

**Exclusive and inclusive muon pair production in collisions of relativistic nuclei**

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The exclusive production of one  $\mu^+\mu^-$  pair in collisions of two ultrarelativistic nuclei is considered. We present a simple method for the calculation of the Born cross section for this process based on an improved equivalent photon approximation. We find that the Coulomb corrections to this cross section (corresponding to multiphoton exchange of the produced  $\mu^\pm$  with the nuclei) are small, whereas the unitarity corrections (corresponding to the exchange of light-by-light blocks between nuclei) are large. This is in sharp contrast to the exclusive  $e^+e^-$  pair production where the Coulomb corrections to the Born cross section are large, whereas the unitarity corrections are small. We calculate also the cross section for the production of one  $\mu^+\mu^-$  pair and several  $e^+e^-$  pairs in the leading logarithmic approximation. Using this cross section we find that the inclusive production of  $\mu^+\mu^-$  pair coincides in this approximation with its Born value.

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**I. INTRODUCTION**

Lepton pair production in ultrarelativistic nuclear collisions were discussed in numerous articles (see Ref. [1] for a review and references therein). For the Relativistic Heavy Ion Collider (RHIC) and CERN Large Hadron Collider (LHC) the charge numbers of nuclei  $Z_1 = Z_2 \equiv Z$  and their Lorentz factors  $\gamma_1 = \gamma_2 \equiv \gamma$  are given in Table I.

The cross section of one  $e^+e^-$  pair production in Born approximation, described by the Feynman diagram of Fig. 1, was obtained many years ago [2]. Because the Born cross section  $\sigma_{\text{Born}}^{e^+e^-}$  is huge (see Table I), the  $e^+e^-$  pair production can be a serious background for many experiments. It is also an important issue for the beam lifetime and luminosity of these colliders [3]. It means that various corrections to the Born cross section, as well as the cross section for  $n$ -pair production, are of great importance. At present, there are a lot of controversial and incorrect statements in articles devoted to this subject. The corresponding references and critical remarks can be found in Refs. [1,4,5].

Because the parameter  $Z\alpha$  may be not small ( $Z\alpha \approx 0.6$  for Au-Au and Pb-Pb collisions), the whole series in  $Z\alpha$  has to be summed to obtain the cross section with sufficient accuracy. The exact cross section for one pair production  $\sigma_1$  can be split into the form

$$\sigma_1 = \sigma_{\text{Born}} + \sigma_{\text{Coul}} + \sigma_{\text{unit}}, \quad (1)$$

where two different types of corrections need to be distinguished. The Coulomb correction  $\sigma_{\text{Coul}}$  corresponds to multiphoton exchange of the produced  $e^\pm$  with the nuclei (Fig. 2); it was calculated in Ref. [4]. The unitarity correction  $\sigma_{\text{unit}}$  corresponds to the exchange of light-by-light blocks between nuclei (Fig. 3); it was calculated in [5]. It was found in Refs. [4] and [5] that the Coulomb corrections are large,

whereas the unitarity corrections are small (see Table II). The results of Ref. [5] were confirmed recently in Ref. [6] by a direct summation of the Feynman diagrams.

In this article we present our calculations related to the exclusive and inclusive muon pair production. This process may be easier to observe experimentally than  $e^+e^-$  pair production described above. It should be stressed that the calculation scheme, as well as, the final results for the  $\mu^+\mu^-$  pair production are quite different than those for the  $e^+e^-$  pair production.

In the next section we calculate the Born cross section for one  $\mu^+\mu^-$  pair production using the improved equivalent photon approximation with an accuracy about 5%. In Sec. III we present the Coulomb and unitarity corrections to the exclusive production of one muon pair. In this section we also obtain the probability for the production of one muon and several electron pairs in collisions of nuclei at a given impact parameter  $\rho$ . This result allows us to calculate the inclusive production of one muon pair in Sec. IV. In Sec. V we calculate the cross section for two muon pair production in lowest order. The conclusion of the article is given in Sec. VI. In the Appendix we consider the relatively simple process of one muon pair production by a real photon off a nucleus  $\gamma Z \rightarrow Z\mu^+\mu^-$ , which serves as a good test of the approach used. A short preliminary version of this article was published in Ref. [7].

**II. BORN CROSS SECTION FOR ONE  $\mu^+\mu^-$  PAIR PRODUCTION**

The production of one  $\mu^+\mu^-$  pair

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^- \quad (2)$$

TABLE I. Colliders and cross sections for the lepton pair production.

Collider	Z	$\gamma$	$\sigma_{\text{Born}}^{e^+e^-}$ (kb)	$\sigma_{\text{Born}}^{\mu^+\mu^-}$ (b)
RHIC, Au-Au	79	108	36.0	0.23
LHC, Pb-Pb	82	3000	227	2.6
LHC, Ar-Ar	18	3400	0.554	0.0082

in the Born approximation is described by the Feynman diagram of Fig. 1. When two nuclei with charges  $Z_1e$  and  $Z_2e$  and four-momenta  $P_1$  and  $P_2$  collide with each other, they emit equivalent (virtual) photons with the four-momenta  $q_1, q_2$ , energies  $\omega_1, \omega_2$  and their virtualities  $Q_1^2 = -q_1^2, Q_2^2 = -q_2^2$ . Upon fusion, these photons produce a  $\mu^+\mu^-$  pair with the total four-momentum  $q_1 + q_2$  and the invariant mass squared  $W^2 = (q_1 + q_2)^2$ . In addition to this we denote

$$(P_1 + P_2)^2 = 4E^2 = 4M^2\gamma^2, \quad \alpha \approx 1/137$$

and use the system of units in which  $c = 1$  and  $\hbar = 1$ .

### A. General formulae

The Born cross section of the process [Eq. (2)] can be calculated to a good accuracy using the equivalent photon approximation (EPA) in the improved variant presented, for example, in Ref. [8]. Let the numbers of equivalent photons be  $dn_1$  and  $dn_2$ . The most important contribution to the production cross section stems from photons with very small virtualities  $Q_i^2 \ll \mu^2$  where  $\mu$  is the muon mass. Therefore to a good approximation, the photons move in opposite directions, and  $W^2 \approx 4\omega_1\omega_2$ . In this very region the Born differential cross section  $d\sigma_B$  for the process considered is related to the cross section  $\sigma_{\gamma\gamma}$  for the process with real photons:  $\gamma\gamma \rightarrow \mu^+\mu^-$  by the equation

$$d\sigma_B = dn_1 dn_2 d\sigma_{\gamma\gamma}(W^2), \quad W^2 \approx 4\omega_1\omega_2. \quad (3)$$

The number of equivalent photons are (see Eq. (D.4) in Ref. [8])

$$dn_i(\omega_i, Q_i^2) = \frac{Z_i^2 \alpha}{\pi} \left(1 - \frac{\omega_i}{E_i}\right) \frac{d\omega_i}{\omega_i} \times \left(1 - \frac{Q_{i\text{min}}^2}{Q_i^2}\right) F^2(Q_i^2) \frac{dQ_i^2}{Q_i^2}, \quad (4)$$

where

$$Q_i^2 \geq Q_{i\text{min}}^2 = \frac{\omega_i^2}{\gamma^2} \quad (5)$$

TABLE II. Coulomb and unitarity corrections to the  $e^+e^-$  pair production.

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$
RHIC, Au-Au	-25%	-4.1%
LHC, Pb-Pb	-14%	-3.3%
LHC, Ar-Ar	-1.06%	-0.025%

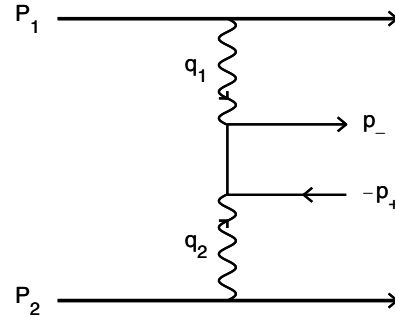


FIG. 1. The Feynman diagram for the lepton pair production in the Born approximation.

and  $F(Q^2)$  is the electromagnetic form factor of the nucleus. It is important that the integral over  $Q^2$  converges rapidly for  $Q^2 > 1/R^2$ , where

$$R = 1.2 A^{1/3} \text{fm} \quad (6)$$

is the radius of the nucleus with  $A \approx M/m_p$  the number of nucleons ( $R \approx 7$  fm,  $1/R \approx 28$  MeV for Au and Pb). Because  $Q_{\text{min}}^2 \lesssim 1/R^2$ , the main contribution to the cross section is given by virtual photons with energies

$$\omega_i \lesssim \gamma/R. \quad (7)$$

Therefore, we can use the spectrum of equivalent photons neglecting terms proportional to  $\omega_i/E_i$  given by:

$$dn_i(\omega_i, Q_i^2) = \frac{Z_i^2 \alpha}{\pi} \frac{d\omega_i}{\omega_i} \left(1 - \frac{\omega_i^2}{\gamma^2 Q_i^2}\right) F^2(Q_i^2) \frac{dQ_i^2}{Q_i^2}. \quad (8)$$

After the transformation

$$\frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} = \frac{d\omega_1}{\omega_1} \frac{dW^2}{W^2} \quad (9)$$

we cast the cross section in the form

$$d\sigma_B = \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \frac{d\omega_1}{\omega_1} \left(1 - \frac{\omega_1^2}{\gamma^2 Q_1^2}\right) F^2(Q_1^2) \frac{dQ_1^2}{Q_1^2} \times \left(1 - \frac{\omega_2^2}{\gamma^2 Q_2^2}\right) F^2(Q_2^2) \frac{dQ_2^2}{Q_2^2} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2), \quad (10)$$

where  $\omega_2 \approx W^2/(4\omega_1)$ .

### B. Leading logarithmic approximation (LLA)

Before using the calculation scheme above, it is instructive to present a rougher but simpler approximation—the so-called leading logarithmic approximation (LLA). In the LLA, the equivalent photon spectrum as a function of photon

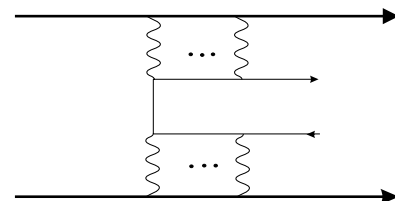


FIG. 2. The Feynman diagram for the Coulomb correction.

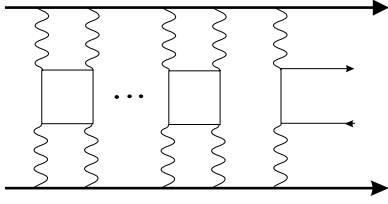


FIG. 3. The Feynman diagram for the unitarity correction.

energy  $dn_i(\omega_i)$  is obtained after integrating  $dn_i(\omega_i, Q_i^2)$  over  $Q_i^2$  in the region between

$$Q_{i\min}^2 \leq Q_i^2 \lesssim 1/R^2 \quad (11)$$

which leads to

$$dn_i(\omega_i) \approx \frac{Z_i^2 \alpha}{\pi} \ln \frac{\gamma^2}{(R\omega_i)^2} \frac{d\omega_i}{\omega_i}. \quad (12)$$

The restriction  $Q_{i\min}^2 \lesssim 1/R^2$  corresponds to the integration interval

$$a = \frac{W^2 R}{4\gamma} \lesssim \omega_1 \lesssim b = \frac{\gamma}{R}, \quad (13)$$

which gives

$$\sigma_B^{\text{LLA}} = \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) \int_a^b \frac{d\omega_1}{\omega_1} \ln \frac{b^2}{\omega_1^2} \ln \frac{\omega_1^2}{a^2}. \quad (14)$$

Because  $\sigma_{\gamma\gamma}(W^2) \approx (4\pi\alpha^2/W^2) \ln(W^2/\mu^2)$  for large values of  $W \gg \mu$ , the main contribution to the Born cross section comes from the region of small values of  $W$  near the threshold. Therefore, within logarithmic accuracy we replace  $W$  by some fixed value  $W_0 \sim 2\mu$  in the lower bound  $a$ . After that the integral over  $W^2$  gives

$$I = \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) = \frac{14\pi\alpha^2}{9\mu^2} \quad (15)$$

and further integration over  $\omega_1$  leads to

$$\int_a^b \frac{d\omega_1}{\omega_1} \ln \frac{b^2}{\omega_1^2} \ln \frac{\omega_1^2}{a^2} = \frac{2}{3} L^3, \quad (16)$$

where

$$L = \ln \frac{\gamma^2}{(W_0 R/2)^2}. \quad (17)$$

As a result, we obtain

$$\sigma_B^{\text{LLA}} = \frac{28}{27\pi} \frac{(Z_1 \alpha Z_2 \alpha)^2}{\mu^2} L^3 \quad (18)$$

in accordance with the result of Landau and Lifshitz [2]. The accuracy of the LLA depends on the choice of the value for  $W_0$ . If we use for numerical estimations  $W_0 = 3\mu$ , then the accuracy of the LLA for the colliders discussed is about 15%.

The same result can be obtained in the framework of the impact parameter-dependent representation, which will also be useful later. For this aim we introduce the probability for muon pair production  $P_B(\rho)$  in the collision of two nuclei at a fixed impact parameter  $\rho$ . For  $\gamma \gg 1$ , it is possible to treat

the nuclei as sources of external fields and to calculate  $P_B(\rho)$  analytically using the same approach as in Ref. [5]. The Born cross section  $\sigma_B$  can then be obtained by the integration of  $P_B(\rho)$  over the impact parameter:

$$\sigma_B = \int P_B(\rho) d^2\rho. \quad (19)$$

We calculate this probability in the LLA using Eq. (3) with

$$dn_i = \frac{Z_i^2 \alpha}{\pi^2} \frac{d\omega_i}{\omega_i} \frac{d^2\rho_i}{\rho_i^2}; \quad \omega_i \ll \frac{\gamma}{R}; \quad R \ll \rho_i \ll \frac{\gamma}{\omega_i}, \quad (20)$$

where  $\rho_i$  is the impact parameter of  $i$ -th equivalent photon with respect to the  $i$ -th nucleus. This allows us to write the above probability in the form

$$\begin{aligned} P_B(\rho) &= \int dn_1 dn_2 \delta(\rho_1 - \rho_2 - \rho) \sigma_{\gamma\gamma}(W^2) \\ &= \frac{28}{9\pi^2} \frac{(Z_1 \alpha Z_2 \alpha)^2}{(\mu\rho)^2} \Phi(\rho). \end{aligned} \quad (21)$$

Depending on the value of  $\rho$  two different forms for  $\Phi(\rho)$  need to be used:

$$\Phi(\rho) = \left( 4 \ln \frac{\gamma}{\mu\rho} + \ln \frac{\rho}{R} \right) \ln \frac{\rho}{R} \text{ for } R \ll \rho \leq \frac{\gamma}{\mu}, \quad (22)$$

$$\Phi(\rho) = \left( \ln \frac{\gamma^2}{\mu^2 \rho R} \right)^2 \text{ for } \frac{\gamma}{\mu} \leq \rho \ll \frac{\gamma^2}{\mu^2 R}. \quad (23)$$

Note that the function  $\Phi(\rho)$  is continuous at  $\rho = \gamma/\mu$  together with its first derivative. As expected the integration of  $P_B(\rho)$  over  $\rho$  in the region  $R < \rho < \gamma^2/(\mu^2 R)$  gives back the result in Eq. (18).

To prove Eqs. (21)–(23), we make the transformation given in Eq. (9) together with the integration over  $W^2$  according to Eq. (15). This gives

$$\begin{aligned} P_B(\rho) &= \frac{14}{9\pi^3} \frac{(Z_1 \alpha Z_2 \alpha)^2}{\mu^2} \int_{\mu^2 R/\gamma}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int \frac{d^2\rho_1}{\rho_1^2 (\rho - \rho_1)^2} \\ &\quad \times \vartheta \left( \frac{\gamma}{\omega_1} - \rho_1 \right) \vartheta \left( \frac{\gamma\omega_1}{\mu^2} - |\rho - \rho_1| \right), \end{aligned} \quad (24)$$

where  $\vartheta(x)$  is the step function. The main contribution to this integral is given by two regions:  $R \ll \rho_1 \ll \rho$  and  $R \ll |\rho - \rho_1| = \rho_2 \ll \rho$ . Moreover, the two regions in the  $\omega_1$  integration with  $\mu < \omega_1 < \gamma/R$  and  $\mu^2 R/\gamma < \omega_1 < \mu$  give the same contributions. As a result we get

$$\begin{aligned} \Phi(\rho) &= 2(J_1 + J_2), \\ J_1 &= \int_{\mu}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} \vartheta(\gamma/\omega_1 - \rho_1) \vartheta(\gamma\omega_1/\mu^2 - \rho), \\ J_2 &= \int_{\mu}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_2}{\rho_2} \vartheta(\gamma/\omega_1 - \rho) \vartheta(\gamma\omega_1/\mu^2 - \rho_2). \end{aligned} \quad (25)$$

Next we consider the two regions of  $\rho$ .

In the region of relatively small impact parameters,  $R \ll \rho \leq \gamma/\mu$ , the second step function does not impose any

limitations, therefore,

$$\begin{aligned} J_1 &= \int_{\mu}^{\gamma/\rho} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} + \int_{\gamma/\rho}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\gamma/\omega_1} \frac{d\rho_1}{\rho_1} \\ &= \ln \frac{\gamma}{\mu\rho} \ln \frac{\rho}{R} + \frac{1}{2} \left( \ln \frac{\rho}{R} \right)^2 \\ J_2 &= \int_{\mu}^{\gamma/\rho} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} = \ln \frac{\gamma}{\mu\rho} \ln \frac{\rho}{R}. \end{aligned}$$

Summing up, we obtain (22).

In the region of relatively large impact parameters,  $\gamma/\mu \leq \rho \ll \gamma^2/(\mu^2 R)$ , we have

$$J_1 = \int_{\mu^2\rho/\gamma}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\gamma/\omega_1} \frac{d\rho_1}{\rho_1} = \frac{1}{2} \left( \ln \frac{\gamma^2}{\mu^2 R \rho} \right)^2, \quad J_2 = 0,$$

therefore, the sum gives (23).

We compare Eqs. (21)–(23) for  $\Phi(\rho)$  with the numerical calculations based on the exact matrix element calculated with the approach as outlined in Ref. [9]. We find good agreement for Pb-Pb collisions: the discrepancy is less than 10% at  $\mu\rho > 10$  and it is less than 15% at  $\mu\rho > 2\mu R = 7.55$ .

### C. More refined calculation

In the calculation below we use for the form factor of the nucleus the simple approximation of a monopole form factor, which corresponds to an exponentially decreasing charge distribution, whose mean squared radius  $\sqrt{\langle r^2 \rangle}$  is adjusted to the experimental value:

$$F(Q^2) = \frac{1}{1 + Q^2/\Lambda^2}, \quad \Lambda^2 = \frac{6}{\langle r^2 \rangle}. \quad (26)$$

For lead and gold, the parameter is  $\Lambda \approx 80$  MeV. This approximate form of the form factor enables us to perform some calculations analytically, which otherwise could only be done numerically.

The equivalent photon spectrum  $dn_i(\omega_i)$  is obtained after integrating  $dn_i(\omega_i, Q_i^2)$  over  $Q_i^2$  (the upper limit of this integration can be set to be equal to infinity in a good approximation, due to the fast convergence of the integral at  $Q^2 > \Lambda^2$ ):

$$dn_i(\omega_i) = \frac{Z_i^2 \alpha}{\pi} f\left(\frac{\omega_i}{\Lambda\gamma}\right) \frac{d\omega_i}{\omega_i}. \quad (27)$$

Here the function

$$f(x) = (1 + 2x^2) \ln\left(\frac{1}{x^2} + 1\right) - 2 \quad (28)$$

is large for small values of  $x$ ,

$$f(x) \approx \ln \frac{1}{x^2} - 2 = \ln \frac{1}{(ex)^2} \quad \text{at } x \ll 1, \quad (29)$$

but drops very quickly for large  $x$  in accordance with Eq. (7):

$$f(x) < \frac{1}{6x^4} \quad \text{for } x > 1. \quad (30)$$

Finally we obtain

$$\begin{aligned} \sigma_B &= \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} G(W^2) \sigma_{\gamma\gamma}(W^2) \\ &= \frac{(Z_1 \alpha Z_2 \alpha)^2}{\pi \mu^2} J(\gamma\Lambda/\mu), \end{aligned} \quad (31)$$

where

$$G(W^2) = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{\omega} f\left(\frac{\omega}{\Lambda\gamma}\right) f\left(\frac{W^2}{4\Lambda\gamma\omega}\right). \quad (32)$$

Because  $\omega_i < E$  and  $\omega_1\omega_2 \sim \mu^2$  we have  $\omega_{\min} \sim \mu^2/E$  and  $\omega_{\max} = E$ . However, due to the fast decrease of  $f(x)$  for  $x > 1$  one can extend these limits up to  $\omega_{\min} = 0$  and  $\omega_{\max} = \infty$  without any lack of accuracy, therefore,

$$G(W^2) = 2 \int_0^{\infty} f(x_1) f(x_2) dy, \quad x_{1,2} = \frac{W}{2\Lambda\gamma} e^{\pm y}. \quad (33)$$

A numerical evaluation of the integrals in Eqs. (31)–(33) yields the function  $J(\gamma\Lambda/\mu)$  presented in Fig. 4 and cross sections  $\sigma_{\text{Born}}^{\mu^+\mu^-}$  given in Table I.

Corrections to the photon spectrum are represented by terms in Eqs. (3) and (8) of the order of  $Q_i^2/W^2$  (see Eqs. (E.1) in Ref. [8]), which are dropped before the integration over  $Q_i^2$  is done. After the integration with weight  $1/Q_i^2$  the relative value of these corrections becomes of the order of

$$\eta_1 = \frac{\Lambda^2}{W^2 L}. \quad (34)$$

Thus, for the collisions considered here one can estimate the accuracy of the calculations on the level  $\eta_1 \sim 5\%$ . Another test of accuracy of the approach used is given in Appendix.

Note that in the LLA the function  $G(W^2)$  is just

$$G^{\text{LLA}}(W^2) = \frac{2}{3} \left[ \ln \frac{\gamma^2}{(eW/2\Lambda)^2} \right]^3 \approx \frac{2}{3} \left[ \ln \frac{\gamma^2}{(WR/2)^2} \right]^3 \quad (35)$$

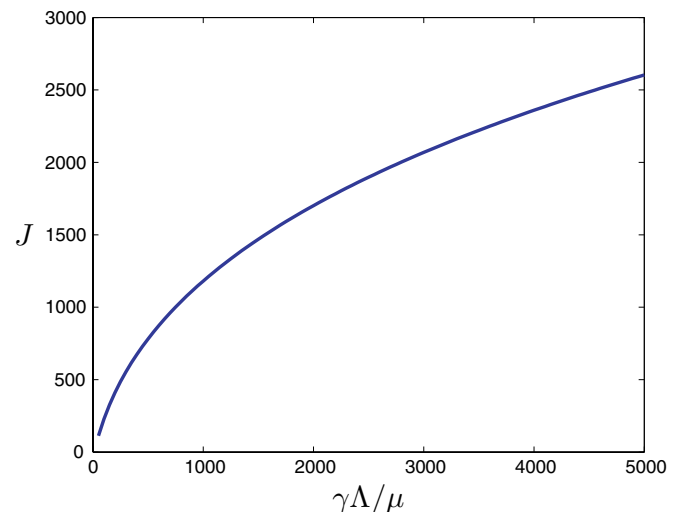


FIG. 4. (Color online) The function  $J(\gamma\Lambda/\mu)$  from Eq. (31).

in accordance with Eq. (16) (taking into account that  $\Lambda/e \approx 1/R$ ).

### III. COULOMB AND UNITARITY CORRECTIONS

The Coulomb corrections correspond to Feynman diagram of Fig. 2. In the case of the muon pair production by exchange photons we can neglect these corrections because of a dipole interaction of the photon field with the muon pair. Indeed, the form factor  $F(\mathbf{k}_{i\perp}^2)$  cuts out all exchange photon momenta  $\mathbf{k}_{i\perp}^2 > 1/R^2$ . Therefore, the “remaining” long-wave photons with  $\mathbf{k}_{i\perp}^2 \lesssim 1/R^2$  cannot resolve a dipole formed by a muon pair, with a typical space separation of the pair components of the order of  $1/\mu$ . For this reason, any additional photon exchange brings in an additional suppression factor  $\sim 1/(R\mu)^2$  in the cross section. In another words, due to the restriction of the transverse momenta of additionally exchanged photons to the range below  $1/R$ , the effective parameter of the perturbation series is not  $(Z\alpha)^2$  but  $(Z\alpha)^2/(R\mu)^2$ . In addition, the contribution of the additional exchange photons is suppressed by a logarithmic factor, because the cross section for two-photon production mechanism (the Born cross section) is proportional to  $L^3$ , whereas the cross section for the multiple-photon production mechanism is proportional only to  $L^2$ . Therefore, the real parameter describing the suppression of the Coulomb correction is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}, \quad (36)$$

which corresponds to Coulomb corrections of less than 1%. The example considered in Appendix confirms this estimate.

The unitarity correction  $\sigma_{\text{unit}}$  to the one muon pair production corresponds to the exchange of light-by-light blocks between the two nuclei (Fig. 3). We start with a more general process—the production of one  $\mu^+\mu^-$  pair and  $n$  electron-positron pairs ( $n \geq 0$ ) in a collision of two ultrarelativistic nuclei

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^- + n(e^+e^-) \quad (37)$$

taking into account the unitarity corrections, which correspond to the exchange of the blocks of light-by-light scattering via the virtual lepton loops. The corresponding cross section  $d\sigma_{1+n}$  can be calculated by a simple generalization of the results obtained in Ref. [6] for the  $n$ -pair process without muon pair production:  $Z_1 + Z_2 \rightarrow Z_1 + Z_2 + n(e^+e^-)$ . This multiple pair production process was studied in a number of articles; see [1] for a review. It was found that the probability is to a good approximation given by a Poisson distribution with the deviations found to be small. Indeed, it is not difficult to show that the basic equations for the latter process should be modified as follows. In Eq. (26) of Ref. [6] the additional factor

$$\tilde{B}^\mu(\boldsymbol{\rho}, \mathbf{r}_{n+1}) e^{-L[A^\mu(\boldsymbol{\rho})/2 + i\varphi^\mu(\boldsymbol{\rho})]} \quad (38)$$

appears under the integral, where  $L = \ln(\gamma_1\gamma_2)$  and the functions  $\tilde{B}^\mu$ ,  $A^\mu$ , and  $\varphi^\mu$  are the same as the functions  $\tilde{B}$ ,  $A$ , and  $\varphi$  in Eq. (27) of Ref. [6] but with the replacement of electrons by muons. As a result, Eq. (31) of Ref. [6] is

replaced by

$$\frac{d\sigma_{1+n}}{d^2\rho} = LA_1^\mu(\rho) \frac{[LA_1(\rho)]^n}{n!} e^{-LA_1^\mu(\rho) - LA_1(\rho)}, \quad (39)$$

where  $LA_1^\mu(\rho) \approx P_B(\rho)$  is the probability for one muon pair production in the Born approximation, as discussed in Sec. II B. In the region of interest,  $\rho > 2R$ , the function  $A_1^\mu(\rho)$  is small,

$$LA_1^\mu(\rho) \ll 1, \quad A_1^\mu(\rho) \ll A_1(\rho), \quad (40)$$

therefore, we can rewrite Eq. (39) in the simpler form

$$\frac{d\sigma_{1+n}}{d^2\rho} = P_{1+n}(\rho), \quad P_{1+n}(\rho) = P_B(\rho) \frac{[\bar{n}_e(\rho)]^n}{n!} e^{-\bar{n}_e(\rho)}, \quad (41)$$

where  $\bar{n}_e(\rho) = LA_1(\rho)$  is the average number of  $e^+e^-$  pairs produced in collisions of the two nuclei at a given impact parameter  $\rho$ . The result that the probabilities for the different processes factorizes is due to the independence of the individual processes. For a general discussion of the validity of this factorization together with possible violations we refer to Ref. [10].

In particular, we get the cross section for the exclusive one  $\mu^+\mu^-$  pair production, including the unitarity correction, as

$$\sigma_{1+0} = \int P_B(\rho) e^{-\bar{n}_e(\rho)} d^2\rho. \quad (42)$$

This expression can be rewritten in the form

$$\sigma_{1+0} = \sigma_B + \sigma_{\text{unit}}, \quad (43)$$

where

$$\sigma_B = \int P_B(\rho) d^2\rho \quad (44)$$

is the Born cross section discussed in Sec. II and

$$\sigma_{\text{unit}} = - \int [1 - e^{-\bar{n}_e(\rho)}] P_B(\rho) d^2\rho \quad (45)$$

corresponds to the unitarity correction for the one muon pair production.

A rough estimation of  $\sigma_{\text{unit}}$  can be done as follows. The main contribution to  $\sigma_{\text{unit}}$  comes from the region

$$R \ll \rho \ll 1/m_e \quad (46)$$

in which the function  $\bar{n}_e(\rho) \approx \bar{n}_e(2R)$  and the integral (45) can be calculated in LLA. It gives

$$\sigma_{\text{unit}} \approx - \frac{28}{27\pi} \frac{(Z_1\alpha Z_2\alpha)^2}{\mu^2} [1 - e^{-\bar{n}_e(2R)}] J_{\text{unit}}, \quad (47)$$

where

$$J_{\text{unit}} = 6 \int_{2R}^{1/m_e} \Phi(\rho) \frac{d\rho}{\rho}. \quad (48)$$

As a result, we find  $\sigma_{\text{unit}} \approx -1.2$  b for the Pb-Pb collisions at LHC, which corresponds approximately to (−50)% of the Born cross section.

It is seen that unitarity corrections are large, in other words, the exclusive production of one muon pair differs considerably from its Born value.

#### IV. INCLUSIVE PRODUCTION OF ONE $\mu^+\mu^-$ PAIR

The experimental study of the exclusive muon pair production seems to be a very difficult task. Indeed, this process requires that the muon pair should be registered without any electron-positron pair production including  $e^\pm$  emitted at very small angles. Otherwise, the corresponding cross section will be close to the Born cross section.

To prove this, let us consider the process (37), whose probability is given by Eq. (41). The corresponding cross section is

$$\sigma_{1+n} = \int P_{1+n}(\rho) d^2\rho. \quad (49)$$

It is clearly seen from this equation that after summing up over all possible electron pairs we obtain the Born cross section

$$\sum_{n=0}^{\infty} \sigma_{1+n} = \sigma_B. \quad (50)$$

Therefore, there is a very definite prediction: the inclusive muon pair production coincides with the Born limit. This direct consequence of calculations, which take into account strong field effects, may be easier to test experimentally than the prediction for cross sections of several  $e^+e^-$  pair production.

#### V. TWO MUON PAIR PRODUCTION

The cross section of the process

$$Z_1 Z_2 \rightarrow Z_1 Z_2 + \mu^+ \mu^- \mu^+ \mu^- \quad (51)$$

can be calculated in lowest order in  $\alpha$  according to

$$\sigma_2 = \frac{1}{2} \int [P_B(\rho)]^2 d^2\rho \quad (52)$$

with the integration region  $\rho \geq 2R$ . But in this region the probability  $P_B(\rho)$  is given to a good accuracy by Eqs. (23)–(25). From this we get  $\sigma_2 = 1.24$  mb for Pb-Pb collisions at LHC.

#### VI. CONCLUSION

The exclusive production of one  $\mu^+\mu^-$  pair in collisions of two ultra-relativistic nuclei is considered. We present a simple method for the calculation of the Born cross section for this process.

We found that the Coulomb corrections to this cross section are small, while the unitarity corrections are large. This is in sharp contrast to the exclusive  $e^+e^-$  pair production where the Coulomb corrections to the Born cross section are large while the unitarity corrections are small.

We calculate also the cross section for the production of one  $\mu^+\mu^-$  pair and several  $e^+e^-$  pairs in LLA. Using this cross section we found that the inclusive production of  $\mu^+\mu^-$  pair coincides in this approximation with its Born value.

Let us discuss the relation of the cross sections obtained for the muon pair production with the the differential cross section of the  $e^+e^-$  pair production in the region of large transverse momenta for the  $e^\pm$ , for example at  $p_{\perp\pm} \gtrsim 100$  MeV. It is clear

that for the  $e^+e^-$  pair production in this region, the situation is similar to the case considered for  $\mu^+\mu^-$  pair production.

We expect that the inclusive production of a single  $e^+e^-$  pair with large transverse momenta of  $e^\pm$  (together with several unobserved  $e^+e^-$  pairs in the region of small transverse momenta of  $e^\pm$  of the order of  $m_e$ ) coincides with the Born limit.

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#### APPENDIX

To tests the approach used in Sec. II C, we consider the simpler case of the muon pair production by a real photon with the energy  $\omega$  off a nucleus

$$\gamma Z \rightarrow Z\mu^+\mu^-. \quad (A1)$$

This cross section was calculated by Ivanov and Melnikov in Ref. [11] using the same expression [Eq. (26)] for the form factor of the nucleus and assuming  $\Lambda^2/(2\mu)^2 \ll 1$ . The corresponding formula for the Born contribution and the first Coulomb correction is

$$\sigma_{\gamma Z} = \frac{28}{9} \frac{Z^2 \alpha^3}{\mu^2} (l - C_1 - C_2), \quad (A2)$$

where

$$l = \ln \frac{2\omega\Lambda}{\mu^2} - \frac{57}{14}, \quad C_1 = \frac{12}{35} \left( \frac{\Lambda}{2\mu} \right)^2, \quad (A3)$$

$$C_2 = 0.92(Z\alpha)^2 C_1.$$

Therefore, the relative magnitude of the Coulomb correction is given by

$$\eta_2 = \frac{C_2}{l}, \quad (A4)$$

which confirms the estimate in Eq. (36).

In the equivalent photon approximation, the cross section is given by

$$d\sigma_{\gamma Z}^{\text{EPA}} = dn_2 \sigma_{\gamma\gamma}(W^2), \quad (A5)$$

which has the form

$$\sigma_{\gamma Z}^{\text{EPA}} = \frac{Z^2 \alpha}{\pi} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} f \left( \frac{W^2}{2\omega\Lambda} \right) \sigma_{\gamma\gamma}(W^2). \quad (A6)$$

The main contribution to this integral is given by the region near the lower limit, where the argument of the function  $f$  is small and therefore  $f$  can be replaced by its approximate expression (29):

$$f \left( \frac{W^2}{2\omega\Lambda} \right) = 2 \ln \frac{\omega\Lambda}{2\mu^2} - 2 - 2 \ln \frac{W^2}{4\mu^2}. \quad (A7)$$

After that the cross section can be calculated without difficulties as

$$\sigma_{\gamma Z}^{\text{EPA}} = 2 \frac{Z^2 \alpha}{\pi} \left[ \left( \ln \frac{\omega\Lambda}{2\mu^2} - 1 \right) I - I_1 \right] = \frac{28}{9} \frac{Z^2 \alpha^3}{\mu^2} l, \quad (A8)$$

where  $I$  is given by Eq. (15) and

$$I_1 = \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) \ln \frac{W^2}{4\mu^2} = \frac{(43 - 28 \ln 2)\pi\alpha^2}{9\mu^2}. \quad (\text{A9})$$

Comparing this expression with the one of Eq. (A2), we find that those terms, which are omitted in the EPA, have a relative magnitude of the order of

$$\eta_1 = \frac{C_1}{I}, \quad (\text{A10})$$

this expression confirms the estimate [Eq. (34)].

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- [1] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Yu. Kharlov, *Phys. Rep.* **364**, 359 (2002).
- [2] L. D. Landau and E. M. Lifshitz, *Phys. Z. Sowjet.* **6**, 244 (1934); G. Racah, *Nuovo Cimento* **14**, 93 (1937).
- [3] S. R. Klein, *Nucl. Instrum. Methods A* **459**, 51 (2001).
- [4] D. Yu. Ivanov, A. Schiller, and V. G. Serbo, *Phys. Lett.* **B454**, 155 (1999).
- [5] R. N. Lee, A. I. Milstein, and V. G. Serbo, *Phys. Rev. A* **65**, 022102-1 (2002); *Proceedings of the International Conference on the Structure and Interactions of the Photon, PHOTON 2001*, Ascona, Switzerland, 2–7 September 2001, edited by M.-N. Kienzle-Focacci and M. Wadhwa (World Scientific, New Jersey-London-Singapore-Hong Kong, 2002), p. 339.
- [6] E. Bartoš, S. R. Gevorkyan, E. A. Kuraev, and N. N. Nikolaev, *Phys. Lett.* **B538**, 45 (2002).
- [7] K. Hencken, E. A. Kuraev, and V. G. Serbo, *Acta Phys. Polon. B* **37**, 969 (2006).
- [8] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, *Phys. Rep. C* **15**, 181 (1975).
- [9] K. Hencken, D. Trautmann, and G. Baur, *Phys. Rev. A* **51**, 1874 (1995).
- [10] G. Baur, K. Hencken, A. Aste, D. Trautmann, and S. R. Klein, *Nucl. Phys.* **A729**, 787 (2003).
- [11] D. Ivanov and K. Melnikov, *Phys. Rev. D* **57**, 4025 (1998).