# **Superscaling in a dilute Fermi gas and the nucleon momentum distribution in nuclei**

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The superscaling observed in inclusive electron scattering is described within the dilute Fermi gas model with interaction between the particles. The comparison with the relativistic Fermi gas (RFG) model without interaction shows an improvement in the explanation of the scaling function  $f(\psi')$  in the region  $\psi' < -1$ , where the RFG result is  $f(\psi') = 0$ . It is found that the behavior of  $f(\psi')$  for  $\psi' < -1$  depends on the particular form of the general power-law asymptotics of the momentum distribution  $n(k) \sim 1/k^{4+m}$  at large *k*. The best agreement with the empirical scaling function is found for  $m \approx 4.5$  in agreement with the asymptotics of  $n(k)$  in the coherent density fluctuation model where  $m = 4$ . Thus, superscaling gives information about the asymptotics of  $n(k)$  and the *NN* forces.

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### **I. INTRODUCTION**

The concepts of *y* scaling  $[1-10]$  and superscaling (based on the  $\psi'$ -scaling variable) [\[10–18\]](#page-4-0) have been used for extensive analyses of the vast amount of inclusive electron scattering world data (see also Ref. [\[19\]](#page-4-0)). These analyses showed the existence of high-momentum components in the nucleon momentum distribution (MD)  $n(k)$  at momenta  $k > 2$  fm<sup>-1</sup> due to the presence of nucleon-nucleon (*NN*) correlations. Scaling of the first kind (i.e., no dependence on the momentum transfer) can be observed at excitation energies below the quasielastic (QE) peak. Scaling of second kind (i.e., no dependence on the mass number) is excellent in the same region. When scaling of both first and second types occurs, one says that superscaling takes place. It was shown (e.g., in Refs. [\[15–18\]](#page-4-0)) that the physical reason for superscaling phenomena is the specific high-momentum tail of  $n(k)$ , which is similar for all nuclei.

It has been shown in Refs. [\[20,21\]](#page-4-0) that because of the contribution introduced by inelastic scattering, together with the correlation contribution and meson exchange currents [\[22,](#page-4-0) [23\]](#page-4-0), scaling of both the first and, to a lesser extent, the second kind is violated at energies above the QE peak.

The theoretical concept of superscaling has been introduced in Refs. [\[10,11\]](#page-4-0) using the properties of the relativistic Fermi gas (RFG) model. The Fermi momentum for the RFG was used as a physical scale to define the proper scaling variable  $\psi'$  for each nucleus. As emphasized in Ref. [\[13\]](#page-4-0), however, the actual dynamical physics content of the superscaling phenomenon is more complex than that provided by the RFG model. In particular, the extension of the superscaling property to large negative values of  $\psi'$  ( $\psi' < -1$ ) is not predicted by the RFG model. The QE scaling function in the RFG model  $f_{\text{RFG}}^{QE}(\psi') = 0$  for  $\psi' \le -1$ , whereas the experimental scaling function  $f^{QE}(\psi')$  has been observed for large negative values of  $\psi'$  up to  $\psi' \approx -2$  in the data for  $(e, e')$  processes. Thus, it has been necessary to consider superscaling in theoretical approaches that go beyond the RFG model. One of them is

the coherent density fluctuation model (CDFM) (e.g., Refs. [\[24,25\]](#page-4-0)) which gives a natural extension of the Fermi gas case to realistic finite nuclear systems. It was shown in Refs. [\[15–18\]](#page-4-0) that in the CDFM, both basic quantities—density and momentum distributions—are responsible for the scaling and superscaling phenomena in nuclei. The results of the CDFM for the QE scaling function  $f(\psi')$  agree with the available experimental data at different transferred momenta and energies below the QE peak position, showing superscaling for  $\psi$  < 0, including  $\psi' \lesssim -1$ , and going well beyond the RFG model. Second, as pointed out in  $[16,18]$ , the nucleon momentum distribution for various nuclei obtained in Ref. [\[26\]](#page-4-0) (and with the modification in Ref. [\[18\]](#page-4-0)) within a theoretical approach based on the light-front dynamics (LFD) method (e.g., Refs. [\[27,28\]](#page-4-0)) can also be used to describe both *y*- and  $\psi'$ -scaling data.

The superscaling analyses of inclusive electron scattering from nuclei (for energies of several hundred MeV to a few GeV) have been extended in Ref. [\[29\]](#page-4-0) to include not only QE processes but also the region where  $\Delta$  excitation dominates. Both QE- and  $\Delta$ -region scaling functions  $f^{\text{QE}}(\psi')$ and  $f^{\Delta}(\psi_{\Delta}')$  were deduced from phenomenological fits to electron-nuclei scattering data. Generally, the specific features of the scaling function should be accounted for by reliable microscopic calculations that take final-state interactions into account. In particular, the scaling function  $f^{QE}(\psi')$  with asymmetric shape obtained in Refs. [\[30,31\]](#page-4-0) by using a relativistic mean field (RMF) for the final states agrees well with the experimental scaling function.

The features of the superscaling phenomenon in inelastic electron scattering have induced studies of neutrino scattering from nuclei on the same basis. The neutrino-nucleus interactions have been studied using the superscaling analyses of few-GeV inclusive electron scattering data in a method proposed in Ref. [\[29\]](#page-4-0) to predict the inclusive *νA* and  $\bar{v}A$  cross sections for the case of  ${}^{12}C$  in the nuclear resonance region. Various other theoretical considerations (e.g., those in Refs. [\[32–43\]](#page-5-0)) have

<span id="page-1-0"></span>been devoted to studies of both neutral- [\[32–36\]](#page-5-0) and chargechanging [\[29,30,](#page-4-0)[34,35,37–43\]](#page-5-0) neutrino-nucleus scattering.

The CDFM and the LFD method have been extended [\[18\]](#page-4-0) from the QE to the  $\Delta$ -excitation region of the inclusive electron scattering, and the QE scaling functions calculated in both methods were used to calculate and predict charge-changing neutrino-nucleus cross sections of the ( $\nu_{\mu}$ ,  $\mu^{-}$ ) and ( $\bar{\nu}_{\mu}$ ,  $\mu^{+}$ ) reactions on  ${}^{12}C$  at energies from 1 to 2 GeV. The asymmetry in the CDFM QE scaling function was introduced in a phenomenological way. These analyses make it possible to gain information about the nucleon correlation effects on both nucleon momentum and local density distributions. It became clear that only the detailed knowledge of the behavior of  $n(k)$ at high momenta in realistic nuclear systems could lead to quantitative agreement with the experimental scaling function. On the other hand, the behavior of the latter gives valuable information about the *NN* correlation effects on the tail of the momentum distribution. So, it was shown in Ref. [\[16\]](#page-4-0) within the CDFM that the *y*- and  $\psi'$ -scaling data are informative for  $n(k)$  at momenta up to  $k \approx 2{\text -}2.5 \text{ fm}^{-1}$ , and it was concluded that further experiments are necessary in studies of the high-momentum components of the nucleon momentum distributions.

The aim of the present work is to consider in more detail the connection between the *NN* forces in nuclear media and their effect on the components of  $n(k)$  from one side and, from the other side, the role of  $n(k)$  on the behavior of the QE scaling function. For this purpose, we use first the MD in a hard-sphere dilute Fermi gas model (HSDFG) (e.g., Refs. [\[44–48\]](#page-5-0)) to calculate the scaling function. Second, we attempt to throw light on the connection between the generally established high-momentum asymptotics of *n*(*k*) [\[44,45,49–51\]](#page-5-0) and the QE scaling function. The latter makes it possible to establish (at least approximately) the particular form of the power-law decrease of  $n(k)$  at large values of  $k$ . This makes it possible to extract additional information about the *NN* forces from the description of the superscaling phenomenon.

The theoretical scheme and the results of calculations are given in Sec. II. The conclusions are summarized in Sec. [III.](#page-3-0)

# **II. THEORETICAL SCHEME AND RESULTS OF CALCULATIONS**

In the first part of this section, we consider the hard-sphere dilute Fermi gas, i.e., the low-density Fermi gas whose particles interact via a repulsive hard-core potential (see, e.g., Refs. [\[44–48\]](#page-5-0)), and we use the nucleon momentum distribution in such a system to calculate the scaling function  $f^{\text{HSDFG}}(\psi')$ . The quantities of interest in the HSDFG model can be expanded in powers of the parameter  $k_F c$ , where  $c$ (*>*0) denotes the hard-core radius in the *NN* interactions or it is identified with the scattering length in free space, and  $k_F$  is the Fermi momentum. In Ref.  $[44]$ , the value of  $k_F c$  is adopted to be equal to 0*.*70*,* which corresponds to an *NN* core radius of  $c = 0.50$  fm and a typical value of the Fermi momentum  $k_F = 1.40$  fm<sup>-1</sup>. As was pointed out by Migdal [\[46\]](#page-5-0),  $n(k)$  in the normal Fermi gas is discontinuous at the Fermi momentum. The analytical expressions for the dimensionless  $n(k)$  in the

HSDFG obtained in Refs. [\[44,48\]](#page-5-0) have the form

$$
n(k) = n_{<}(k) + n_{>}(k) \text{ with } \begin{cases} n_{<}(k) = 0 \text{ for } k > k_F, \\ n_{>}(k) = 0 \text{ for } k < k_F. \end{cases} (1)
$$
At  $k < k_F$ :

$$
n_{<}(k) = 1 - \frac{\nu - 1}{3\pi^{2} x} (k_{F} c)^{2} \left[ (7 \ln 2 - 8)x^{3} + (10 - 3 \ln 2)x + 2 \ln \frac{1 + x}{1 - x} - 2 (2 - x^{2})^{3/2} \ln \frac{(2 - x^{2})^{1/2} + x}{(2 - x^{2})^{1/2} - x} \right],
$$
\n(2)

where  $x = k/k_F$  and  $v = 4$  [\[44\]](#page-5-0) is adopted. At  $1 < x < \sqrt{2}$ :

$$
n_{>}(k) = \frac{\nu - 1}{6\pi^{2}x}(k_{F}c)^{2} \Biggl\{ (7x^{3} - 3x - 6) \ln \frac{x - 1}{x + 1} + (7x^{3} - 3x + 2) \ln 2 - 8x^{3} + 22x^{2} + 6x - 24 + 2(2 - x^{2})^{3/2} \Biggr[ \ln \frac{2 + x + (2 - x^{2})^{1/2}}{2 + x - (2 - x^{2})^{1/2}} + \ln \frac{1 + (2 - x^{2})^{1/2}}{1 - (2 - x^{2})^{1/2}} - 2 \ln \frac{x + (2 - x^{2})^{1/2}}{x - (2 - x^{2})^{1/2}} \Biggr] \Biggr\}.
$$
\n(3)

At 
$$
\sqrt{2} < x < 3
$$
:  
\n
$$
n_{>}(k) = \frac{v - 1}{6\pi^2 x} (k_F c)^2 \left\{ (7x^3 - 3x - 6) \ln \frac{x - 1}{x + 1} -8x^3 + 22x^2 + 6x - 24 + (7x^3 - 3x + 2) \right\}
$$
\n
$$
\times \ln 2 - 4(x^2 - 2)^{3/2} \left[ \arctan \frac{(x + 2)}{(x^2 - 2)^{1/2}} + \arctan \frac{1}{(x^2 - 2)^{1/2}} - 2 \arctan \frac{x}{(x^2 - 2)^{1/2}} \right] \right\}.
$$
\n(4)

At 
$$
x > 3
$$
:

$$
n_{>}(k) = 2\frac{v-1}{3\pi^2 x}(k_F c)^2 \left\{ 2\ln\frac{x+1}{x-1} - 2x + (x^2 - 2)^{3/2} \times \left[ 2\arctan\frac{x}{(x^2 - 2)^{1/2}} - \arctan\frac{x-2}{(x^2 - 2)^{1/2}} - \arctan\frac{(x+2)}{(x^2 - 2)^{1/2}} \right] \right\}.
$$
 (5)

The momentum distribution in the HSDFG model is presented in Fig. [1](#page-2-0) for  $k_F c = 0.70$ .

Following the definition of the  $\psi'$ -scaling function given by Barbaro *et al.* [\[11\]](#page-4-0), one can obtain for the case of the HSDFG

$$
f^{\text{HSDFG}}(\psi') = \frac{3}{2} \int_{|\xi|/\eta_F}^{\infty} x n(x) \, dx,\tag{6}
$$

<span id="page-2-0"></span>

FIG. 1. Momentum distribution  $n(k)$  in hard-sphere dilute Fermi gas [\[44\]](#page-5-0) as a function of  $x = k/k_F$ .

where  $\eta_F = k_F / m_N$ ,  $m_N$  being the nucleon mass, and

$$
\zeta = \psi' \left\{ \left[ \sqrt{1 + \eta_F^2} - 1 \right] \left[ 2 + \psi'^2 \left( \sqrt{1 + \eta_F^2} - 1 \right) \right] \right\}^{1/2}.
$$
\n(7)

In Eqs. [\(6\)](#page-1-0) and (7), the dimensionless scaling variable  $\psi^2$ (in units of the Fermi energy) has the physical meaning of the smallest kinetic energy that one of the nucleons responding to an external probe can have [\[11\]](#page-4-0). Since  $\eta_F^2 \ll 1$ , we write Eq.  $(6)$  as

$$
f^{\text{HSDFG}}(\psi') \simeq \frac{3}{2} \int_{|\psi'|}^{\infty} x n(x) dx.
$$
 (8)

It can be seen from Eq. (8) that, as expected, the HSDFG system under consideration also *exhibits superscaling*. Equation (8) was obtained using the following approximations [\[43\]](#page-5-0):

- (i) The Fermi momentum distribution of the initial nucleon in the nucleus  $\frac{3}{4\pi k_F^3} \theta(k_F - |k|)$  is replaced by the MD (with dimension), namely,  $\int P(\vec{k}, E) dE$ , where  $P(\vec{k}, E)$ is the spectral function.
- (ii) For  $k > k_F$ , the step function  $\theta(k_F |k|)$  is retained to take into account approximately Pauli blocking for the final nucleon.

In Figs. 2 and 3, we give the results for the HSDFG scaling function (in logarithmic and linear scale, respectively) calculated for different values of  $k_F c$  from 0.70 to 0.28 and compared with the result for the scaling function in the RFG model. One can see that the HSDFG scaling function is extended for large negative values of  $\psi'$  in contrast to the case of the RFG scaling function, but there is no good agreement with the experimental data. One can also see in the figures the step behavior of the scaling function which reflects the discontinuity of  $n(k)$  at  $k = k_F$ . For these reasons, we next consider the relation between the asymptotic behavior of the momentum distribution and the  $\psi'$ -scaling function.

A relationship between the *NN* force in nuclear medium  $\tilde{V}_{NN}(k)$  and the asymptotic behavior of the momentum distribution  $n(k)$  was derived with great generality in



FIG. 2. Scaling function  $f(\psi')$  in HSDFG calculated for different values of  $k_F c$  in comparison with RFG model results. Grey area shows experimental data from Ref. [\[13\]](#page-4-0).

Refs. [\[49–51\]](#page-5-0). In those articles, Amado and Woloshyn showed that the asymptotics of  $n(k)$  at large values of  $k$  is a power-law decrease, that is,

$$
n(k) \xrightarrow[k \to \infty]{} \left[ \frac{\tilde{V}_{NN}(k)}{k^2} \right]^2, \tag{9}
$$

where  $\tilde{V}_{NN}(k)$  is the Fourier transform of the *NN* interaction  $\tilde{V}_{NN}(r)$ . In the case of  $\delta$  forces (i.e., in the HSDFG), the asymptotics is  $n(k)$  ∼ 1/k<sup>4</sup> [\[45\]](#page-5-0). It is still unknown if *k* or  $k/A$  must be large for Eq. (9) to apply  $[49-51]$ . In principle, it is shown in Ref.  $[44]$  that  $n(k)$  in the HSDFG decreases faster than  $k^{-4}$ , typically like  $\sim 1/k^{4+m}$ , where *m* > 0. In Fig. [4,](#page-3-0) we show  $x^4n(x)$  as a function of  $x = k/k_F$ . It can be seen that the HSDFG *n*(*k*) decreases approximately like  $\sim 1/k^{4+m}$  with a small value of *m*, because the result for  $x^4n(x)$  is almost a straight line which decreases slowly with the increase of  $x \equiv k/k_F > 1$ .

Next in this work we study the question about the general feature of the *NN* force  $\tilde{V}_{NN}(k)$  that results in an  $n(k)$  with a power-law behavior that best agrees with the scaling function. To this aim, we assume a *NN* potential  $\tilde{V}_{NN}(r)$  different from a *δ* function and calculate the scaling function using different asymptotics for  $n(k)$  in the dilute Fermi gas at  $k > k<sub>F</sub>$ .



FIG. 3. Same as Fig. 2, but on a linear scale for  $f(\psi')$ .

<span id="page-3-0"></span>

FIG. 4. Momentum distribution in HSDFG *n*(*x*) multiplied by  $x^4 = (k/k_F)^4$ .

Therefore, we look for the proper value of *m*. For  $k < k_F$ , we use  $n(k)$  [Eq. [\(2\)](#page-1-0)] from Ref. [\[44\]](#page-5-0), but for  $k > k_F$ , we use

$$
n(k) = N \frac{1}{k^{4+m}} \quad \text{for} \quad k > k_F. \tag{10}
$$

The value of *N* is obtained by the total normalization of  $n(k)$  and is equal to

$$
N = \frac{0.24}{3} (1 + m) k_F^{4+m}.
$$
 (11)

The factor 0*.*24 corresponds to the result for the part of the normalization (for  $k > k_F$ ) from the total normalization condition:

$$
\frac{3}{4\pi k_F^3} \int n(\vec{k})d^3\vec{k} = 1.
$$
 (12)

Finally, from Eq. [\(8\)](#page-2-0) one can obtain the following expression for the scaling function:

$$
f(\psi') = 0.12 \left(\frac{1+m}{2+m}\right) \frac{1}{|\psi'|^{2+m}}.
$$
 (13)



FIG. 5. Scaling function in a dilute Fermi gas calculated using Eq. (13) for different values of *m* in the asymptotics of the momentum distribution  $n(k)$  ∼ 1/ $k^{4+m}$  given in comparison with the RFG result. Grey area shows experimental data from Ref. [\[13\]](#page-4-0).

In Fig. 5, we present the results for the scaling function [Eq. [\(8\)](#page-2-0)] for different values of *m*, compared with the RFG model result. One can see that agreement with the experimental QE scaling function is achieved when the value  $m \approx 4.5$  is used in Eqs.  $(10)$ ,  $(11)$ , and  $(13)$ . This means that the power-law decrease of  $n(k)$  which gives an optimal agreement with the data is

$$
n(k) \approx \frac{1}{k^{8.5}}.\tag{14}
$$

We should note that this particular form of the power-law asymptotics is close to that obtained in the CDFM [\[25\]](#page-4-0)

$$
n(k) \sim \frac{1}{k^8},\tag{15}
$$

i.e., it corresponds to  $n(k) \sim 1/k^{4+m}$  with  $m = 4$ . The inverse Fourier transform of  $V_{NN}(k)$  for  $m = 4$  and  $m = 5$  gives  $V_{NN}(r) \sim 1/r$  and  $V_{NN}(r) \sim 1/r^{1/2}$ , respectively.

We would like to emphasize the consistency of both the optimal asymptotics of  $n(k)$  for the dilute Fermi gas found in this work  $[Eq. (14)]$  with that in the CDFM  $[Eq. (15)]$ . As was shown in Refs. [\[15–18\]](#page-4-0), the calculated QE scaling function  $f(\psi')$  in the CDFM agrees well with the experimental scaling function. This fact shows that the behavior of the QE scaling function depends mainly on the particular form of the power-law asymptotics of the nucleon momentum distribution. This is proved in our work by the similarities of the result for the case of an interacting dilute Fermi gas with that obtained in the CDFM as a model accounting for *NN* correlations in realistic finite nuclear systems.

#### **III. CONCLUSIONS**

The results of the present work can be summarized as follows:

- (i) The superscaling observed in inclusive electron scattering from nuclei is considered within the model of dilute Fermi gas with interactions between particles. The latter gives an improvement over the results of the relativistic noninteracting Fermi gas model, allowing one to describe the QE scaling function for  $\psi' < -1$ , whereas the RFG model gives  $f(\psi') = 0$  in this region.
- (ii) It is established that the hard-sphere (with *δ*-forces between nucleons) approximation for the dilute Fermi gas is quite a rough one. The use of more realistic *NN* forces leading to  $m \simeq 4.5$  instead of  $m = 0$  (for *δ*-force) in the well-known power-law asymptotics of the momentum distribution  $n(k) \sim 1/k^{4+m}$  at large *k* leads to a good explanation of the data for the *ψ* -scaling function in inclusive electron scattering from a wide range of nuclei.
- (iii) The asymptotics of  $n(k) \sim 1/k^{8.5}$  found in the dilute Fermi gas by optimal fit to the data for  $f(\psi')$  is similar to that in the CDFM ( $\sim 1/k^8$ ) [\[25\]](#page-4-0) which, being a theoretical correlation model, describes the superscaling in the quasielastic part of the electron-nucleus scattering. Thus, the momentum distribution in the dilute Fermi gas model with realistic *NN* forces can serve as an "effective"

<span id="page-4-0"></span>momentum distribution (a steplike one with a discontinuity) which gives a similar result for  $f(\psi')$  as the correlation methods for realistic finite nuclear systems. It can be concluded that the momentum distribution with asymptotics from  $\sim 1/k^8$  to  $\sim 1/k^{8.5}$  is the proper one for explaining the phenomenological shape of the scaling function obtained from inclusive QE electron scattering.

As already mentioned, the superscaling is due to the *specific high-momentum tail* of  $n(k)$  similar for all nuclei which is known to be caused by the short-range and tensor correlations related *to peculiarities of the NN forces* near their core. The main result of the present work might be the observation that the values of  $f(\psi')$  for  $\psi' < -1$  depend on *the particular form* of the power-law asymptotics of  $n(k)$  at large *k* which is related to *a corresponding particular behavior* of the *in-medium NN forces* around the core. Namely, we point out that the power-law decrease of  $n(k)$  as  $\sim 1/k^{4+m}$  with  $m \approx 4.5$  in the interacting dilute Fermi gas is the proper one, and it is close to that obtained in CDFM  $(m = 4 \, [25])$ which describes the superscaling correctly as well. The *NN*

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force for  $m = 4$  is expected to go as  $V_{NN}(r) \sim 1/r$  and for  $m = 5$  to go as  $V_{NN}(r) \sim (1/r)^{1/2}$ . Hence, the present study allows one to conclude that the important property of the repulsive short-range core [leading to *NN* correlations and high-momentum tail of  $n(k)$ ] is that it goes to infinity for  $r \rightarrow 0$  as  $1/r$  or softer. The link between the asymptotic behavior of  $n(k)$  and *NN* forces implies that inclusive QE electron scattering from nuclei provides important information on the *NN* forces in the nuclear medium.

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