# Investigation of the pairing effect using newly evaluated empirical studies for 14–15 MeV neutron reaction cross sections

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The asymmetry term effects for the cross sections of $(n$, charged particle) and $(n, 2n)$ reactions at 14–15 MeV neutron incident energy have been investigated. The effects of pairing and odd-even nucleon numbers in new data and in the formula of Tel *et al.* [J. Phys. G. **29**, 2169 (2003)] are discussed. We have determined three different parameters groups by the classification of nuclei into even-even, even-odd, and odd-even $(n, d)$ reactions. In addition, since there are not enough experimental data available, we have considered two different parameters groups by the classification of nuclei into odd-$A$ and even-$A$ $(n, t)$ reaction cross sections. The empirical formulas with two parameters for the evaluation of the $(n, d)$ and $(n, t)$ reactions cross sections are discussed in the present study.

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## I. INTRODUCTION

Neutron scattering cross sections and emission differential data have a critical importance on fusion reactor neutronics. These data can be extensively used for the investigation of the structural materials of the fusion reactors, radiation damage of metals and alloys, tritium breeding ratio, neutron multiplication and nuclear heating in the components, neutron spectrum, reaction rates in the blanket, and neutron dosimetry. There are several new technological applications in the fields of fast-neutron research, including accelerator-driven incineration and transmutation of long-lived radioactive nuclear wastes into short-lived or stable isotopes [1]. Especially in accelerator-driven subcritical systems for fission energy production and/or nuclear waste transmutation as well as in intermediate energy accelerator-driven neutron sources, ions and neutrons with energies beyond 20 MeV (the upper limit of existing data files that were produced for d-$^3$H fusion applications) will interact with materials [2]. Moreover, there are several biomedical applications (e.g., production of radioisotopes and cancer therapy).

Because of these cases, we need to accumulate new data about the cross sections of nuclear reactions for energies around 20 MeV [3,4]. Therefore, more many experiments have been carried out to obtain and detect neutrons for different energy ranges. For instance, the method of energy measurement by means of velocity determination is a widely used technique, known as time of flight (TOF) [5]. Additionally, data obtained from various techniques are necessary to develop additional nuclear theoretical calculation models to explain nuclear reaction mechanisms and the properties of the excited states for different energy ranges. Furthermore, the experimental neutron cross-section data at around 14–15 MeV energy and particle emission spectra have a profound importance for understanding the binding energy systematics, the basic nucleon-nucleus interaction, nuclear structure, and refined nuclear models.

The energy around 14–15 MeV is enough to excite the nucleus for reactions such as $(n, p)$, $(n, d)$, $(n, 2n)$, $(n, t)$ and $(n, \alpha)$. The systematic experimental studies for the cross sections of $(n$, charged particle) such as $\sigma(n, p)$ and $\sigma(n, \alpha)$ at 14–15 MeV have been studied over the years for a large number of nuclei [6–9]. Moreover, applications of statistical and thermodynamical methods to the calculation of nuclear process for heavy nuclei go back to the fundamental work of Weisskopf [10]. However, numerous empirical and semi-empirical formulas with different parameters for cross-section calculations of the reactions $(n, p)$, $(n, t)$, $(n, \alpha)$, and $(n, 2n)$ at different neutron energies have also been proposed by several authors [10–16], and Tel *et al.* [17] suggested using these new experimental data to reproduce a new empirical formula of the cross sections of the $(n, p)$ reactions. This formula depends on the asymmetry parameter, $s = (N - Z)/A$, and has the pairing effect on the binding energy systematics of the nuclear shell model. Tel *et al.* also obtained a new appropriate coefficient by using this formula for $(n, 2n)$ and $(n, \alpha)$ reactions [18].

In the present study, $(n$, charged particle) and $(n, 2n)$ reaction cross sections and the change of pairing effect with respect to the asymmetry parameter were investigated for 14–15 MeV incident neutron energies and for odd-even properties of target nuclei (Figs. 1–5). The detailed cross-section formulas were obtained for $(n, p)$, $(n, \alpha)$, and $(n, 2n)$ reactions in previous studies [17–20]. Therefore, for this study, only empirical formulas of the $(n, d)$ and $(n, t)$ cross sections were obtained at the energy range of 14–15 MeV for different parameter groups by using the Tel *et al.* formula [17]. Three different parameter groups classified into even-even, even-odd, and odd-even for $(n, d)$ reaction cross sections and two different parameter groups classified into odd-$A$ and even-$A$ for $(n, t)$ reaction cross sections have been determined. This allows an examination of the odd-even effect on basic nucleon-nucleus interactions by considering new experimental data and new cross section formulas developed by Tel *et al.* for the $(n, p)$, $(n, d)$, $(n, 2n)$, $(n, t)$, and $(n, \alpha)$ reactions. The results obtained have also been investigated and compared with the other theoretical and experimental results proposed in earlier studies [21–24].

 

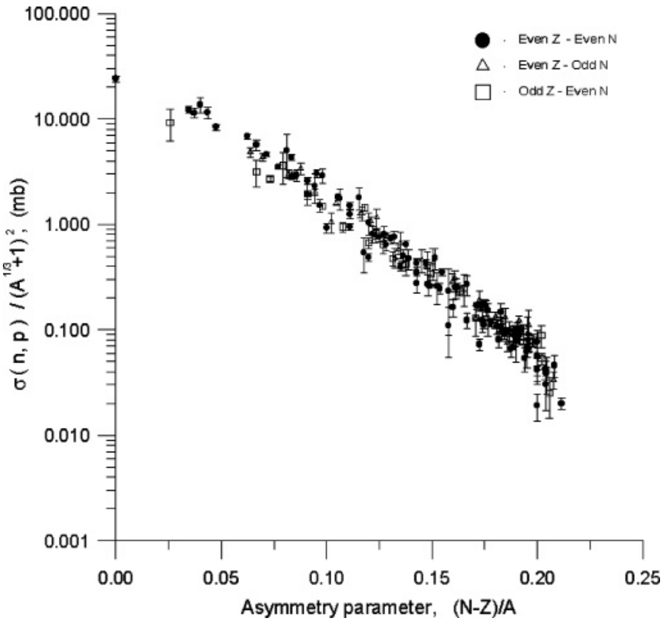


FIG. 1. Systematics of $(n, p)$ reaction cross sections (in millibarns) induced by 14–15 MeV neutrons. Experimental data were taken from Refs. [1,3,4,21].

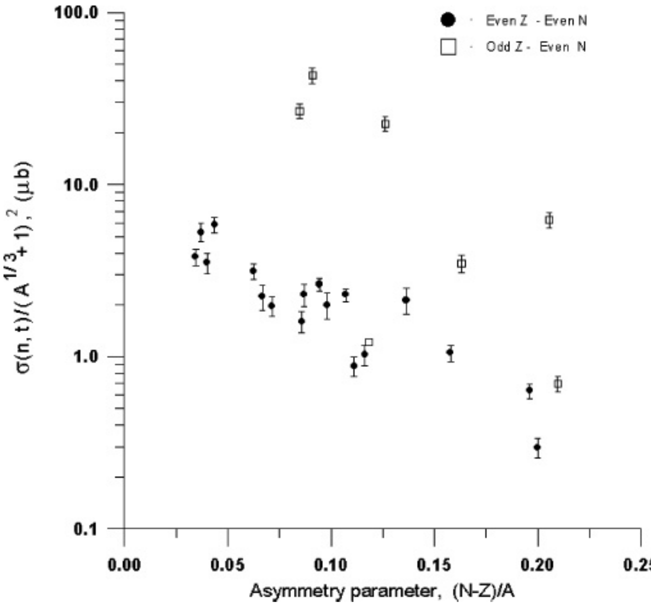


FIG. 3. Systematics of $(n, t)$ reaction cross sections (in microbarns) induced by 14–15 MeV neutrons. Experimental data were taken from Refs. [3,4,23].

## II. EMPIRICAL FORMULAS FOR NEUTRON CROSS SECTIONS

Recently nuclear data files have been prepared to study neutron transport, nuclear heating and gas production, and radiation damage for materials irradiated by fast neutrons [4]. These data files include information about total cross sections, elastic and inelastic cross sections for threshold reactions, and energy and angular distributions for secondary neutrons, protons, and $\alpha$ particles. However, nuclear models are frequently needed to provide estimations of neutron-induced reaction cross sections, especially when experimental data are scarce or very difficult to acquire. If the calculations on nuclear models are carried out using global parameters that have not been sufficiently validated by experimental data, the obtained numerical results can be quite unreliable. The calculations employed in various model codes have shown that the results may vary depending on the codes and input parameters when no experimental data are available [25–29]. Furthermore, considerable experimental data have been published on the

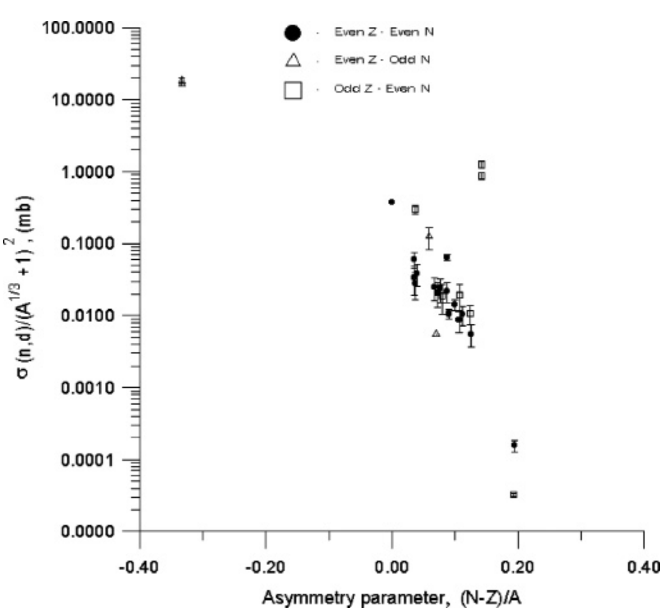


FIG. 2. Systematics of $(n, d)$ reaction cross sections (in millibarns) induced by 14–15 MeV neutrons. Experimental data were taken from Refs. [3,4].

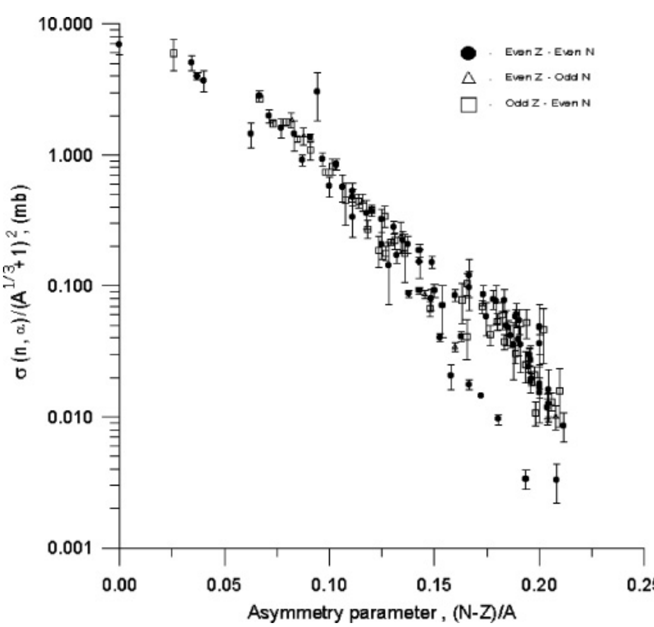


FIG. 4. Systematics of $(n, \alpha)$ reaction cross sections (in millibarns) induced by 14–15 MeV neutrons. Experimental data were taken from Refs. [1,3,4,21].

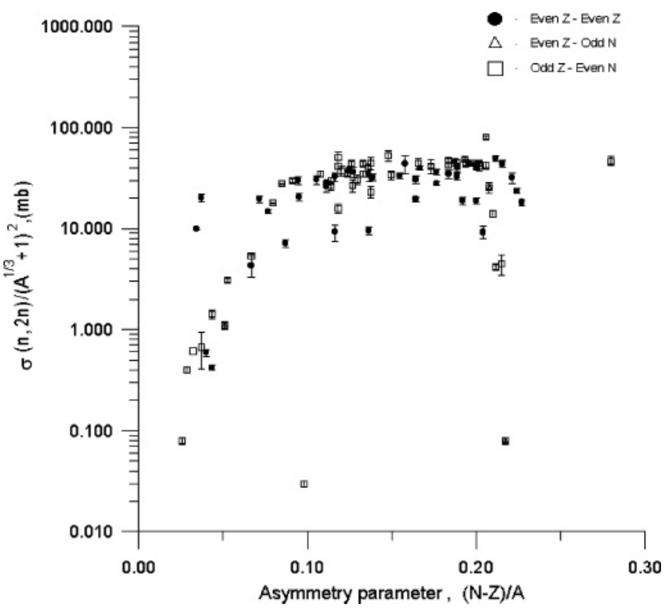


FIG. 5. Systematics of $(n, 2n)$ reaction cross sections (in millibarns) induced by 14–15 MeV neutrons. Experimental data were taken from Refs. [1,3,4,21].

($n$, charged particle) and $(n, 2n)$ reaction cross sections induced by 14–15 MeV neutrons [3]. According to previous reports [11–16], the cross sections for many nuclei significantly vary with the mass number $A$, neutron number $N$, and proton number $Z$ of the target nucleus. In addition, effects attributable to the asymmetry parameter, $s = (N - Z)/A$, as well as to the isotopic, isotonic, and odd-even properties of nuclei, have been observed in the data.

The empirical cross sections of reactions induced by fast neutrons can be approximately expressed as follows:

$$\sigma(n, x) = C\sigma_{ne} \exp[as], \tag{1}$$

where $\sigma_{ne}$ is the neutron nonelastic cross section, and the coefficients $C$ and $a$ are the fitting parameters determined from the least-squares method for different reactions. The nonelastic cross sections have been measured intensely for many nuclides in the MeV range, enabling us to determine their variation with atomic mass. The neutron nonelastic cross section $\sigma_{ne}$ is given

$$\sigma_{ne} = \pi r_0^2 (A^{1/3} + 1)^2, \tag{2}$$

where $r_0 = 1.2 \times 10^{-13}$ cm.

Equation (1) represents the product of two factors, each of which might be assigned to a stage of nuclear reaction within the framework of the statistical model of nuclear reactions. The exponential term represents the escape of the reaction products from a compound nucleus. It has a strong $(N - Z)/A$ dependence implied by Eq. (1). This case has already been shown by Betak *et al.* [1]. for neutron-induced reaction cross sections. There are also several formulas describing the isotopic dependence of cross sections for different reactions at a neutron energy of 14.5 MeV. The measured cross sections exhibit a large gradient for the lighter masses ($Z \leqslant 30$) with increasing asymmetry parameter and then become almost constant for medium- and heavy-mass nuclei (starting from $A \leqslant 100$).

The best fit can be obtained with the new free parameters to provide the minimum value of the following expression:

$$\chi^2 = \frac{1}{N} \sum_i^N \left( \frac{\sigma_{\mathrm{exp}}^i - \sigma_{\mathrm{cal}}^i}{\Delta\sigma_{\mathrm{exp}}^i} \right)^2 \tag{3}$$

where $\sigma_{\mathrm{exp}}^i$ and $\sigma_{\mathrm{cal}}^i$ are the experimental and the calculated cross sections, respectively, and $\Delta\sigma_{\mathrm{exp}}^i$ is the error associated with $\sigma_{\mathrm{exp}}^i$. Details of the results of the best fitting parameters and the values of $\chi^2$ can be found for $(n, p)$ reactions in Refs. [14,16,17,23] and for $(n, \alpha)$ and $(n, 2n)$ reactions in Refs. [16,18–20].

## III. RESULTS AND DISCUSSION

Odd-even and nucleon binding energy systematics have been compared with $(n, p)$, $(n, d)$, $(n, t)$, $(n, \alpha)$, and $(n, 2n)$ measured cross sections with the empirical fits of Tel *et al.* as shown in Figs. 1–5. The $(n, p)$ experimental data include only proton emission and does not include $(n, np)$, $(n, pn)$, and $(n, 2p)$ cross sections. Also the $(n, d)$ experimental data include only $(n, d)$ cross sections and do not include $(n, np)$ or $(n, pn)$ cross sections. Asymmetry term effects for ($n$, charged particle) and $(n, 2n)$ reactions have been investigated and are given in Figs. 1–4 and Fig. 5, respectively. The $(n, p)$, $(n, d)$, $(n, t)$, and $(n, \alpha)$ reaction cross sections include the Coulomb effect as seen in Figs. 1–4. In these reactions, it can also be seen that the reaction cross sections decrease with increasing asymmetry parameter (Figs. 1–4). Unlike $(n, p)$, $(n, d)$, $(n, t)$, and $(n, \alpha)$ reaction cross sections, there is no Coulomb effect

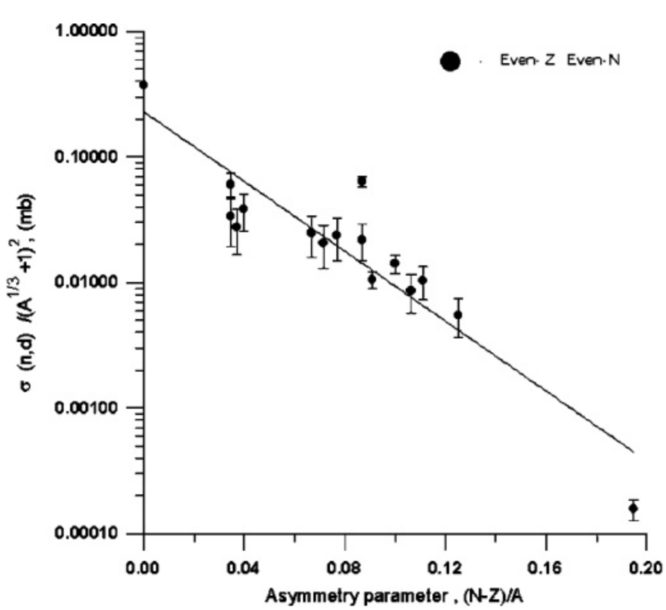


FIG. 6. Systematics of $(n, d)$ reaction cross sections (in millibarns) for even-$Z$, even-$N$ nuclides induced by 14–15 MeV neutrons. The experimental points were fitted with $\sigma_{n,d} = 0.23(A^{1/3} + 1)^2 \exp[-32.04\ s]$ and the correlation coefficient was determined as $R^2 = 0.84$. Experimental data were taken from Refs. [3,4,31–34].

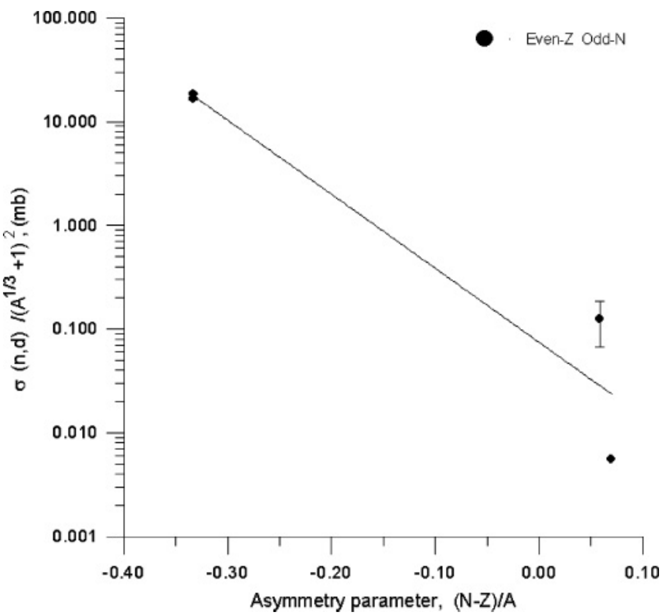


FIG. 7. Systematics of $(n, d)$ reaction cross sections (in millibarns) for even-$Z$, odd-$N$ nuclides induced by 14–15 MeV neutrons. The experimental points were fitted with $\sigma_{n,d} = 0.074(A^{1/3} + 1)^2 \exp[-16.44\ s]$ and the correlation coefficient was determined as $R^2 = 0.90$. Experimental data were taken from Refs. [3,4].

for $(n, 2n)$ reactions and the reaction cross sections increase as the asymmetry parameter increases (Fig. 5). Betak *et al.* [21] showed that the experimental $(n, p)$ cross sections as a functions of $A$ have a linear dependence on a semi-log plot and $(n, p)$ cross sections decrease with increasing $A$ for thin targets on $^{40-48}$Ca isotopes [1] and $^{112,115,117}$Sn isotopes.

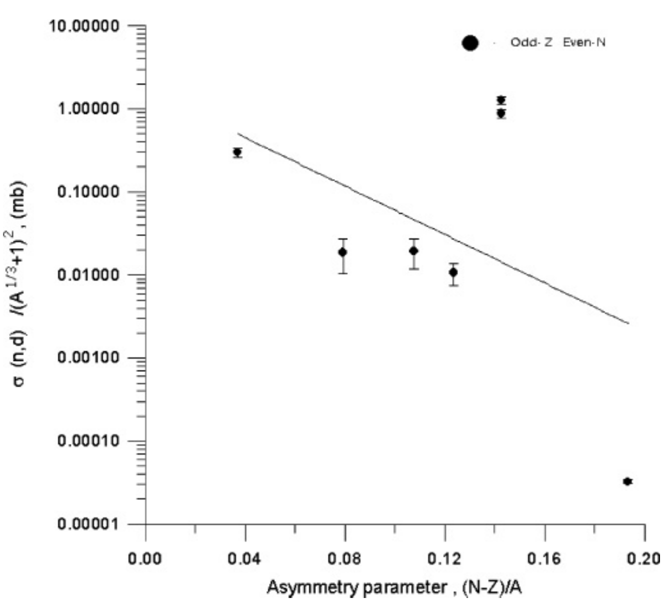


FIG. 8. Systematics of $(n, d)$ reaction cross sections (in millibarns) for odd-$Z$, even-$N$ nuclides induced by 14–15 MeV neutrons. The experimental points were fitted with $\sigma_{n,d} = 1.74(A^{1/3} + 1)^2 \exp[-33.58\ s]$ and the correlation coefficient was determined as $R^2 = 0.71$. Experimental data were taken from Refs. [3,4].

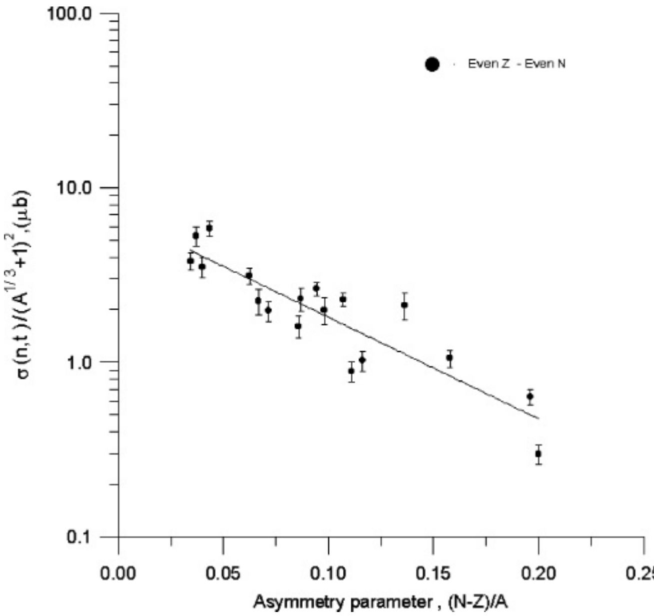


FIG. 9. Systematics of $(n, t)$ reaction cross sections (in microbarns) for even-$Z$, even-$N$ nuclides induced by 14–15 MeV neutrons. The experimental points were fitted with $\sigma_{n,t} = 6.94(A^{1/3} + 1)^2 \exp[-13.41\ s]$ and the correlation coefficient was determined as $R^2 = 0.80$. Experimental data were taken from Refs. [3,4,23].

This trend originates from the systematic variation of the neutron and proton separation energies. The neutron separation energies, $S_n$, decrease with increasing $A$ whereas proton separation energies, $S_p$, increase with increasing $A$ [1,22]. For the present study, it can be seen that odd-even effects

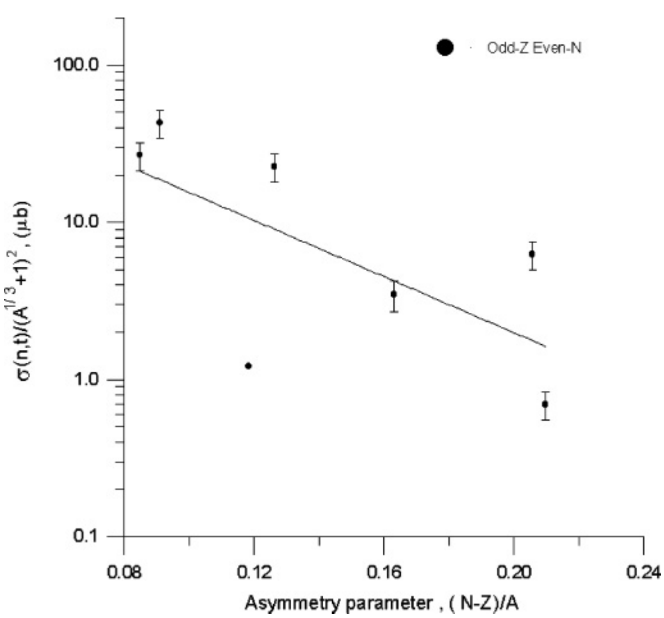


FIG. 10. Systematics of $(n, t)$ reaction cross sections (in microbarns) for odd-$Z$, even-$N$ nuclides induced by 14–15 MeV neutrons. The experimental points were fitted with $\sigma_{n,t} = 121.21(A^{1/3} + 1)^2 \exp[-20.54\ s]$ and the correlation coefficient was determined as $R^2 = 0.73$. Experimental data were taken from Refs. [3,4,23].

TABLE I. The coefficients $C$ and $a$ and the empirical formulas for $(n, d)$ and $(n, t)$ reactions.

| $Z$ | $N$ | $C$ | $a$ | $\sigma(n,x) = C\sigma_{ne}\exp[as]$ |
|---|---|---|---|---|
| Even | Even | 0.23 | −32.04 | $\sigma_{n,d} = 0.23(A^{1/3}+1)^2\exp[-32.04\ s]$ |
| Even | Odd | 0.074 | −16.44 | $\sigma_{n,d} = 0.074(A^{1/3}+1)^2\exp[-16.44\ s]$ |
| Odd | Even | 1.74 | −33.58 | $\sigma_{n,d} = 1.74(A^{1/3}+1)^2\exp[-33.58\ s]$ |
| Even | Even | 6.94 | −13.41 | $\sigma_{n,t} = 6.94(A^{1/3}+1)^2\exp[-13.41\ s]$ |
| Odd | Even | 121.21 | −20.54 | $\sigma_{n,t} = 121.21(A^{1/3}+1)^2\exp[-20.54\ s]$ |

are strong for $(n, p)$, $(n, d)$, $(n, t)$, and $(n, \alpha)$ reaction cross sections from Figs. 1–4.

According to Levskovskii's formula [6], the proton and $\alpha$ emission probabilities increase with increasing relative proton number. The same relation can be also expected for $d$, $^3$H, and $n$ emissions. Besides, pre-equilibrium process are important mechanisms in nuclear reactions induced by light projectiles with incident energies above 10 MeV [25–28]. The pre-equilibrium reaction effects strongly depend on the asymmetry parameter. Particularly, in region I ($A = 40$–62), the $(n, p)$ reaction is possible with a compound process whereas this reaction is possible with a pre-equilibrium process in region III ($A = 90$–160). Moreover, in intermediate region II ($A = 63$–89) this reaction is also governed by both processes in regions I and III [30]. Depending upon the asymmetry parameter, the same effects can be observed for $(n, \alpha)$ reaction cross sections in the present study (Fig. 4). However, in cases with a smaller asymmetry parameter, $(n, p)$ and $(n, \alpha)$ cross sections were found to be higher (Figs. 1 and 4). Similar results can also be observed for $(n, d)$ and $(n, t)$ cross sections (Figs. 2 and 3).

In Figs. 1–5, reaction cross sections were classified according to odd-even properties depending upon the asymmetry parameter. As can be obviously seen from Figs. 1–5, reaction cross sections separate from each other (relative to odd-even properties) as the asymmetry parameter rises. This separation is most prominent for $(n, d)$ and $(n, t)$ reactions. These cases shows a strong pairing effect depending upon the target nuclei mass number $A$. The weak odd-even effect can be seen in Fig. 5 for $(n, 2n)$ reaction cross sections. These results exhibit a fair agreement with those of the studies carried out by Habbani and Osman on the semi-empirical theoretical calculations for $(n, 2n)$ cross sections [16].

Detailed investigations for $(n, p)$, $(n, \alpha)$, and $(n, 2n)$ reaction cross sections have been performed in previous studies [17,18]. In the present study, we have only investigated the $(n, d)$ and $(n, t)$ reaction cross sections at the energy of 14–15 MeV in two different groups by using the Tel *et al.* formula, as can be seen in Figs. 6–10. We have determined three different parameter groups classified into even-even, even-odd, and odd-even for $(n, d)$ reactions. Because there are not enough experimental data for $(n, t)$ reactions, only two different parameter groups have been classified (odd-$A$and even-$A$). As can be clearly seen from Figs. 2 and 3, the $(n, d)$ and $(n, t)$ reaction cross sections separate from each other with increasing asymmetry parameter. Therefore, we could not perform a good fit for this case. However, for the $(n, d)$ and $(n, t)$ cross-sections values, a good fit was achieved by considering the even-even correction (Figs. 6 and 9).

We have used 27 experimental $(n, d)$ cross-section data points for different nuclei taken from Refs. [3,4] for the fitting procedure. The nuclei used in this study have mass numbers of $A = 3$–181, atomic numbers of $Z = 2$–73, and neutron numbers $N = 1$–108. In Figs. 6–8, we have introduced three formulas by fitting two parameters for each formula presented by considering the pairing effect of the nuclear shell model. For this purpose, three different groups have been determined (classified into even-even, even-odd, and odd-even for these reaction cross sections parameters depending on asymmetry).

The coefficients $C$ and $a$ were determined by a least-squares fitting method and the empirical formulas obtained by fitting two parameter for $(n, d)$ and $(n, t)$ reactions are given in Table I and also in Figs. 6–8 and 9 and 10, respectively.

We have used 25 experimental $(n, t)$ reaction cross-section data points taken from Refs. [3,4,23] for the fitting procedure. We have used nuclei with mass numbers of $A = 46$–209, atomic numbers of $Z = 22$–83, and neutron numbers of $N =$ 24–126. In Figs. 9 and 10, the experimental $(n, t)$ cross sections of these nuclides have been fitted depending on the asymmetry parameter. In Figs. 9 and 10, we have introduced two formulas by fitting two parameters for each formula presented by considering the pairing effect of the nuclear shell model. We could not use the even-even, even-odd, and odd-even systematics because of the lack of sufficient experimental data.

Obtaining more reliable results and developing additional nuclear reaction mechanisms and nuclear models depends on obtaining more experimental data for the neutron scattering and emission differential cross sections using new technology. Precise knowledge of the systematics for different neutron-induced reactions is of great importance for understanding the binding energy systematics of the nuclear shell model. Studies of present kind will help to improve and clarify our understanding of the binding energy systematics of the nuclear shell model and to provide estimation of unknown data for the development of nuclear reaction theories.

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