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Mass distribution from a quark matter equation of state

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We analyze the equation of state in terms of quasiparticles with continuously distributed mass. We seek for a description of the entire pressure-temperature curve at vanishing chemical potential in terms of a temperature independent mass distribution. We point out properties indicating a mass gap in this distribution, conjectured to be related to confinement.

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According to the proposal of Jaffe and Witten, a successful quantum Yang-Mills theory must have a mass gap [1]. In heavy ion collisions a deconfined phase is expected to form, and the produced quark-gluon plasma (QGP) is described by quantum chromodynamics (QCD). Thus one can expect the appearance of such a mass-gap in the spectral function of the basic QGP degrees of freedom, namely quarks and gluons. In this paper we perform a quantitative analysis on some results from lattice QCD on the equation of state (EoS) and reconstruct it from a mass distribution of noninteracting quasiparticles. We present strong indications for a mass gap in this distribution.

We have used earlier a mass distribution for massive quarks and developed a coalescence picture [2] to describe hadronization of deconfined quark matter, and reproduced final state hadron ratios and transverse spectra successfully. That model was based on an earlier coalescence model [3,4], where quarks and gluons had finite effective masses without any width. The apparent entropy reduction problem by coalescence with an associated reduction (confinement) of color degrees of freedom can be resolved by assuming sufficiently massive partons around the hadronization temperature in the precursor matter. The necessary mass scale for quarks is about 300–350 MeV and even higher (about 700 MeV) for gluons [5], thus we could assume that in the prehadronization stage the heavy gluons decay into quark-antiquark pairs [3]. Recently, partonic level models of heavy ion reactions also utilized the quark coalescence picture successfully [6,7].

Considering quark coalescence as the mechanism of hadronization one has to deal with the question, how to make a hadron with a mass lower than the sum of two parton masses. In order to solve this problem we have introduced distributed mass partons into our hadronization model [2]. Having in medium partons in quark matter as precursors of emerging hadrons in mind, we connect now the distributed mass parton picture to a simplified treatment of spectral functions.

The equation of state of an interacting system, when analyzed in terms of quasiparticles, is coded in the spectral function $\rho(\omega, \vec{p})$. Our ansatz to this assumes a particular form:

$$\rho(\omega, \vec{p}) = 2\pi \frac{w(m)}{2m} \Theta(m^2)(\Theta(\omega) - \Theta(-\omega)) \tag{1}$$

with $m^2 = \omega^2 - \vec{p}^2$ and $\Theta(x)$ being the step function. A continuous $w(m)$ mass distribution describes a finite width ansatz for the spectral function. The normalization of the spectral function, $\int_{-\infty}^{+\infty} \rho(\omega, \vec{p}) \omega d\omega/\pi = 1$, requires $\int_0^\infty w(m)\mathrm{dm} = 1$. The conventional quasiparticle approach on the other hand often explores a Breit-Wigner form of the spectral density [9],

$$\rho(\omega, \vec{p}) = \frac{\gamma}{E} \left(\frac{1}{(\omega - E)^2 + \gamma^2} - \frac{1}{(\omega + E)^2 + \gamma^2} \right) \tag{2}$$

with $E^2 = M^2 - \gamma^2 + \vec{p}^2$ and temperature dependent parameters $M(T)$ and $\gamma(T)$.

Thermodynamical consistency of the quasiparticle picture imposes constraints on the mass distribution, $w(m)$, in particular on its dependence on the temperature or on other medium parameters [8]. In this article we investigate the possibility of a temperature independent mass distribution and therefore neglect the mean field term for consistency. The total pressure at vanishing chemical potential is given as the following integral:

$$p(T) = \int_0^\infty w(m) p(m, T) dm. \tag{3}$$

One may suppose that only a single mass scale occurs in the mass distribution, so it can be expressed by a dimensionless distribution:

$$w(m) = \frac{1}{T_c} f\left(\frac{m}{T_c}\right). \tag{4}$$

The normalization integral for w is inherited by the shape (form factor) function $f(t)$:

$$\int_0^\infty w(m) dm = \int_0^\infty f(t) dt = 1. \tag{5}$$

The quark gluon plasma at vanishing chemical potential has the pressure

$$p(T) = \sigma(z) \kappa T^4, \tag{6}$$

with $z = T_c/T$ and κ being the Stefan-Boltzmann constant. In the Boltzmann approximation the fixed m-contributions are given by the Bessel K-function, $p(m, T) \propto T^4 \Phi(m/T)$ with [23] $\Phi(u) = u^2 K_2(u)/2$. Deviations in $\Phi(m/T) = p(m, T)/p(0, T)$ due to using Bose or Fermi distributions as a function of m/T never exceed 6% Thus in the distributed

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mass model the $\sigma(z)$ function in this approximation is given by the integral

$$\sigma(z) = \int_0^\infty f(t)\frac{(zt)^2}{2}K_2(zt)dt. \quad (7)$$

This may be recognized as the so-called Meijer K-transform [11,12] (a generalized Laplace transform) of the $f(t)$ function. The inverse of this transformation yields the mass distribution function in terms of the observed $\sigma(z)$ values:

$$f(t) = \frac{2}{i\pi}\int_{c-i\infty}^{c+i\infty}\sigma(z)\frac{I_2(zt)}{zt}dz. \quad (8)$$

This raises a peculiar question: is it possible to show, that to any $\sigma(z)$ function extracted from an equation of state (e.g., from lattice QCD calculations) there exists a unique mass distribution $f(t)$ with the mass scale parameter kept temperature and chemical potential independent? In this case the shape of the mass distribution is not arbitrary. Of course, the Meijer K-transform is invertible, but one has to check whether the $f(t)$ function obtained by Eq. (8) is positive semidefinite and normalized to unity. The normalization is the easier problem, the $\sigma(0)$ limit being directly the integral of the $f(t)$ function due to the small argument behavior of the Bessel K-function. It can, however, be difficult to arrive at a nowhere negative $f(t)$ by knowing $\sigma(z)$ only at some points on the real z-axis.

Before investigating any particular ansatz for $\sigma(z)$ let us consider an important general property. There is a relation between the integration moments of this quantity (the scaled pressure) and the normalized mass distribution, $f(t)$:

$$M_n = \int_0^\infty z^{n-1}\sigma(z)dz = I_n\int_0^\infty f(t)t^{-n}dt \quad (9)$$

with

$$I_n = \frac{1}{2}\int_0^\infty u^{n+1}K_2(u)du = 2^n\Gamma\left(2+\frac{n}{2}\right)\Gamma\left(\frac{n}{2}\right), \quad (10)$$

where $\Gamma(x)$ is Euler's Gamma function. This is finite for positive n and divergent for zero or negative integer values. We conclude that as long as the M_n moments of the EoS curve are finite so must be the inverse mass moments of the mass distribution. Since due to construction $\sigma(0) = 1$ and $\sigma(z)$ is rapidly decreasing due to confinement for large $z = T_c/T$ (low temperature), any mass distribution reconstructing the equation of state of QCD must be suppressed for low masses.

Let us now discuss how to obtain a particular functional form for $\sigma(z)$. The high-temperature (small z) expansion of $\Phi(zt)$ leads to

$$\frac{p(T)}{\kappa T^4} = 1 - \frac{\langle m^2\rangle}{4T^2} + \frac{\langle m^4\rangle}{16T^4}\left(\frac{3}{4}-\gamma\right) + \frac{\left\langle m^4 \ln\frac{2T}{m}\right\rangle}{16T^4} + \frac{\langle m^6\rangle}{192T^6}\left(\frac{17}{12}-\gamma\right) + \frac{\left\langle m^6 \ln\frac{2T}{m}\right\rangle}{192T^6} + \dots \quad (11)$$

with γ being the Euler-Mascheroni constant. Accidentally the perturbative QCD pressure (taken as the finite part at the scale $2\pi T$) shows a similar structure,

$$\frac{p(T)}{\kappa T^4} = 1 - a_2g^2 + a_4g^4 + b_4g^4\ln\frac{2\pi T}{\Lambda} + \dots \quad (12)$$

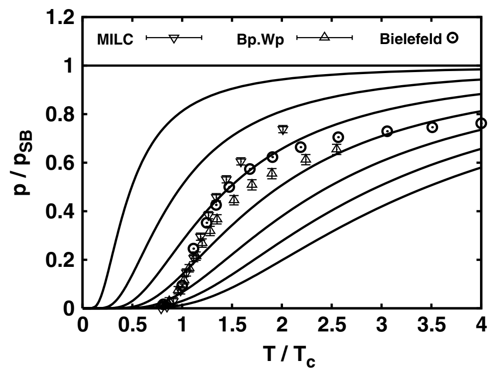

FIG. 1. The pressure normalized to the massless Stefan-Boltzmann value as a function of the scaled temperature T/T_c for different constant mass relativistic gases (full lines in order for $M/T_c = 0, 1, 2, 3, 4, 5, 6,$ and 7) and from lattice QCD data of Refs. [13] (down triangles), [14] (open circles), and [15] (up triangles).

($a_2 \approx 0.072, a_4 \approx 0.061, b_4 \approx 0.008$ for $N_f = 3$ based on Ref. [17]). It is possible to fit this form as a high-T asymptotics by assuming a scaling of expectation values, like $\langle m^2\rangle = cg^2T^2, \langle m^4\rangle = c'g^4T^4$, etc. This is the basis of the traditional quasiparticle picture [16], at the same time it predicts a width changing with the temperature. This view assumes a temperature-dependent mass distribution, $w(m, T)$, which—for the sake of thermodynamical consistency—would require a temperature-dependent mean field pressure, $-B(T)$ to be taken into account.

There is, however, another possibility, which we would like to pursue in the present article. The low-argument expansion Eq. (11) fails if the expectation values, like $\langle m^2\rangle, \langle m^4\rangle$, etc., are divergent. In fact this assumes a high-mass tail of the $w(m)$ distribution not decaying faster than m^{-3}. As we shall point out later, our numerical efforts to obtain $w(m)$ agree with this statement.

Figure 1 presents the normalized pressure for relativistic Boltzmann gases with several fixed masses. The lattice QCD EoS data of the Budapest-Wuppertal group [13] and of the Bielefeld group [14] seem to lie everywhere below the curve for the mass $M = 3T_c$, recent MILC data [15] below the curve for $M = 2.5T_c$. As a consequence, if one accepts this property also for lower temperatures where actually no reliable simulations are available, the mass spectrum $w(m)$ would not contain any mass lower than $M = 3T_c$ or $M = 2.5T_c$, respectively. This property can be verified by rigorous mathematical estimates of upper bounds for $w(m)$ on the interval $0 < m < M$ [22].

The pressure of hot QCD has been recently calculated up to $\mathcal{O}(g^6\ln(1/g))$ [17]. The result contains formally $\ln(2\pi T/\Lambda)$ terms, but according to the suggestion of the authors the coupling $g(\Lambda)$ should be taken at $\Lambda \approx 6.47T$ [18] and this way the temperature dependence of the normalized pressure stems from the temperature dependence of the coupling strength, renormalized relative to a scale proportional to the temperature. Agreement with lattice QCD eos data is achieved at the highest computable level only, with an extra fit of a constant which is not calculable perturbatively.

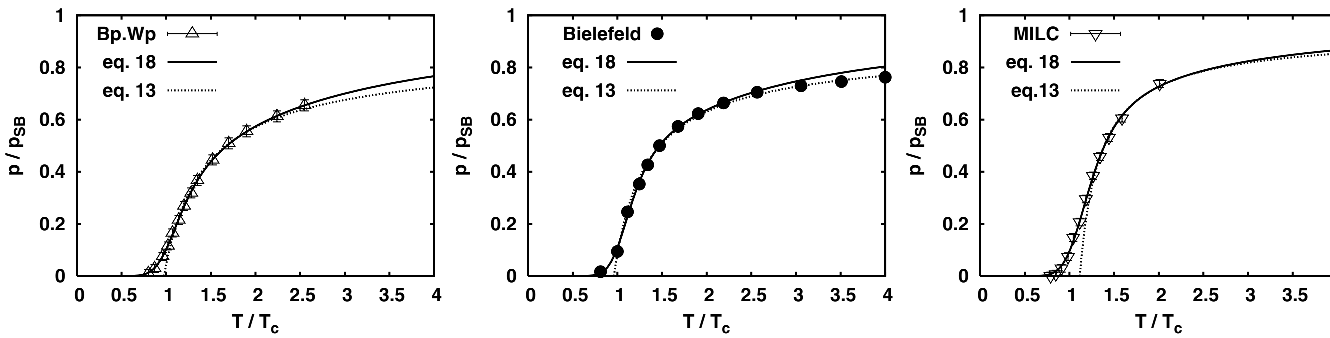

FIG. 2. The lattice QCD pressure normalized to the massless Stefan-Boltzmann value as a function of the temperature T from lattice EoS results of Refs. [13] (above), [14] (middle), and [15] (bottom). Our fits are indicated by the continuous lines, the $1 - K/\ln(\eta T/T_c)$-type fits by the dotted lines.

It is intriguing that for practical purposes the $\mathcal{O}(g^2)$ formula by using the one-loop renormalized $g(T) = 1/b\ln(T/\bar{\Lambda})$ can be also fitted to lattice data by fitting $\bar{\Lambda}$. This results in a formula

$$\frac{p(T)}{\kappa T^4} = 1 - \frac{K}{\ln(\eta T/T_c)} \tag{13}$$

reaching zero pressure at $T \approx T_c$. Fits to different lattice QCD equations of state leads to quantitative, but no qualitative differences. For the data of Ref. [13] we obtain $K = 0.54, \eta = 1.76$, for Ref. [14] $K = 0.43, \eta = 1.6$, and for Ref. [15] $K = 0.22, \eta = 1.12$ (cf. dotted lines on Fig. 2).

There are some theoretical signs on the other hand, that the $g(T) = 1/b\ln(T/\Lambda)$ formula is not necessarily the really high temperature limit prediction of QCD. The finite temperature renormalization group result is of type [10]

$$\frac{1}{\alpha(Q^2, T^2)} = b\ln\frac{Q^2}{Q_0^2} + c\left(\frac{T^2}{Q^2} - \frac{T^2}{Q_0^2}\right) \tag{14}$$

with $b = 1/\alpha(Q_0^2, T^2) + b_0$ and $b_0 = (11N_c/2 - 2N_f/3)/(4\pi)$ being the one-loop perturbative beta function coefficient. This is usually considered in the $Q_0^2 \gg T^2$ limit and then, assuming a sharp thermal distribution of Q^2 values, $Q^2 = (a\pi T)^2$ is taken. This leads to the conjecture

$$\frac{1}{\alpha((a\pi T)^2, T^2)} = \frac{1}{\alpha(T^2)} = b_0\ln\frac{T^2}{\Lambda^2}. \tag{15}$$

Since Q_0 was large and Λ is around T_c, also the coefficient a is taken as a large number. None of these assumptions is established by the QCD itself. The assumption $Q_0^2 \gg T^2$ contradicts the $T \to \infty$ limit, the spread of a thermal distribution of possible Q^2 values also increases like T^2, and finally there is always a non-negligible influence of low Q^2 physics on the coupling at any finite temperature. In fact calculating the thermal distribution of Q^2/T^2 between two massless, Boltzmann-distributed particles one obtains easily

$$P\left(\frac{Q^2}{T^2}\right) = \frac{1}{128}\left(\frac{Q^3}{T^3}K_1\left(\frac{Q}{T}\right) + 2\frac{Q^2}{T^2}K_2\left(\frac{Q}{T}\right)\right). \tag{16}$$

for $Q^2 > 0$. The probability of having $Q^2 = 0$ is finite at any temperature, $P(0) = 3/64$ and this is the maximum of $P(x)$. As a consequence higher twist effects which are not infrared safe (like $\ln Q^2$, $1/Q^2$, etc.) might destroy the often quoted logarithmic scaling of the coupling constant with temperature. It is probably the best to consider a $K/\ln(\eta T/T_c)$-type formula [Eq. (13)] as a standard, but not the only possible fit to the lattice EoS, with some parameters of nonperturbative origin.

In order to evaluate the inverse Meijer transform, Eq. (8), one has to approximate the lattice QCD data by an analytic $\sigma(z)$ function. A family of mass distributions can be Meijer-transformed analytically:

$$\begin{aligned} w(m) &= \frac{1}{\Gamma(\nu)\Gamma(2-\nu)}\frac{2\lambda}{m^3}(m^2-\lambda^2)^{1-\nu}\Theta(m-\lambda), \\ \sigma(T) &= \frac{2}{\Gamma(\nu)}\left(\frac{\lambda}{2T}\right)^{\nu}K_{\nu}\left(\frac{\lambda}{T}\right) \end{aligned} \tag{17}$$

with $0 < \nu < 2$. The $\nu = 2$ limit belongs to a Dirac-delta mass distribution, the $\nu = 0$ limit to the Bessel function K_0 with logarithmic asymptotics for high temperature (small argument). All these ansatze contain a mass-gap, the distributions being zero for $m < \lambda$. The $\nu = 1/2$ value leads to the particularly simple EoS: $\sigma = e^{-\lambda/T}$.

We found that an overall fit in the range of known lattice data is also achieved by the analytic ansatz

$$\sigma(z) = \exp(-\lambda z)\frac{1 + e^{-a/b}}{1 + e^{(z-a)/b}} \tag{18}$$

with $z = T_c/T$, $\lambda = 1.05$, $a = 0.90$, $b = 0.11$ for data from [13], $\lambda = 0.87$, $a = 0.90$, $b = 0.10$ for data from [14], and $\lambda = 0.56$, $a = 0.83$, $b = 0.10$ for data from [15]. We note that in Refs. [13,14] $T_c \approx 170$ MeV, but in Ref. [15] $T_c \approx 190$ MeV was taken. These fits are demonstrated in Fig. 2 where the different sets of lattice QCD data are compared with the fitted $\sigma(z) = p/p_{\rm SB}$ curves of Eq. (18).

In this article we investigate the lattice QCD EoS data of Refs. [13–15] closely, but they qualitatively agree with other results on this issue. The rise at moderately high temperatures (low z) cannot be accommodated by quantum statistical effects, but it can be characterized as the effect of an exponential factor $\exp(-\lambda T_c/T)$ in the range from T_c to $2.5T_c$ (cf. Fig. 2). While this moderately high-temperature behavior is well fitted by the pure exponential $\sigma(z)$ function, the part below T_c is more reduced. The $\sigma(z) = 1 - K/\ln(\eta/z)$ form is also able to fit $T > T_c$ data, but it goes to negative values at a finite temperature, which is unphysical. Our exponential fit is overall positive. A most satisfying extrapolation would interpolate between these two functions.

In Fig. 2 lattice data from Refs. [13] (a), [14] (b), and [15] (c) on $p/p_{\rm SB} = \sigma(z)$ as a function of the temperature T/T_c are plotted. In a one-loop resummed pQCD motivated approach, using a mass directly proportional to the temperature the approach to one is logarithmic, $1 - K/\ln(\eta T/T_c)$ (dotted lines). The exponential behavior, on the other hand, supports the presence of a lowest mass in high-temperature QCD. While this fact depends on the low-temperature drop of the pressure curve, it is not easy to consolidate the effect due to quantitatively different pressure curves presented by different lattice QCD calculations. Although we do not intend to review lattice QCD EoS calculations in this paper, we note that the investigated simulations differ in the corresponding value of the physical pion mass ($m_\pi \approx 140$ MeV for [13], $m_\pi \approx 540$ MeV for [14], and $m_\pi \approx 300$ MeV for [15]). There can be further differences of technical nature, which we do not feel to be able to comment on. In our further analysis we choose the data of the Budapest-Wuppertal group [13] to seek for a corresponding mass distribution, but of course the same exercise can be done for other sets of pressure data, too.

To evaluate the integral given by Eq. (8), we choose a simple path parallel to the imaginary z axis, $z = c + i\omega$. With numerical integration we obtain an $f(t)$ mass distribution shown in Fig. 3 by full boxes. Fluctuations at small masses are due to limitations of the applied numerical method. The part of the mass distribution shown here reconstructs the $T > T_c$ part of the pressure curve nicely, but it fails to approximate the pressure at $T < T_c$. In the following we seek to understand this phenomenon.

One can obtain simple analytic approximations for the $f(t)$ function by expanding the expression for $\sigma(z)$, Eq. (18). However, requiring a convergent expansion, one arrives at two distinct series expansions: one for $z < a$ and another one for $z > a$,

$$\begin{aligned} \sigma(z) &= e^{-z\lambda} + \dots \quad \text{for} \quad z < a, \\ \sigma(z) &= e^{-z(\lambda+1/b)}(1 + e^{a/b}) + \dots \quad \text{for} \quad z > a. \end{aligned} \tag{19}$$

The inverse Meijer K-transform of the simple exponential, exp($-\lambda z$), can be given based on an analytically known integral [cf. Eq. (17) for $\nu = 1/2$]:

$$f(t) = \frac{4\lambda}{t^2\pi}\sqrt{1 - \frac{\lambda^2}{t^2}}. \tag{20}$$

The above expression is valid for $t \geqslant \lambda$, for smaller $t = m/T_c$ values $f(t)$ is identically zero. Hence the t-integration in the Meijer K-transform, when determining the pressure contribution, starts at $t = \lambda$. Physically this corresponds to a lowest mass in the continuous spectrum, to a mass gap. Since both the approximations to $T \leqslant T_c$ and to $T \geqslant T_c$ parts of the pressure contain a leading exponential factor [λ and $\lambda + 1/b$, respectively, cf. Eq. (19)], EoS data seem to support a lowest value of a continuous mass spectrum both in moderately low and moderately high temperature quark matter (see the full line in Fig. 3).

Actually, requiring $z > a$ is equivalent to a Hagedorn limiting temperature $T_H = T_c/a$, and in fact transforms back nearly to an exponentially rising mass spectrum part. In this regime the QCD matter also has been fitted by a hadron resonance gas [19].

Substituting the respective $f(t)$-s for $T < T_c$ and $T > T_c$ into Eq. (20) we calculate the pressure from Eq. (7). These two curves are shown in Fig. 4, together with the lattice QCD results of Ref. [13]. A numerical method designed to obtain an overall non-negative (probability like) $f(t)$ distribution, which fits well some $\sigma(z_k) = s_k$ points, is represented by the maximum entropy method (MEM). We applied this method to the lattice QCD EoS data discussed in this paper in order to obtain a mass distribution: both by using a MEM program designed to invert the Meijer K-transform and also by searching numerically for the inverse Laplace transform of $\sigma(z)$ first. We failed, however, to obtain better numerical results then by evaluating the complex integral Eq. (8) as discussed above.

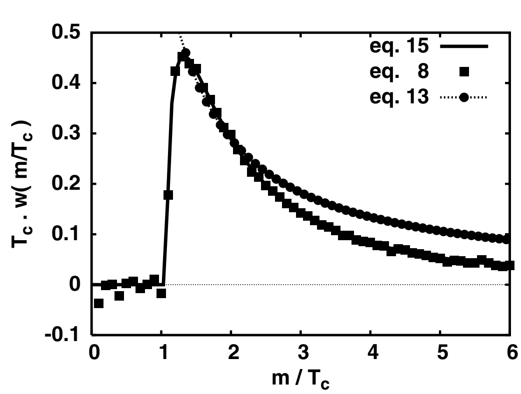

FIG. 3. The mass distribution function obtained by evaluating the complex integral from Eq. (8) (boxes) and using the analytic fit Eq. (18) to lattice QCD EoS data of Ref. [13]. The full line corresponds to Eq. (20), circles show the curve obtained by using the $1 - K/\ln(\eta T/T_c)$ type fit.

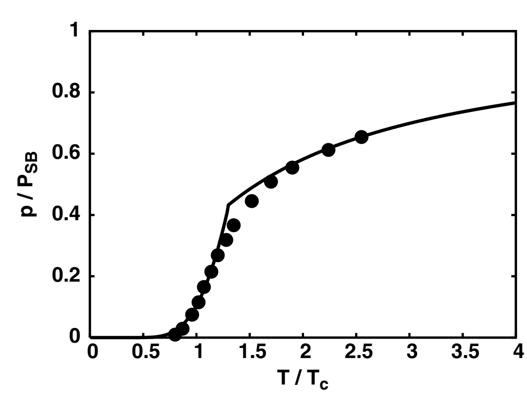

FIG. 4. The normalized lattice QCD pressure and the pressure fitted to convergent series expansions, Eq. (19), obtained by numerically re-integrating $f(t)$ functions given by Eq. (20).

For calculating quark number susceptibilities or higher order Taylor coefficients the Boltzmann approximation becomes unreliable; starting at the fourth order the Boltzmannian term no longer dominates the Fermi distribution. Experience with the hadronic resonance gas model supports the expectation that dependence on chemical potential also can be interpreted in terms of clustered but noninteracting components [19–21].

In conclusion, we have analyzed lattice QCD pressure data in terms of a continuous, temperature independent mass distribution. We find a strong indication for a finite mass gap in such quasiparticle models, the details depending on the low temperature behavior of the pressure curve. Since all simulation data are below the $M = 2.5T_c$ curve, the immediate conclusion would be that the $p(T)$ curve can be fitted by components with higher mass only. Allowing for a milder drop of the pressure at low temperature the lowest mass may be lower, we presented an example with $M \approx \lambda T_c$ with fitted λ-values near to one. In general for any $p(T)$ curve showing finite T^{n+1}-weighted integrals for p/p_0 the low-m behavior of $w(m)$ is restricted by finite integrals of $m^{-n}w(m)$. Since a single-mass $p(T)/p_0$ curve cannot fit the lattice QCD equation of state obtained by any of the groups calculating it, these data demand a finite width mass distribution.

For the physical problem of quark matter we have learned from the above analyses that either the mass distribution is temperature dependent and then the thermodynamical description is rather complex then, or there is a mass gap compatible to the equation of state unless the pressure rises again at low temperatures (where we have presently no simulation data, but the idea of a noninteracting pion gas would correspond to a pressure higher than zero).

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