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Quark matter in neutron stars within the Nambu-Jona-Lasinio model and confinement

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The quark matter equation of state (EoS) derived from the standard Nambu-Jona-Lasinio (NJL) model is soft enough to render neutron stars (NS) unstable at the onset of the deconfined phase, and no pure quark matter can be actually present in its interior. Since this is a peculiarity of the NJL model, we have studied a modified NJL model with a momentum cutoff which depends on the density. This procedure, which improves the agreement between QCD and NJL model at large density, modifies the standard NJL equation of state, and then it is potentially relevant for the stability analysis of neutron stars. We show that also within this approach, the NS instability still persists, and that the vacuum pressure, as a signal of quark confinement, has a fundamental role for the NS stability. In this respect, our conclusions point to a relationship between confinement and NS stability.

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I. INTRODUCTION

In the core of astrophysical compact objects, like neutron stars (NS) or protoneutron stars, nuclear matter is expected to reach a density which is several times nuclear saturation density. Calculations based on microscopic equation of states (EoS), which include only nucleonic degrees of freedom, show that the central density of the most massive neutron stars can be from seven to ten times the nuclear saturation density [1]. In such configurations the nucleons are closely packed, and to consider them as separate entities becomes highly questionable. Unfortunately, it is difficult to calculate accurately the transition point from nucleonic to quark matter, and only rough estimates have been given in the literature [2]. The microscopic theory of the nucleonic equation of states has reached a high degree of sophistication, and different many-body methods have been developed. They show a substantial agreement among each others [3], and therefore the main uncertainty of the transition point stays on the quark matter EoS. Assuming a first order phase transition, as suggested by lattice calculations, one can use different models for quark matter to estimate the transition point, and calculate the compact object configuration. This approach has been followed by several authors, and a vast literature exists on this subject. The applications of quark matter models to the study of the NS structure have used different versions of the MIT bag model [4], the color-dielectric model [5], and different formulations of the NJL model [6]. In general, it seems that the maximum mass of NS that contain quark matter in their interior is bounded to be less than 1.6–1.7 solar masses. However, more recently it has been shown in [7] that if one corrects the MIT bag model by introducing additional terms suggested by perturbative QCD, the maximum mass can reach values close to 1.9 solar masses, similar to the ones obtained with nucleonic degrees of freedom. Despite the similarity of the results on the value of the maximum NS mass, the predictions on the NS configurations can differ substantially from model to model. The most striking difference is in the NS quark matter content, which can be extremely large in the case of EoS related to the MIT bag model or the color-dielectric model, but it is vanishingly small in the case of the original version of the NJL model [6,8]. In the latter case it turns out that, as soon as quark matter appears at increasing NS mass, the star becomes unstable towards collapse to a black hole, with only the possibility of a small central region with a mixed phase of nucleonic and quark matter. This result can be quite relevant for the physics of NS, since the NJL model is the only model which is based on phenomenological low energy data, i.e., on hadron properties. The main drawback of the model is the absence of confinement, since the gap equations which determine the quark masses as a function of density cannot incorporate a confining potential. One then assumes that the chiral phase transition marks also the confinement transition, as indicated by all lattice calculations. More recently a confining potential has been introduced in the NJL model [9], which is simply switched off at the chiral phase transition. In this case indeed no sharp instability of the NS is observed at the onset of quark matter in the central core. This is suggestive of a connection between the presence of confinement and the possibility of NS with a quark matter central core. However it is not clear if the instability of the NS is related or not to confinement for (at least) two reasons. Indeed, the introduction of the confining potential requires several parameters and, moreover, the NJL model at large density suffers of another drawback pointed out in [10]. In fact, it has been shown that the standard NJL model is not able to reproduce the correct QCD behavior of the gap for large density, and therefore a different cut-off procedure at large momenta has been proposed. More precisely, a density dependent cutoff has been introduced, and this strongly modifies the standard NJL model thermodynamics. Therefore, the stability analysis of NS by the new EoS, which follows from this modified treatment of NJL model at large density, is a preliminary step to understand whether confinement is an essential ingredient in the stabilization mechanism.

It is the purpose of this paper to clarify this point and to identify the origin of the NS instability at the quark onset within the original NJL model, which sharply distinguishes this model from all the others. We show that with the density dependent cut-off procedure the NS instability still persists and that the vacuum pressure, as signal of quark confinement, has a fundamental role for the NS stability, as yet observed in

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the MIT bag model. In this respect, our conclusions point to an indirect relationship between confinement and NS stability.

II. NJL AT LARGE DENSITY

The NJL model provides a good phenomenological description of low energy QCD based on the Lagrangian (for the two flavor case)

$$\mathcal{L}_{\rm NJL} = i\bar\psi\gamma^\mu\partial_\mu\psi + g\left[\left(\bar\psi\psi\right)^2 + \sum_{\alpha=1}^{N_f^2-1}(\bar\psi\tau^\alpha i\gamma_5\psi)^2\right]. \quad (1)$$

As pointed out in [10], the model is not able to reproduce the correct behavior of the solution of QCD gap equation at large density. Indeed, at zero temperature, the relevant dynamical degrees of freedom have momenta in the shell $|p| \simeq \mu + \delta$, where μ is the chemical potential and δ is the cutoff evaluated from the Fermi momentum. Therefore the standard cutoff Λ on the momentum, ($|p| < \Lambda$), forces $\delta \simeq 0$ when $\mu \simeq \Lambda$, and this gives an unphysical reduction of the number of dynamical states of the system.

To solve this problem a new procedure has been proposed, which simultaneously preserves the low energy properties of the theory, and allows one to write a cut-off independent gap equation at zero temperature and density. The idea is the introduction of a cut-off dependence in the four-fermion coupling constant, g, by

(i) keeping fixed f_π to its experimental value and deriving the constituent mass M as a function of Λ, $M(\Lambda)$, from the expression for f_π:

$$f_\pi = 3M^2\int\frac{d^3p}{(2\pi)^3}\frac{1}{E_p^3}\theta(\Lambda - |\vec p|). \quad (2)$$

(ii) determining the coupling $g(\Lambda)$ which solves the gap equation, at $\mu = 0$ and zero temperature,

$$M(\Lambda) = m + 4N_fN_c\, g(\Lambda)\int_\Lambda\frac{d^3p}{(2\pi)^3}\frac{M(\Lambda)}{\sqrt{p^2+[M(\Lambda)]^2}} \quad (3)$$

for any value of Λ. As shown in Fig. 1 of [10], $\Lambda^2 g(\Lambda)$ smoothly decreases with Λ.

Then, according to the procedure developed in [10], at finite density, a μ dependent cutoff $\Lambda(\mu)$ is introduced, which, in turn, implies a μ dependent coupling constant. In particular, a linear dependence in $\Lambda(\mu)$ is taken, which provides a better agreement with high density QCD.

On the other hand this procedure strongly changes the thermodynamics and the EoS of the system at finite density with respect to the standard NJL approach. In fact, the functional form of the thermodynamical potential Ω is not modified by a μ dependent cutoff, and therefore the pressure is

$$p = -V^{-1}\Omega(\Lambda(\mu), g(\mu), \mu, T = 0), \quad (4)$$

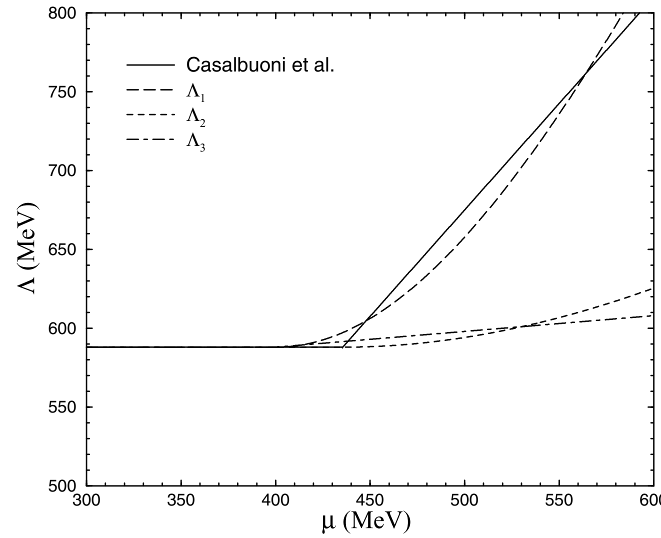

FIG. 1. The cutoff is displayed as a function of the chemical potential. Several functional dependencies have been adopted, as discussed in the text.

where V indicates the three-dimensional volume of the system and Ω, in the following, will be computed in the usual mean field approximation. The particular choice of $\Lambda(\mu)$ considered in [10]:

$$\Lambda(\mu) = \begin{cases}\Lambda_0 & \mu < \Lambda_0/(c+1) \approx 435\,\text{MeV}\\ (c+1)\mu & \mu > \Lambda_0/(c+1) \approx 435\,\text{MeV},\end{cases} \quad (5)$$

where $\Lambda_0 \sim 600\,\text{MeV}$ is the NJL cutoff in the standard treatment, and $c = \delta/\mu = 0.35$ (see [10]), introduces a discontinuity in the density $\rho = -\frac{1}{V}(\partial\Omega/\partial\mu)$, and in the corresponding EoS. This definition of the density ρ takes care of the dependence of Λ on μ, and ensures thermodynamical consistency.

Since there is no compelling restriction to a specific functional form of $\Lambda(\mu)$, we replace Eq. (5) with some smooth interpolations that have no discontinuous densities and grow with different slopes for large values of the chemical potential. In Fig. 1 we plot the cutoff of Eq. (5) together with three different ansatz:

$$\Lambda_1(\mu) = \begin{cases}\Lambda_0 & \mu \lesssim 400\,\text{MeV}\\ \sqrt{9(\mu-400)^2+\Lambda_0^2} & \mu \gtrsim 400\,\text{MeV},\end{cases} \quad (6)$$

$$\Lambda_2(\mu) = \begin{cases}\Lambda_0 & \mu \lesssim 435\,\text{MeV}\\ (c+1)\mu\left[\log\left(\frac{(c+1)\mu}{\Lambda_0}\right)-1\right]+2\Lambda_0 & \mu \gtrsim 435\,\text{MeV},\end{cases} \quad (7)$$

$$\Lambda_3(\mu) = \begin{cases}\Lambda_0 & \mu \lesssim 397\,\text{MeV}\\ 0.1\mu + 548 & \mu \gtrsim 403\,\text{MeV}.\end{cases} \quad (8)$$

The numerical coefficients in Eqs. (6), (7), and (8), are adjusted in order to keep the curves in Fig. 1 either close to the one of Eq. (5), or close to the constant standard cutoff Λ_0. In particular, for Λ_3, in the interval $397\,\text{MeV} < \mu < 403\,\text{MeV}$, a continuous quadratic interpolation between the two straight lines reported in Eq. (8), is taken. We stress that the adopted

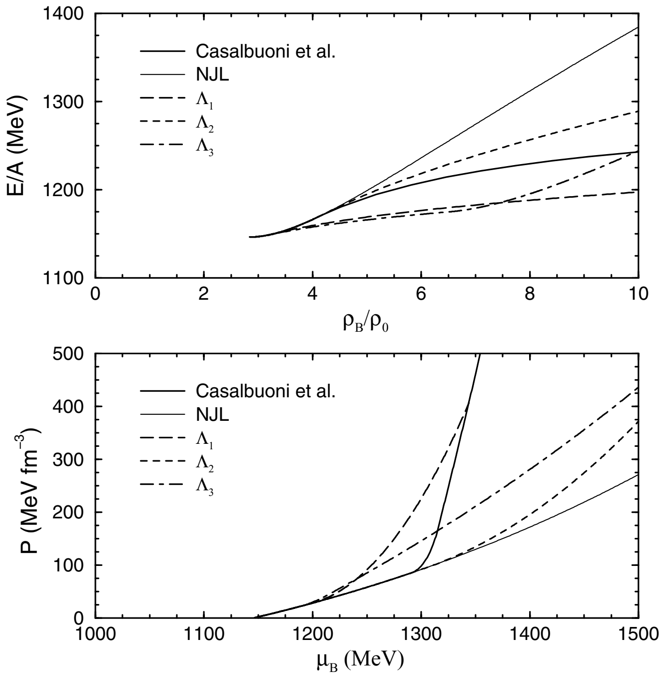

FIG. 2. The energy per baryon is shown as a function of the baryon density (upper panel), whereas the pressure as a function of the baryon chemical potential is displayed in the lower panel, for different choices of the cutoff. See text for details.

smoothing procedure is not intended to propose a method alternative to the one presented in [10], but it has the only purpose of making this approach suitable for our application to NS, and avoiding spurious discontinuities in the density.

Considering different slopes in the μ dependence of the cutoff is an important issue, because this implies different growth behaviors of the pressure, and therefore of the density, as a function of the baryon chemical potential μ_B, and moreover different dependence of the energy per baryon on the baryon density ρ_B, which is crucial to determine the critical value of ρ_B in the transition from nuclear to quark matter.

In Fig. 2 we display in the upper panel the energy per baryon E/A versus ρ_B/ρ_0 for the pure quark phase (ρ_0 indicates the nuclear saturation density, $\rho_0 = 0.17\,\text{fm}^{-3}$), and the pressure as a function of μ_B in the lower panel for the various $\Lambda_i(\mu)$.

For completeness, we also show the results obtained using a constant cutoff, as in the standard NJL model (thin solid line), for which the parametrization of Ref. [8] has been used. As we shall see in the next section, the stability analysis of the NS is influenced by the various behavior of E/A plotted in Fig. 2.

III. NJL AND NEUTRON STAR STABILITY

The transition to quark matter in the core of NS has been studied by many authors by matching the baryonic EoS, as determined by microscopic many-body calculations involving nucleons only, and the quark EoS as determined by simple models which simulate QCD. The crossing of the two EoS in the pressure vs. chemical potential plane marks the transition point between the two phases. At the transition the two phases are in mechanical and chemical equilibrium. Below the transition point, i.e., at lower density, the nucleonic phase is favored, while above the point the quark matter is favored. This corresponds to the Maxwell construction for a first order phase transition, as suggested by the indications coming from lattice calculations [11]. Solving the Tolmann-Oppenheimer-Volkoff (TOV) equations [12], one can then calculate the NS configuration and the corresponding gravitational mass as a function of the central density or of the corresponding NS radius. In a previous work [6], where the NJL model was used, it was found that the quark onset at the center of the NS as the mass increases marks an instability of the star, i.e., the NS collapses to a black hole at the transition point since the quark EoS is unable to sustain the increasing central pressure due to gravity. For the nucleonic sector a microscopic EoS derived with the Brueckner-Bethe-Goldstone (BBG) method was used [13]. The uncertainty in the baryonic EoS is relatively small and not relevant to the present analysis. In fact, the results are in line with similar calculations within the NJL model, where other nucleonic EoS are used [14]. For simplicity here we restrict the calculations to the two flavor case.

In the calculations presented in this paper, for the hadronic phase we have adopted a nucleonic equation of state obtained within the BBG approach [15], using the Argonne v_{18} two-body potential [16], supplemented by the Urbana phenomenological three-body force [17]. For the quark phase, we use the standard parametrization of the NJL model [8], and our proposed prescriptions of the cutoff described in Sec. II. In Fig. 3 we display the complete EoS, i.e., the pressure as function of the baryon density for the cases discussed above. The thick solid line displays the equation of the state for the pure hadron phase, treated in the BBG approach. The thin monotonically increasing curves represent the quark matter branch of the EoS, whereas the plateaus are a consequence of the first order Maxwell construction. The transition density

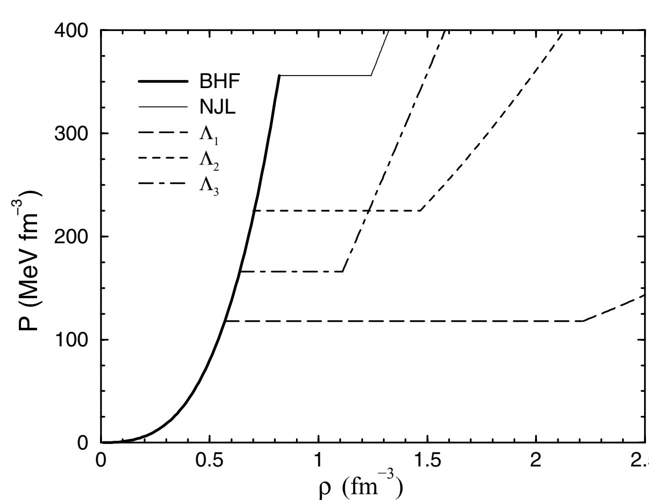

FIG. 3. The complete EoS is displayed, i.e., the pressure as function of the baryon density. See text for details.

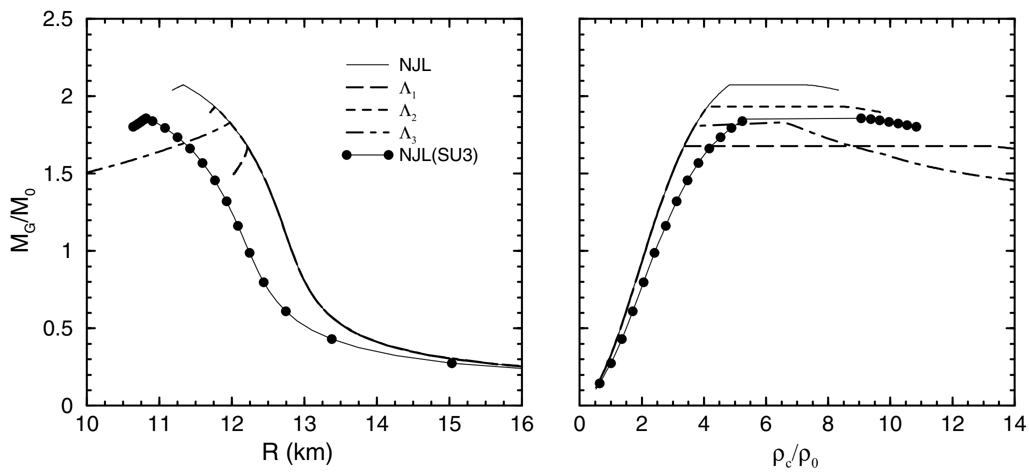

FIG. 4. The gravitational mass (in units of the solar mass $M_\odot = 2 \times 10^{33} g$) is plotted as function of the radius (left panel) and the central density (right panel), for different choices of the cut-off behavior.

range shown by the plateau depends on the adopted cut-off procedure.

In Fig. 4 we report the neutron star masses as a function of the radius (left panel), and of the central density (right panel) for the different choices of the density dependent cutoff discussed above. For comparison also the results for the standard NJL model (three flavor) of Ref. [6] are reported (full circles). In all cases one can see that at the maximum mass the plot is characterized by a cusp, which corresponds to the instability mentioned above. The plateau which appears in each of the curves reported in the right panel is a consequence of the Maxwell construction. The width of the plateau is the jump of the central density at the onset of quark matter.

It is likely that if a mixed phase, according to the Glendenning construction [18], had been used, then the plot would be smooth, but in any case no pure quark matter is expected to be allowed in a stable NS.

Therefore the improvement of the NJL model by the density dependent cutoff does not solve the NS stability problem, and the origin of the instability should be related to the other missing dynamical ingredient, i.e., the quark confinement.

To have an indication on the role of confinement, let us consider the pressure as a function of energy density for the NJL model, as reported in Fig. 5. One can see that in the relevant range, since the chiral symmetry is restored, the model behaves like the MIT bag model (i.e., a free Fermi gas) with a bag constant $B_{\rm NJL}$ close to about 140 MeV fm^{-3}. It is well known that at such value of the bag constant the maximum mass within the MIT bag model is much smaller than the value at the maximum of the curves in Fig. 4. In general, the maximum mass of NS increases at decreasing value of B approximately as $B^{-1/2}$ [19].

It looks likely that the same mechanism is acting in the three flavor case. In fact we have seen that the new cut-off prescription $\Lambda = \Lambda(\mu)$ tends to soften the quark EoS with respect to the standard NJL. Since the latter displays already the NS instability discussed above even with three quark flavors [6], we can expect an enhancement of the instability in the present case. Within the NJL model the value of the effective bag constant $B_{\rm NJL}$ is dictated by the low energy phenomenology and it cannot be tuned. Indeed, the physical content of the model demands that the pressure in vacuum is zero, since there is no confinement, and then the constant pressure $B_{\rm NJL}$ above the chiral transition is necessarily present and determined uniquely by the parameters of the model [20]. If one adds by hand a confining potential which is switched off at the chiral phase transition [9], then of course the instability can be removed since the effective bag constant would be correspondingly reduced.

In any case, this shows that the instability is closely linked to the lack of confinement in the original NJL model.

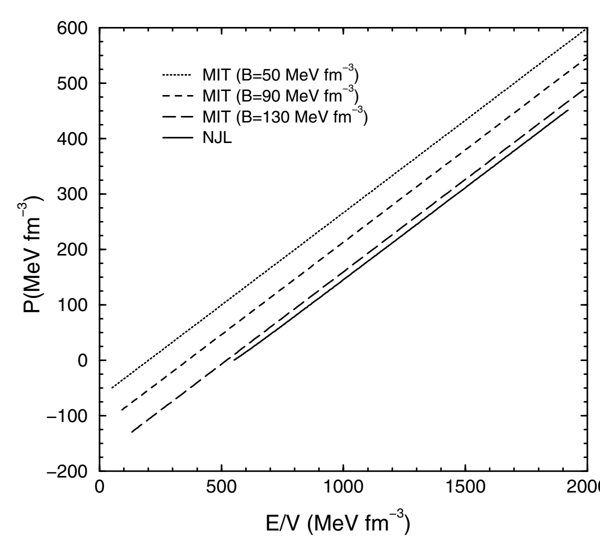

FIG. 5. The pressure is plotted as function of the energy density for the two-flavor case.

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