radiation is cut off, $p_\perp^{\mathrm{assoc}} > \hat{\omega}$, and the two-peak structure is not present.

The experimental findings support these qualitative expectations. Clearly, a realistic comparison with experimental data would need a much more sophisticated analysis, including not only the probability of multiple splittings but also hadronization and the contribution from the vacuum, which are ignored here. Our results are very encouraging, though, as the distributions we obtain for single splitting resemble very much the experimental findings, both in shape and in centrality dependence.

Let us also comment here that the use of the multiple-soft scattering approximation [4–6,9] in the medium-induced gluon radiation, Eq. (3), as well as a large value of the transport coefficient $\hat{q}$ are essential in our estimates: using only the first order in the opacity expansion, the angle decreases with increasing medium length as $\sin\theta \sim 1/\sqrt{L}$ [21,22], making the radiation more collinear with increasing centrality [in contrast with Eq. (2) which is independent of the centrality]. Also essential to our approach is the use of exclusive distributions, constructed by means of the Sudakov suppression factor. As can be checked from Eq. (3), the inclusive distribution gives a $\cos\Phi$ when translating to experimental variables because of the Jacobian of the transformation (12) [21]; i.e., a maximum would be present at $\Delta\Phi = \pi$ instead of a dip.

On the other hand, one alternative explanation for the non-Gaussian shape found at RHIC is in terms of shock waves produced by the highly energetic particle into the medium. In this picture, a large amount of the energy lost must be transferred to the medium, which thermalizes almost instantaneously. The energy deposition needed for the sound modes to become visible in the spectrum has been found to be quite large [23]. To our knowledge, no attempt has been made so far to describe the centrality dependence of the shape of the azimuthal correlations in this approach. Given the fact that the two formalisms described above rely on completely different hypotheses, finding experimental observables that could distinguish between them is certainly an issue which deserves further investigation. New data on three-particle correlations are expected to shed some light on the problem. Here, we just notice that our Y configurations, which by kinematics should dominate when $p_\perp^{\rm trigg} \sim p_\perp^{\rm assoc}$, would lead to signatures similar to the ones from the shock wave model. In the most general case, however, different configurations, not considered in our simple analysis, and characterized by different radiation angles for both gluons, would produce a smeared signal.

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[1] J. Adams *et al*. (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005); B. B. Back *et al*. (PHOBOS Collaboration), Nucl. Phys. **A757**, 28 (2005); I. Arsene *et al*. (BRAHMS Collaboration), Nucl. Phys. **A757**, 1 (2005); K. Adcox *et al*. (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005).
[2] M. Shimomura (PHENIX Collaboration), Nucl. Phys. **A774**, 457 (2006).
[3] C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **93**, 042301 (2004).
[4] R. Baier *et al*., Nucl. Phys. **B484**, 265 (1997).
[5] B. G. Zakharov, JETP Lett. **65**, 615 (1997).
[6] U. A. Wiedemann, Nucl. Phys. **B588**, 303 (2000).
[7] M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001).
[8] X. N. Wang and X. F. Guo, Nucl. Phys. **A696**, 788 (2001).
[9] C. A. Salgado and U. A. Wiedemann, Phys. Rev. D **68**, 014008 (2003).
[10] H. Stoecker, Nucl. Phys. **A750**, 121 (2005); J. Casalderrey-Solana, E. V. Shuryak, and D. Teaney, J. Phys. Conf. Ser. **27**, 22 (2005); J. Ruppert and B. Muller, Phys. Lett. **B618**, 123 (2005).
[11] S. S. Adler *et al*. (PHENIX Collaboration), Phys. Rev. Lett. **97**, 052301 (2006); N. Grau (PHENIX Collaboration), Nucl. Phys. **A774**, 565 (2006).
[12] K. J. Eskola *et al*., Nucl. Phys. **A747**, 511 (2005); A. Dainese, C. Loizides, and G. Paic, Eur. Phys. J. C **38**, 461 (2005).
[13] R. Baier *et al*. J. High Energy Phys. 09 (2001) 033.
[14] G. Marchesini and B. R. Webber, Nucl. Phys. **B238**, 1 (1984).
[15] For more detail on the role of Sudakov form factors in hadron-collision Monte Carlo simulations, see R. K. Ellis, W. J. Stirling, and B. R. Webber, *QCD and Collider Physics* (Cambridge University, Cambridge, 1996).
[16] N. Borghini and U. A. Wiedemann, CERN-PH-TH-2005-100 (unpublished), hep-ph/0506218.
[17] T. Renk and J. Ruppert, Phys. Lett. **B646**, 19 (2007); Phys. Rev. C **73**, 011901(R) (2006).
[18] S. S. Adler (PHENIX Collaboration), Phys. Rev. D **74**, 072002 (2006).
[19] K. J. Eskola and H. Honkanen, Nucl. Phys. **A713**, 167 (2003).
[20] J. Adams *et al*. (STAR Collaboration), Phys. Rev. Lett. **97**, 162301 (2006); T. Peitzmann (STAR Collaboration), talk at Hard Probes 2006 Conference, Asilomar, CA, June 2006 (unpublished).
[21] I. Vitev, Phys. Lett. **B630**, 78 (2005).
[22] A. Majumder and X. N. Wang, Phys. Rev. C **73**, 051901(R) (2006).
[23] J. Casalderrey-Solana, E. V. Shuryak, and D. Teaney (unpublished), hep-ph/0602183.