Negative parity states and some electromagnetic transition properties of even-odd erbium isotopes

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The negative parity states and some electromagnetic transition properties of even-odd erbium isotopes (^{159,161,163,165}Er) were studied within the framework of the interacting boson-fermion model. The single fermion is assumed to be in one of the $lh_{9/2}$, $3p_{3/2}$, $2f_{5/2}$, and $3p_{1/2}$ single-particle orbits. It was found that the calculated negative parity state energy spectra of the even-odd erbium isotopes agree quite well with the experimental data. The $B(E2)$ values were also calculated and compared with the experimental data.

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I. INTRODUCTION

The interacting boson approximation represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques, which have also found recent application to problems in atomic, molecular, and low-energy physics [1,2]. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy.

The interacting boson model (IBM) [3] and its extension to the odd-A nuclei, the interacting boson-fermion model (IBFM) [4], have proved to be able to give a successful description of widely varying classes of nuclei situated away from closed-shell configurations. Here, we apply the IBFM model to account for even-odd erbium isotopes.

Detailed work has been done on the structure of erbium nuclei in recent years. Gill *et al.* [5] studied the (n,γ) reaction for ^{168}Er and obtained a number of new levels for the first time. Alfter *et al.* [6] determined $M1/E2$ multipole mixing ratios of erbium isotopes by experiment. Minkov *et al.* [7] derived analytic expressions for the energies and $B(E2)$ transition probabilities in the states of the ground and γ bands of heavy deformed nuclei within a collective vector-boson model with SU(3) dynamical symmetry. Barrett *et al.* [8] calculated the multipole mixing ratios of ^{168}Er within the framework of the interacting boson approximation. Guo *et al.* [9] calculated the energies of excited states and the values of $B(E2)$ of $^{159-163}$Er by using the IBFM. Yazar *et al.* [10] explored the energy levels and the electric quadrupole transition probabilities $B(E2; I_i \to I_f)$ and γ-ray $E2/M1$ mixing ratios for selected transitions of $^{162-164-166-168-170}$Er.

In recent years, many negative parity states of the even-odd nuclei, such as even-odd erbium isotopes, have been found experimentally. It is generally believed that such negative parity low-spin states can be explained in particle-core coupled-type models. $^{159-165}$Er has 68 protons and 91–97 neutrons; it is thus appropriate to describe $^{159-165}$Er in the interacting boson fermion approximation (IBFA) by the coupling of a single fermion to the $^{158-164}$Er even-even core. Over the major shell $N = 82$, there are four available negative parity single-particle levels, the $lh_{9/2}$, $3p_{3/2}$, $2f_{5/2}$, and $3p_{1/2}$. For the boson core, the IBM-1 basis states are used. To describe the negative parity states, however, it is necessary to consider the inclusion of all four negative parity single-particle levels. The inclusion of multilevel possibilities into the IBFM has been analyzed by Scholten [11], who developed a formalism based on the BCS (Bardeen, Cooper, and Schrieffer) equations. The single-particle energies were calculated by using the relations given by Ref. [12], and related formulas [Eqs. (10), (11), and (12)] are given in Sec. III.

The aim of the present work is to do a systematic study of the Er isotopes within the IBFA-1 model to give a comprehensive view of these isotopes in a rather simple way. As the model we are using has been extensively described recently [4] we shall present here only the results of the calculation and refer the reader to that paper for details of the model. We restrict the discussion to the negative parity states of the ^{165}Er isotope because the negative parity states of ^{159,161,163}Er isotopes were presented elsewhere [13]. The results of the IBFM multilevel calculations for ^{159,161,163,165}Er are presented for energy levels for which transitions probabilities were compared with the corresponding experimental data in Sec. III.

II. ELECTROMAGNETIC TRANSITION PROBABILITIES OF AN EVEN-EVEN CORE

The IBM [14] provides a unified description of collective nuclear states in terms of a system of interacting bosons. The $^{159-165}$Er isotopes have 68 protons 91–97 neutrons, which fill the orbits above major shell closure at $N = 82$, characterized by 9–15 particle-like neutron states. It is thus appropriate to describe $^{159-165}$Er in the IBFM model by coupling of a single fermion (neutron) to the $^{158-164}$Er even-even core. Within the IBM, these structure or shape changes correspond to the system moving among the vibrational SU(5), γ-unstable O(6), and rotational SU(3) limits. The $^{158-164}$Er nuclei have been considered as a transitional nucleus from SU(3) to O (6) [15]. The IBM-1 Hamiltonian we used to describe the $^{158-164}$Er nuclei has the standard form as given in Ref. [16]. The calculations were done by using the computer codes PHINT for energies and BEFM for $B(E2)$ values, both written by Scholten [11]. The IBM-1 Hamiltonian is nonlinear in the parameters. To obtain the values of the parameters that give

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the best fit we have to calculate for each energy level the difference between its experimental and calculated values. Then we have to sum over the squares of all these differences to find a local minimum to this summation. The least-square-fit procedure was used to find the best fit to the three lowest bands (ground-state, γ-state, and β-state bands) of the erbium isotope under consideration. We find that the calculated energy states that were obtained in the present work are largely consistent with experimental data although the γ states and β states may be considered to show irregularities.

The IBM of Arima and Iachello [1,2] has become widely accepted as a tractable theoretical scheme of correlating, describing, and predicting low-energy collective properties of complex nuclei. In this model it is assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels, one with angular momentum $L = 0$ (s boson) and another with $L = 2$ (d boson). In the original form of the model known as IBM-1, proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure associated with it. In terms of s- and d-boson operators the most general IBM-I Hamiltonian can be expressed as [14]

$$H = \varepsilon_s(s^\dagger s) + \varepsilon_d(d^\dagger \cdot d) + \sum_{L=0,2,4} c_L[(d^\dagger d^\dagger)^{(L)} \cdot (dd)^{(L)}] + (1/2)v_0\left[(d^\dagger d^\dagger)_0^{(0)} s^2 + (s^\dagger)^2 (dd)_0^{(0)}\right] + \sqrt{1/2}v_2[(d^\dagger d^\dagger)^{(2)} ds]_0^{(0)} + [s^\dagger d^\dagger (dd)^{(2)}]_0^{(0)} + (1/2)u_0(s^\dagger)^2 s^2 + 1/\sqrt{5}u_2 s^\dagger s(d^\dagger \cdot d). \quad (1)$$

This Hamiltonian contains two one-body terms, (ε_s and ε_d) and seven two-body interactions [$c_L(L = 0, 2, 4)$, $v_L(L = 0, 2)$, $u_L(L = 0, 2)$], where ε_s and ε_d are the single-boson energies, and c_L, v_L and, u_L describe the two-boson interactions. However, it turn out that for fixed boson number N, only one of the one-body terms and five of the two-body terms are independent, as can be seen by noting $N = n_s + n_d$. The Hamiltonian can be rewritten in terms of the Casimir operators of U(6) groups. In that case, one says that the Hamiltonian H has a dynamical symmetry. These symmetries are called SU(5) vibrational, SU(3) rotational, and O(6) γ-unstable.

The values of the interaction parameters for the ^{164}Er isotope in the IBM-1 Hamiltonian (in terms of code PHINT notation) that gave the best fit to the experimental data are $EPS = 0.0208$, $ELL = 0.0155$, $QQ = -0.0267$, $OCT = 0.0057$, $CHQ = -1.130$ and $HEX = 0.0143$ MeV.) The values of the interaction parameters for the ^{158,160,162}Er isotopes were given in Ref. [13]. The calculated energy levels of ^{158,160,162,164}Er isotopes are shown in Fig. 1 and compared with the experimental levels.

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. The most important

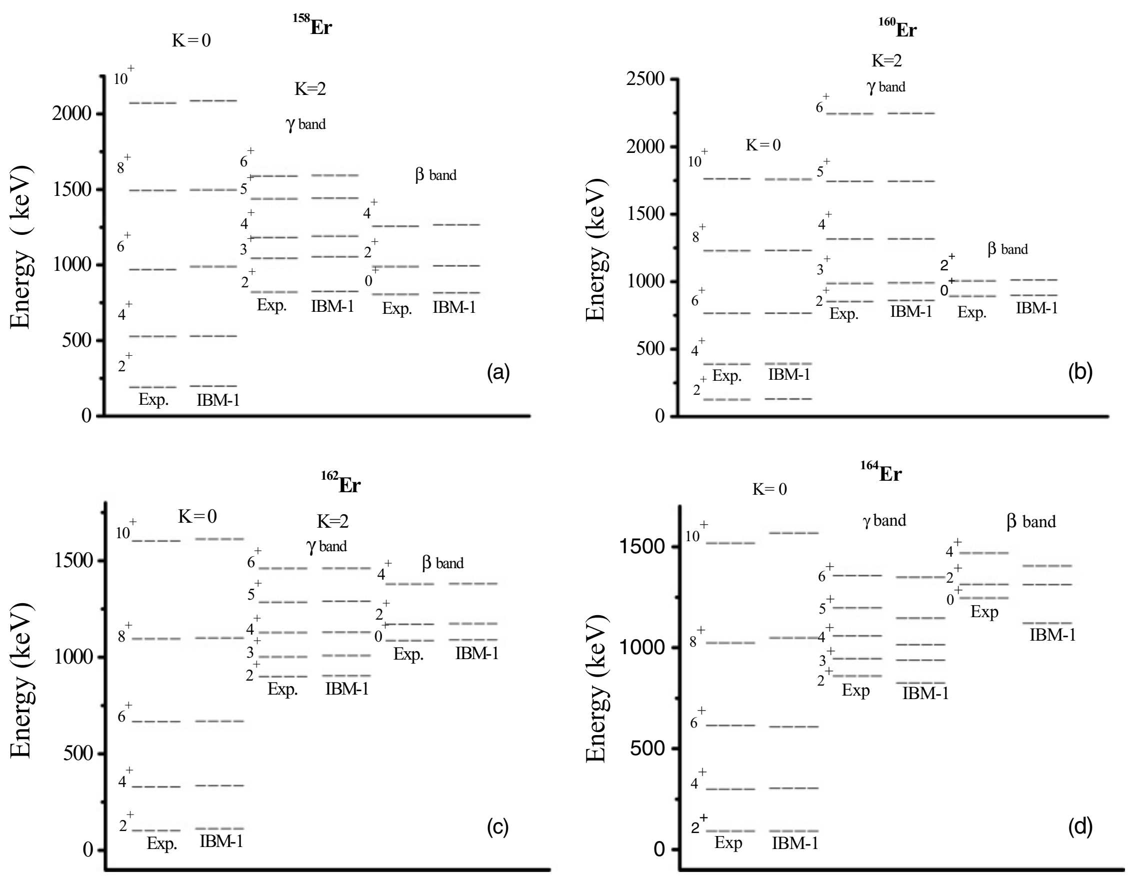

FIG. 1. The three lowest rotational bands in spectra of (a) ^{158}Er [13], (b) ^{160}Er [13], (c) ^{162}Er [13] and (d) ^{164}Er. In each band the experimental data are plotted on the left and calculated values on the right.

electromagnetic features are the $E2$ transitions. The $B(E2)$ values were calculated by using the $E2$ operator. The $E2$ transition operator must be a Hermitian tensor of rank two and therefore the number of bosons must be conserved. Since with these constraints there are two operators possible in the lowest order, the general $E2$ operator can be written as [14]

$$\begin{aligned} T_m(E2) &= e_B Q_B, \\ Q_B &= (s^+ x\tilde{d} + d^+ x\tilde{s})^2 + \chi(d^+ x\tilde{d})^{(2)}, \end{aligned} \tag{2}$$

where χ is a parameter shown by microscopic theory to lie between and $\sqrt{7/2}$ and $-\sqrt{7/2}$ determines the structure of the quadrupole operator and is determined empirically, Q_B is the boson quadrupole operator, and e_B is the "effective boson charge". For IBM-1 calculations effective charges e_B were taken as $e_B = 0.13$ eb for ^{164}Er and $e_B = 0.11$ eb for ^{158,160,162}Er isotopes [13]. The $B(E2)$ strength for the $E2$ transitions is given by

$$B(E2; L_i \to L_f) = 1/(2L_i + 1)^{1/2} |\langle L_f \| T_m(E2) \| L_i \rangle|^2. \tag{3}$$

Since the erbium nucleus has a rather rotational character, taking into account the dynamic symmetry location of the even-even erbium nuclei at the IBM phase triangle where their parameter sets are at the O(6)-SU(3) transition region and closer to SU(3) rotational character and possessing good rotational states, we used the multiple expansion form of the Hamiltonian for our approximation. The predicted $B(E2)$ values agree very well with the theoretical ones, which suggests that the wave functions obtained in this work are reliable. Some calculated $B(E2)$ values from the ground-state band are given in Table I.

III. THE INTERACTING BOSON-FERMION MODEL AND ENERGY LEVELS

In the IBFM, odd-A nuclei are described by the coupling of the odd fermionic quasiparticle to a collective boson core. The total Hamiltonian can be written as the sum of three parts:

$$H = H_B + H_F + V_{\mathrm{BF}}, \tag{4}$$

where H_B is the usual IBM-1 Hamiltonian [14] for the even-even core, H_F is the fermion Hamiltonian containing only one-body terms, and V_{BF} is the boson-fermion interaction that describes the interaction between the odd quasinucleon and the even-even core nucleus. V_{BF} is dominated by three terms: a monopole interaction characterized by the parameter A_0, which plays a minor role in actual calculations, and the more important terms arising from the quadrupole interaction [4] characterized by Γ_0 and the exchange of the quasiparticle with one of the two fermions forming a boson [15] characterized by Λ_0. H_F is the fermion Hamiltonian containing only one-body terms and

$$H_F = \sum_{jm} \varepsilon_j a^+_{jm} a_{jm}, \tag{5}$$

where the ε_j are the quasiparticle energies and $a^+_{jm} a_{jm}$ is the creation (annihilation) operator for the quasiparticle in the eigenstate $|jm\rangle$.

The boson-fermion interaction V_{BF} that describes the interaction between the odd quasinucleon and the even-even core nucleus contains, in general, many different terms and is rather complicated, but it has been shown to be dominated by the following three terms:

$$\begin{aligned} V_{\mathrm{BF}} = &\sum_i A_j [(d^+ x\tilde{d})^{(0)} x (a^+_j x\tilde{a}_j)^{(0)}] \\ &+ \sum_{jj} \Gamma_{jj} [Q^{(2)} x (a^+_j x\tilde{a}_j)^{(2)}]^{(0)}_0 \\ &+ \sum_{jjj} \Lambda^j_{jj} : [(d^+ x\tilde{a}_j)^{(j)} x (a^+_j x\tilde{d}_j)^{(j)}]^{(0)}_0 \end{aligned} \tag{6}$$

The first term in V_{BF} is a monopole interaction, which plays a minor role in actual calculations, the dominant terms are the second and third, which arise from the quadrupole interaction. The third term represents the exchange of the quasiparticle with one of the two fermions forming a boson; Talmi [15] has shown that this exchange force is a consequence of the Pauli principle for the quadrupole interaction between protons and neutrons. The remaining parameters in Eq. (6) can be related to the BCS occupation probabilities u_j, ν_j of the single-particle orbits by

$$\Gamma_{jj} = \sqrt{5}\Gamma_0(u_j u_j + \nu_j \nu_j) Q_{jj}, \tag{7}$$

$$\begin{aligned} \Lambda^j_{jj} = &-\sqrt{5}\Lambda_0[(u_j \nu_j + \nu_j u_j) Q_{jj} \beta_{jj} \\ &+ (u_j \nu_j + \nu_j u_j) Q_{jj} \beta_{jj}]/\sqrt{2j+1}, \end{aligned} \tag{8}$$

TABLE I. $B(E2; I \to I-2)$ values for ground-state bands of $^{158-164}$Er isotopes.

N	$B(E2)$ (W.u.)				$B(E2)$ ratios	
	$4_1 \to 2_1$		$2_1 \to 0_1$		$(4_1 \to 2_1)/(2_1 \to 0_1)$	
	Theory	Exp. [17]	Theory	Exp. [17]	Theory	Exp. [17]
90	92 [13]	177(5)	63 [13]	119(5)	1.46 [13]	1.48(5)
92	113 [13]	262(15)	91 [13]	166(7)	1.24 [13]	1.57(15)(7)
94	162 [13]		113 [13]	191(1)	1.43 [13]	
96	261	258(26)	209	218(7)	1.24	1.18(7)(26)

TABLE II. BCS parameters for the multilevel calculations of erbium isotopes (ε_j in MeV).

s.p. orbits	^{159}Er [13]		^{161}Er [13]		^{163}Er [13]		^{165}Er	
	ε_j	ν_j^2	ε_j	ν_j^2	ε_j	ν_j^2	ε_j	ν_j^2
$1h_{9/2}$	2.154	0.883	2.910	0.885	2.154	0.883	1.702	0.883
$3p_{3/2}$	2.70	0.670	1.510	0.234	2.70	0.670	2.190	0.670
$2f_{5/2}$	1.983	0.195	1.533	0.324	1.983	0.195	1.583	0.195
$3p_{1/2}$	2.06	0.068	1.315	0.420	2.06	0.068	1.638	0.068

where Q_{jj} are single-particle matrix elements of the quadrupole operator and

$$\beta_{jj} = (u_j\nu_j + \nu_j u_j)Q_{jj}/(\varepsilon_j + \varepsilon_j - \hbar w) \tag{9}$$

are the structure coefficients of the d boson deduced from microscopic considerations [18] with $\hbar w$ being the energy of a $|D\rangle$ pair relative to an $|S\rangle$ pair [19].

The BCS trial wave function [12]

$$|\phi\rangle = \prod_j (u_j + \nu_j c_j^\dagger \tilde{c}_j^\dagger)|0\rangle \tag{10}$$

clearly mixes components with various numbers of particles. Consequently, one usually requires that expectation values of the particle number operator fulfills the relation

$$\langle\phi|\hat{N}|\phi\rangle = \min, \tag{11}$$

where $n = Z$ or N (proton or neutron number, respectively). This equation can be treated as an auxiliary constraint when minimizing the total energy:

$$\langle\phi|H|\phi\rangle = \min. \tag{12}$$

Minimization in Eq. (12) with subsidiary condition Eq. (11) leads to the well-known expressions for the coefficients and for the BCS wave function Eq. (10); the appropriate considerations will not be repeated here (see, e.g., Ref. [20]).

The BCS occupation probability ν_j and the quasiparticle energy ε_j of each single-particle orbital can be obtained by solving the gap equations:

$$\varepsilon_j = [(E_j - \lambda)^2 + \Delta^2]^{1/2}, \tag{13}$$

$$\nu_j^2 = \frac{1}{2}\left[1 - \frac{(E_j - \lambda)}{\varepsilon_j}\right], \tag{14}$$

where E_j is the single-particle energy calculated from the relations in Ref. [18], λ is the Fermi level energy, and Δ is the pairing gap energy, which was chosen to be $12A^{-1/2}$ MeV [21]. That leaves the strengths A_0, Γ_0, and Λ_0 as free parameters to be varied to give the best fit to the excitation energies.

The whole Hamiltonian was then diagonalized in the model space. The boson interaction parameters were determined from a least-square calculation on the energy spectra of the boson-core nuclei. This left us with two boson-fermion interaction parameters, Γ_0 and Λ_0, and the fermion single-particle energies ε_j in the model. Those parameters were determined by least-square calculation on the energy spectra of the even-odd Er isotopes. It was found that the obtained parameters vary smoothly versus the change of the mass numbers. The calculated wave functions can be used to calculate the $B(E2)$ values of the electromagnetic transitions. This serves as a further test of the model.

The Hamiltonian [Eq. (4)] was diagonalized by means of the computer program ODDA [21] in which the IBFM parameters are identified as $A_0 =$ BFM, $\Gamma_0 =$ BFQ, and $\Lambda_0 =$ BFE. The parameters for the ^{164}Er core were derived in the present work and are given in Sec. II and for the ^{158,160,162}Er core were given in [13] the quasiparticle energies and occupation probabilities used in this work are given in Table II.

The level calculation was used to fit experimental energy levels (up to the spin $11/2^-$ level at 820 keV) with the boson-fermion parameters $A_0 = 0.0$, $\Lambda_0 = 0.413$, and $\Gamma_0 = 0.589$ MeV for the ^{165}Er nucleus. The $11/2^-$ and $9/2^-$ levels at 822 and 689 keV, however, were calculated to be higher than the experimental values and are given in Fig. 2, but the present choice of parameters gives a good agreement for electromagnetic properties. The boson-fermion parameters for other erbium isotopes were given in Ref. [13].

IV. ELECTROMAGNETIC TRANSITION PROBABILITIES OF AN EVEN-ODD CORE

Calculation of electromagnetic transitions is a good test of the nuclear model wave functions. In this section, we discuss the calculation of the $E2$ transition strengths and compare the results with the available experimental data. In general, the electromagnetic transition operators can be written as sums of two terms, the first of which acts only on the boson part of the wave function and second of which acts only on the fermion part. In the IBFM, the $E2$ operator is

$$T^{(E2)} = e_B Q_B^{(2)} + e_F \sum_{jj''} Q_{jj'}(a_j x \tilde{a}_{j'})^{(2)}, \tag{15}$$

where e_B and e_F are the boson and fermion effective charges, respectively. The $E2$ boson and fermion effective charges and the boson gyromagnetic factors are adjustable parameters. The experimental $B(E2)$ values were used to find the best fit with PBEM [11] and to determine the boson effective charge e_B. The fermion effective charge e_F is taken to be equal [22] to e_B. In calculating the $E2$ transition rates, we chose the boson and fermion effective charges as $e_B = e_F$. The gyromagnetic

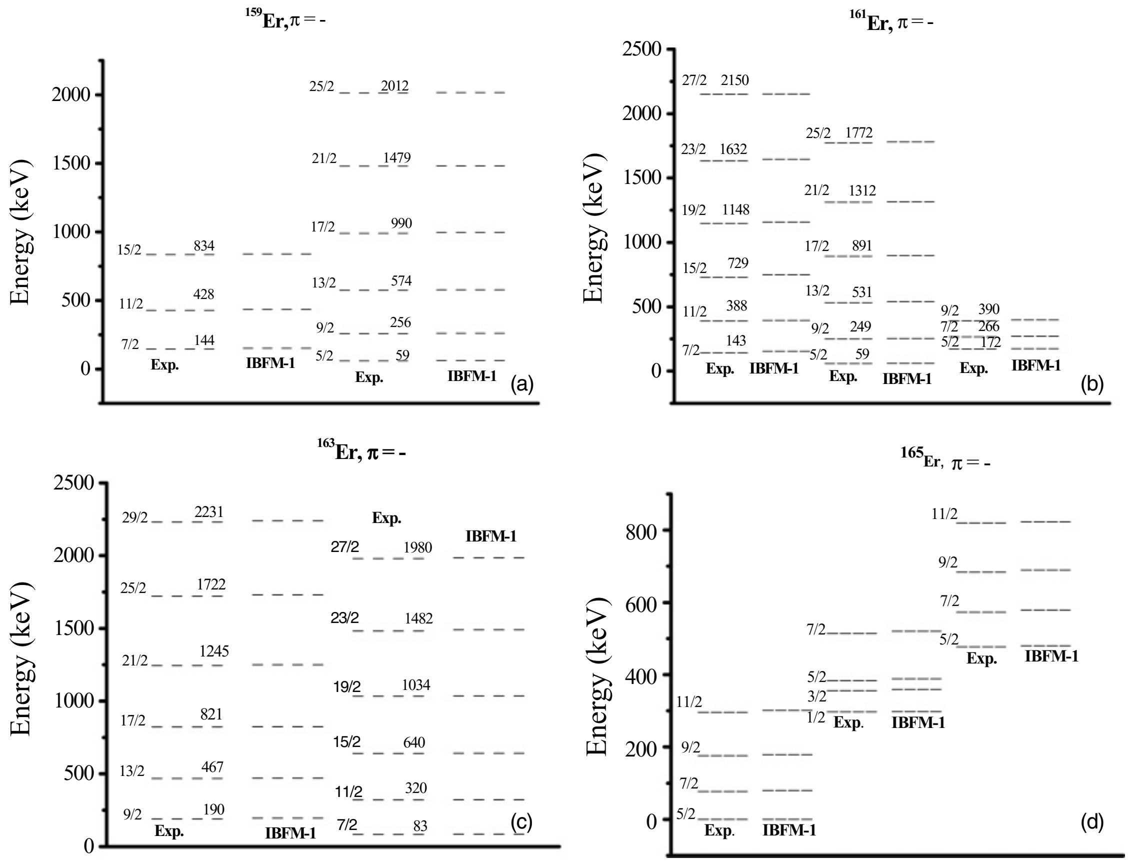

FIG. 2. Comparasion of some calculated energy levels for negative parity with experimental data of (a) ^{159}Er [13], (b) ^{161}Er [13], (c) ^{163}Er [13], and (d) ^{165}Er.

values used for the odd proton are $g_l = 1$ n.m. and $g_s = 3.9095$ n.m. (quenching of 0.7 included).

The negative, low-spin states of $B(E2)$ values, given in Table III, were determined with the boson and fermion effective charges These effective charges are equal to that of the even-even erbium core (Sec. II), which has an SU (3) → O(6) transitional structure with similarities to even-odd erbium isotopes.

TABLE III. The calculated and experimental $B(E2)$ values for ^{159}Er–^{165}Er.

		$B(E2)$ values (W.u.)	
		Theory	Exp [17,23]
^{159}Er	$\frac{7}{2}^-_1 \to \frac{3}{2}^-_1$	82 [13]	>55
^{161}Er	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$	0.52 [13]	>0.59
^{163}Er	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$	285 [13]	310
	$\frac{3}{2}^-_1 \to \frac{7}{2}^-_1$	21 [13]	25
	$\frac{13}{2}^-_1 \to \frac{9}{2}^-_1$	1.67 [13]	>0.43
	$\frac{3}{2}^-_1 \to \frac{5}{2}^-_1$	0.13 [13]	>1.8
^{165}Er	$\frac{7}{2}^-_1 \to \frac{3}{2}^-_1$	82	>55
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$	0.52	>0.59
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$	285	310
	$\frac{3}{2}^-_1 \to \frac{7}{2}^-_1$	21	25
	$\frac{13}{2}^-_1 \to \frac{9}{2}^-_1$	1.67	>0.43
	$\frac{3}{2}^-_1 \to \frac{5}{2}^-_1$	0.13	>1.8

V. SUMMARY AND CONCLUSION

In this paper we have carried out an analysis for the odd-mass erbium isotopes based on the IBFM-1. The nucleus is described by coupling a single fermion to the even-even core of $^{158-164}$Er. The boson core parameters have been obtained from an IBM-1 analysis and the main results for energy levels and quadrupole transition probabilities agree very well with experiment. In general, good agreement was obtained when compared with experiment. The boson-boson interaction parameters were fixed by the calculations on the boson core nuclei and the boson-fermion monopole interaction was omitted ($A_0 = 0.0$); there are only two (Λ_0 and Γ_0) free varying boson-fermion interaction parameters for each even-odd nucleus. The results indicate that the energy spectra of all different quasibands of the even-odd Er isotopes can be reproduced quite well. It is noticed, however, that the results of $B(E2)$ calculations for even-even erbium nuclei were in better agreement with the existing experimental data. Though the observed $B(E2)$ values for the odd Er isotopes are very few, the calculated and experimental $B(E2)$ values are shown in Table III for comparison.

The IBFM was used to calculate electromagnetic properties and, in general, good agreement was obtained when compared with experiment. It is noticed that although the collective degrees of freedom appear well described in the odd-even ^{159,161,163,165}Er nuclei, single-particle degrees of freedom still require improvement.

The IBFM was extended to include a multilevel calculation for ^{159,161,163,165}Er. The present study has shown that the IBFM provides a successful description for the energy level properties of the transitional ^{159,161,163,165}Er nuclei, for which four single-particle levels play a major role.

In general, the calculated values agree with the experimental data reasonably well. The $B(E2)$ values depend quite sensitively on the wave functions, which suggests that the wave functions obtained in this work are reliable. The model may be applied to many other even-odd nuclei and their many other nuclear properties.

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[1] A. Arima and F. Iachello, Ann. Phys. (NY) **99**, 253 (1976).
[2] A. Arima and F. Iachello, Ann. Phys. (NY) **123**, 468 (1979).
[3] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975).
[4] F. Iachello and P. Van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University Press, Cambridge, 1991).
[5] R. L. Gill, R. F. Casten, W. R. Phillips, B. J. Varley, C. J. Lister, J. L. Durell, J. A. Shannon, and D. D. Warner, Phys. Rev. C **54**, 2276 (1996).
[6] I. Alfter, E. Bodenstedt, W. Knichel, and J. Schüth, Nucl. Phys. **A635**, 273 (1998).
[7] N. Minkov, S. B. Drenska, P. P. Raychev, R. P. Roussev, and D. Bonatsos, Phys. Rev. C **60**, 034305 (1999).
[8] B. R. Barrett, S. Kuyucak, P. Navrtil, and P. Van Isacker, Phys. Rev. C **60**, 037302 (1999).
[9] R. S. Guo and L. M. Chen, J. Phys. G **26**, 1775 (2000).
[10] H. R. Yazar and I. Uluer, Pramana-J. Phys. **65** (3), 393 (2005).
[11] O. Scholten, Computer code PHINT, KVT, Groningen, Holland, 1980.
[12] De Voight and M. J. A. Dudek, Rev. Mod. Phys. **55**, 949 (1983).
[13] H. R. Yazar, J. Korean, Phys. Soc. **49** (1), 60 (2006).
[14] F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
[15] I. Talmi, in *Interacting Bose-Fermi System in Nuclei*, edited by F. Iachello (Plenum Press, New York, 1981), p. 329.
[16] O. Scholten, Ph.D. dissertation, University of Groningen, 1980.
[17] R. G. Helmer, Nucl. Data Sheets **72**, 83 (1994).
[18] J. Bardeen, L. N. Cooper, and J. R. Schriefer, Phys. Rev. **108**, 1175 (1957).
[19] B. S. Reehal and R. A. Sorensen, Phys. Rev. C **2**, 819 (1970).
[20] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vols. 1 and 2 (Benjamin, New York, 1969 and 1975).
[21] O. Scholten, Internal Report KVI 252, Computer Code ODDA, University of Groningen, 1980.
[22] O. Scholten, Prog. Part. Nucl. Phys. **14**, 189 (1985).
[23] Nuclear Data Sheets, http://www.nndc.bnl.gov/ensdf/.