

## Density dependence of the symmetry energy and the equation of state of isospin asymmetric nuclear matter

D. V. Shetty, S. J. Yennello, and G. A. Souliotis

*Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA*

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The density dependence of the symmetry energy in the equation of state of isospin asymmetric nuclear matter is studied using the isoscaling of the fragment yields and the antisymmetrized molecular dynamic calculation. It is observed that the experimental data at low densities are consistent with the form of symmetry energy,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^{0.69}$ , in close agreement with those predicted by the results of variational many-body calculation. A comparison of the present result with those reported recently using the NSCL-MSU data suggests that the heavy ion studies favor a dependence of the form,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^\gamma$ , where  $\gamma = 0.6-1.05$ . This constrains the form of the density dependence of the symmetry energy at higher densities, ruling out an extremely “stiff” and “soft” dependences.

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The equation of state (EOS) of isospin asymmetric ( $N \neq Z$ ) nuclear matter is a fundamental quantity that determines the properties of systems as small and light as an atomic nucleus, and as large and heavy as a neutron star [1–3]. The key ingredient in the EOS of asymmetric nuclear matter is the density dependence of the symmetry energy. Theoretical studies [4–8] based on microscopic many-body calculations and phenomenological approaches predict various different forms of the density dependence of the symmetry energy. In general, two different forms have been identified [9]. One, where the symmetry energy increases monotonically with increasing density (“stiff” dependence) and the other, where the symmetry energy increases initially up to normal nuclear density and then decreases at higher densities (“soft” dependence).

Determining the exact form of the density dependence of the symmetry energy is important for studying the structure of neutron-rich nuclei [10–13], and studies relating to astrophysical origin, such as the structure of neutron stars and the dynamics of supernova collapse [14–19]. For example, a “stiff” density dependence of the symmetry energy is predicted to lead to a large neutron skin thickness compared to a “soft” dependence [11,13,20,21]. Similarly, a “stiff” dependence of the symmetry energy can result in rapid cooling of a neutron star, and a larger neutron star radius, compared to a soft density dependence [22,23].

In a heavy ion reaction, the dynamics of the collision between two heavy nuclei is also sensitive to the density dependence of the symmetry energy [24,25]. One can therefore carry out laboratory-based experiments to constrain this dependence. Recently [26], the fragment yields from heavy ion collisions simulated within the antisymmetrized molecular dynamics (AMD) calculation were reported to follow a scaling behavior of the type

$$Y_2(N, Z)/Y_1(N, Z) \propto e^{\alpha N + \beta Z}, \quad (1)$$

where the parameters  $\alpha$  and  $\beta$  are related to the neutron-proton content of the fragmenting source, and  $Y_1$  and  $Y_2$  are the yields from two different reactions. A linear relation

between the isoscaling parameter  $\alpha$ , and the difference in the isospin asymmetry  $(Z/A)^2$  of the fragments, with appreciably different slopes, was predicted for two different forms of the density dependence of the symmetry energy; a “stiff” dependence (obtained from Gogny-AS interaction) and a “soft” dependence (obtained from Gogny interaction).

In this work, we show that the experimentally measured scaling parameter  $\alpha$ , favors a stiff density dependence of the symmetry energy, i.e., Gogny-AS interaction, and can be parametrized as  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^\gamma$ , where  $\gamma = 0.69$ . The present observation is consistent with the EOS of Akmal and Pandharipande obtained from the many-body variational calculations [27,28].

The measurements were carried out at the Cyclotron Institute, Texas A&M University using beams of  $^{40}\text{Ar}$ ,  $^{40}\text{Ca}$ ,  $^{58}\text{Fe}$ , and  $^{58}\text{Ni}$  from the K500 Superconducting Cyclotron on  $^{58}\text{Fe}$  and  $^{58}\text{Ni}$  targets at 25, 30, 33, 40, 45, 47, and 53 MeV/nucleon. Details of the experimental measurements and analysis can be found in Ref. [29].

Figure 1 shows the experimentally determined isoscaling parameter  $\alpha$ , obtained from the fragment yields as a function of the beam energy. The different symbols correspond to various combinations of the reactions chosen for extracting the isoscaling parameters. The solid and the dotted lines are the exponential fits to the data.

As mentioned earlier and shown in [26], the parameter  $\alpha$ , is related to the difference in the fragment isospin asymmetry  $(Z/A)^2$ , through a linear relation of the form

$$\alpha = \frac{4C_{\text{sym}}}{T} [(Z/A)_1^2 - (Z/A)_2^2], \quad (2)$$

where  $C_{\text{sym}}$  is the symmetry energy and  $T$  is the temperature at which the fragments are formed. The quantity  $(Z/A)_1^2 - (Z/A)_2^2$ , is the difference in the isospin asymmetry of the fragments in the two reaction systems. For the present systems, the isospin asymmetry of the fragments were evaluated at  $t = 300$  fm/c of the dynamical evolution from the AMD calculations as discussed extensively in Ref. [29].

Figure 2 shows the  $\alpha$  parameters plotted as a function of the difference in the fragment asymmetry for the beam energy

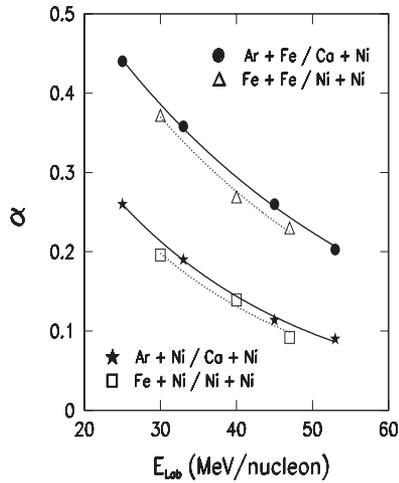


FIG. 1. Experimental isoscaling parameter  $\alpha$ , as a function of the beam energy. The solid circles are from the Ar+Fe and Ca+Ni reactions. The open triangles are from Fe+Fe and Ni+Ni reactions. The solid stars are from Ar+Ni and Ca+Ni reactions. The open squares are from Fe+Ni and Ni+Ni reactions.

of 35 MeV/nucleon. The solid and the dotted lines are the AMD predictions using the “soft” (Gogny) and the “stiff” (Gogny-AS) density dependence of the symmetry energy, respectively. The solid and the hollow symbols (squares, stars, triangles, and circles) are the results of the present measurements for the two different values of the fragment asymmetry, assuming Gogny and Gogny-AS interactions, respectively. Also shown in the figure are the scaling parameters (asterisks, crosses, diamond, and inverted triangle) taken from various other works [30,31] in the literature. One observes

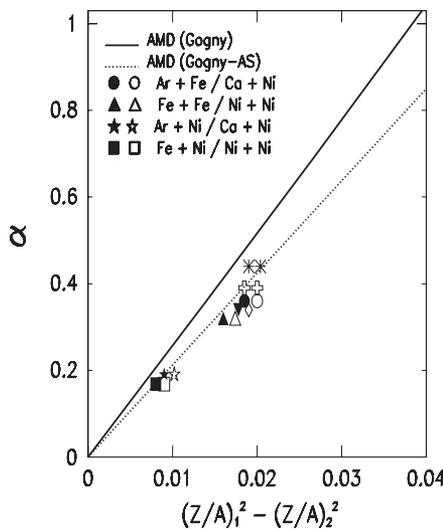


FIG. 2. Scaling parameter  $\alpha$ , as a function of the difference in fragment asymmetry for 35 MeV/nucleon. The solid and the dotted lines are the AMD calculations for the Gogny and Gogny-AS interactions, respectively [26]. The solid and the hollow, squares, stars, triangles, and circles are from the present work as described in the text. The other symbols corresponds to data taken from [30] (asterisks) and [31] (crosses, diamonds, inverted triangles).

from the figure that the experimentally determined  $\alpha$  parameter increases linearly with increasing difference in the asymmetry of the two systems as predicted by the AMD calculation. Also the data points are in closer agreement with those predicted by the Gogny-AS interaction (dotted line) than those from the usual Gogny force (solid line).

It should be mentioned that in the above comparison between the data and the calculation, the corrections for the isoscaling parameter  $\alpha$  due to the secondary deexcitation of the fragments are not taken into account. The slightly lower values of the isoscaling parameters (symbols) from the present measurements with respect to the Gogny-AS values (dotted line) could be due to the small secondary deexcitation effect of the fragments not accounted for in this comparison. It has been reported by Ono *et al.* [32], that the sequential decay effect in the dynamical calculations can affect the  $\alpha$  value by as much as 50%. On the other hand, a dynamical calculation carried out by Tian *et al.*, [33] for the same systems and energy as studied by Ono *et al.*, using the isospin quantum molecular dynamic (IQMD) model, show no significant difference between the primary and the secondary  $\alpha$ .

Due to the large discrepancy that exist in the determination of the primary fragment excitation energy from the dynamical model calculations, it is difficult to estimate the effect of secondary deexcitation in dynamical models at this moment [34]. We have therefore assumed the effect of the sequential decay to be negligible, in the above comparison. A small correction of about 10–15%, as determined from various statistical model studies [35], results in a slight increase in the  $\alpha$  values bringing them even closer to the dotted line. Note the asterisk symbols shown in the figure, and taken from Ref. [30], have already been corrected. The closer agreement of the experimental data with the Gogny-AS type of interaction, therefore, appears to suggest a stiffer density dependence of the symmetry energy rather than the soft Gogny interaction.

Recently, Chen *et al.* [36] also showed, using the isospin dependent Boltzmann-Uehling-Uhlenbeck (IBUU04) transport model calculation, that a stiff density dependence of the symmetry energy parametrized as,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^{1.05}$  explains well the isospin diffusion data [37] from NSCL-MSU (National Superconducting Cyclotron Laboratory at Michigan State University). Their calculation was also based on a momentum-dependent Gogny effective interaction. However, the present measurements on isoscaling gives a slightly softer density dependence of the symmetry energy at higher densities than those obtained by Chen *et al.*

This is clear from Fig. 3, which shows the parametrization of various theoretical predictions of the density dependence of the nuclear symmetry energy in isospin asymmetric nuclear matter. The dot-dashed, dotted, and the dashed curves correspond to those from the momentum dependent Gogny interactions used by Chen *et al.* to explain the isospin diffusion data. These are given as,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^\gamma$ , where,  $\gamma = 1.6, 1.05,$  and  $0.69$ , respectively. The solid curves and the solid points correspond to those from the Gogny and Gogny-AS interactions used to compare with the present isoscaling data. As shown by Chen *et al.*, the dependence parametrized by  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^{1.05}$  (dotted curve) explains the NSCL-MSU data on isospin diffusion quite well. On the

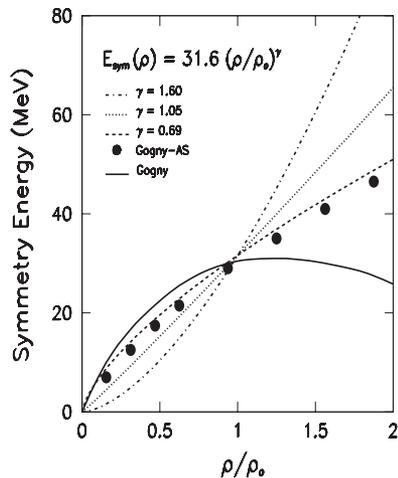


FIG. 3. Parametrization of the various forms of the density dependence of the nuclear symmetry energy used in the analysis of the present measurements on isoscaling data and the isospin diffusion measurements of NSCL-MSU [36]. The various curves are as described in the text.

other hand, the isoscaling data from the present work can be explained well by the Gogny-AS interaction (solid points). Both measurements yield similar results at low densities with significant difference at higher densities. It is interesting to note that by parametrizing the density dependence of the symmetry energy that explains the present isoscaling data, one gets,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^\gamma$ , where  $\gamma = 0.69$ . This form of the density dependence of the symmetry energy is consistent with the parametrization adopted by Heiselberg and Hjorth-Jensen in their studies on neutron stars [38]. By fitting earlier predictions of the variational calculations by Akmal *et al.* [27,28], where the many-body and special relativistic corrections are progressively incorporated, Heiselberg and Hjorth-Jensen obtained a value of  $E_{\text{sym}}(\rho_0) = 32$  MeV and  $\gamma = 0.6$ , similar to those obtained from the present measurements. The present form of the density dependence is also consistent with the findings of Khoa *et al.* [39], where a comparison of the experimental cross-sections in a charge-exchange reaction with the Hartree-Fock calculation using the CDM3Y6 interaction [40], reproduces well the empirical half-density point of the symmetry energy obtained from the present work (see Fig. 2 of Ref. [39]).

The observed difference in the form of the density dependence of the symmetry energy between the present measurement and those obtained by Chen *et al.* is not surprising. Both measurements probe the low density part of the symmetry energy and are thus less sensitive to the high

density region. But the important point to be noted is that both measurements clearly favor a stiff density dependence of the symmetry energy at higher densities, ruling out the very “stiff” (dot-dashed curve) and very “soft” (solid curve) predictions. These results can thus be used to constrain the form of the density dependence of the symmetry energy at supranormal densities relevant for the neutron star studies.

It should be mentioned that the calculations in both the above described works assume a similar value for the symmetry energy at normal nuclear density (about 31 MeV). Although, numerous many-body calculations [4,41–43] and those from the empirical liquid drop mass formula [44,45] predict symmetry energy near normal nuclear density to be around 30 MeV, a direct experimental determination of the symmetry energy does not exist.

Recently, Khoa *et al.* [39], analyzed the experimental cross-section data [46,47] using the isospin dependent CDM3Y6 interaction of the optical potential in a charge exchange  $p(^6\text{He}, ^6\text{Li}^*)n$  reaction. Their analysis probed mainly the surface part of the form factor and hence appropriate for densities close to the normal nuclear density. Based on their results and the Hartree-Fock calculation of asymmetric nuclear matter using the same effective nucleon-nucleon interaction, they estimate the most realistic value of the symmetry energy to be about 31 MeV. An accurate determination of the neutron skin thickness  $\Delta R$ , from the parity-violating electron scattering measurement [20] is however, likely to provide a more precise determination of the symmetry energy near normal nuclear density.

In view of the findings from the present measurements and those of Chen *et al.*, we believe that the best estimate of the density dependence of the symmetry energy that can be presently extracted from heavy ion reaction studies is,  $E_{\text{sym}} \approx 31.6(\rho/\rho_0)^\gamma$ , where  $\gamma = 0.6 - 1.05$ . It must be mentioned that the present comparison between the experimental data and the theoretical calculation is model dependent. Any modification to the compressibility in the equation of state could affect the pressure at subsaturation densities and thus the agreement between the data and the calculation.

*Note added in proof.* Several other authors have now reported similar conclusions using other observables since this article was first submitted.

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