Shell-model calculations of two-neutrino double-β decay rates of ^{48}Ca with the GXPF1A interaction

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The two-neutrino double-β decay matrix elements and half-lives of ^{48}Ca were calculated within a shell-model approach for transitions to the ground state and to the 2^+ first excited state of ^{48}Ti. We use the full *pf* model space and the GXPF1A interaction, which was recently proposed to describe the spectroscopic properties of the nuclei in the nuclear mass region $A = 47$–66. Our results are $T_{1/2}(0^+ \to 0^+) = 3.3 \times 10^{19}$ yr and $T_{1/2}(0^+ \to 2^+) = 8.5 \times 10^{23}$ yr. The result for the decay to the ^{48}Ti 0^+ ground state is in good agreement with experiment. The half-life for the decay to the 2^+ state is two orders of magnitude larger than that obtained previously.

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At present, the double-β ($\beta\beta$) decay is the most sensitive process for direct measurements of the electron neutrino mass [1–4]. For deriving the neutrino mass one needs, on the one hand, experimental half-lives for the neutrinoless $\beta\beta$ ($0\nu\beta\beta$) decay mode and, on the other hand, theoretical values of the nuclear matrix elements (NME) entering these half-lives formulae.

After many years of intense investigations and debate on different nuclear structure methods, accurate calculation of the NME relevant for $\beta\beta$ decay remains a challenging issue. Because many $\beta\beta$ emitters are nuclei with open shells, the proton-neutron random-phase approximation (pnQRPA) and its extensions have been the most used methods to perform such calculations [5–10]. However, due to the significant progress in shell-model (SM) configuration mixing approaches, there are now calculations performed with these methods for several nuclei [11–15]. In spite of their success in getting agreement with the experimental half-lives of the two-neutrino $\beta\beta$ ($2\nu\beta\beta$) decay mode, both pnQRPA- and SM-based approaches have some shortcomings that limit their predictive power for the NME in the case of the more interesting $0\nu\beta\beta$ mode. For example, within pnQRPA methods the NME exhibit a high sensitivity to the renormalization of the particle-particle strength in the 1^+ channel, whereas within SM one has to severely truncate the model space to make the diagonalization procedure tractable. To better understand the source of uncertainties of the NME calculations for $\beta\beta$ decay, a systematic comparison between calculations performed with both types of methods is needed. This comparison will become more feasible as the computational power of the shell-model methods expand to treat larger model spaces. The effective two-body interaction employed is also important, because the $B(\mathrm{GT}^+)$ strengths are especially sensitive to these interactions.

SM calculations for $\beta\beta$ decay can now be carried out rather accurately for ^{48}Ca. Zhao, Brown, and Richter [11] calculated the $2\nu\beta\beta$ NME of ^{48}Ca in a large-basis SM space using the OXBASH code with the MH (Muto and Horie) [12] and MSOBEP [13] two-body interactions. Their predicted $T^{2\nu}_{1/2}$ is smaller than the experimental one. They also made an analysis of the distribution of the $B(\mathrm{GT}^-)$, $B(\mathrm{GT}^+)$, and $M^{2\nu}_{\mathrm{GT}}$ components over the 1^+_k excitation energies in the intermediate nucleus (^{48}Sc), which helps better understand the quenching of the NME for the $2\nu\beta\beta$ decay mode. Caurier, Poves, and Zuker [14] performed a full *pf*-shell calculation of the NME for the $2\nu\beta\beta$ decay mode, for the transitions both to the ground state (g.s.) and to the 2^+_1 of ^{48}Ti. Their calculations were carried out with the ANTOINE code [16]. As an effective interaction they used the Kuo-Brown G matrix [17] with minimal monopole modifications, KB3 [18]. We will discuss their results together with our new results below.

In this article we use the recently proposed GXPF1A two-body effective interaction, which has been successfully tested for the *pf* shell [19–22], to perform $2\nu\beta\beta$ decay calculations for ^{48}Ca. Our goal is to obtain the values of the NME for this decay mode, for transitions both to the g.s. and to the 2^+_1 state of ^{48}Ti, with increased degree of confidence, which will allow us in the next future to address similar calculations for the $0\nu\beta\beta$ decay mode of this nucleus [23]. The $2\nu\beta\beta$ transitions to excited states have longer half-lives, as compared with the transitions to the g.s., due to the reduced values of the corresponding phase spaces. Positive results for the $2\nu\beta\beta$ decay of ^{100}Mo were recently reported [24].

For the $2\nu\beta\beta$ decay mode the relevant NME are of Gamow-Teller type and have the following expressions [1–4]:

$$M^{2\nu}_{\mathrm{GT}}(0^+) = \sum_k \frac{\langle 0_f \| \sigma\tau^- \| 1^+_k \rangle \langle 1^+_k \| \sigma\tau^- \| 0_i \rangle}{E_k + E_0}, \qquad (1)$$

for the g.s.-to-g.s. transition and

$$M^{2\nu}_{\mathrm{GT}}(2^+) = \frac{1}{\sqrt{3}} \sum_k \frac{\langle 2^+_f \| \sigma\tau^- \| 1^+_k \rangle \langle 1^+_k \| \sigma\tau^- \| 0_i \rangle}{(E_k + E_2)^3} \qquad (2)$$

for the g.s.-to-2^+_1 transition. Here E_k is the excitation energy of the 1^+_k state of ^{48}Sc and $E_0 = \frac{1}{2}Q_{\beta\beta}(0^+) + \Delta M$, $E_2 = \frac{1}{2}Q_{\beta\beta}(2^+) + \Delta M$. $Q_{\beta\beta}(0^+)$ and $Q_{\beta\beta}(2^+)$ are the Q values corresponding to the $\beta\beta$ decays to the g.s. and the 2^+_1 excited state of the parent nucleus (^{48}Ti) and ΔM is the ^{48}Ca -^{48}Sc mass difference.

The $\beta\beta$ half-life expression is given by

$$\left[T_{1/2}^{2\nu,J}\right]^{-1} = F_J^{2\nu}\left|M_{\rm GT}^{2\nu}(J)\right|^2, \quad (3)$$

where $F_J^{2\nu}$ are the phase-space factors [1]: 1.044×10^{-17} yr^{-1} MeV^2, corresponding to the g.s.-to-g.s. transition ($J = 0$) and 1.958×10^{-19}yr^{-1} MeV^6, corresponding to g.s.-to-2_1^+ transitions ($J = 2$), respectively.

The calculations were carried out in the full *pf* model space using the CMISHSM shell-model code [25] and the GXPF1A interaction. The most recent effective Hamiltonians, GXPF1 [19,20] and GXPF1A [21], are derived from a microscopic calculation by Hjorth-Jensen based on renormalized G-matrix theory with the Bonn-C interaction [26] and are refined by a systematic fitting of the important linear combinations of two-body matrix elements to low-lying states in nuclei from $A = 47$ to $A = 66$. GXFP1A addresses some shortcomings of the GXPF1 interaction for the region of the neutron-rich Sc, Ti, and Ca isotopes [21] that are relevant for this study. An advantage of using the full *pf* model space is that the Ikeda sum rule is exactly satisfied.

In the calculation of the NME, Eqs. (1) and (2), we used the standard quenching factor of 0.77 for the $\sigma\tau$ operator [14]. We used up to 250 intermediate 1^+ ^{48}Sc states in the sum. They exhaust nearly the entire B(GT) sum rules for the transitions from ^{48}Ti and ^{48}Ca: 1.59 of the exact 1.6 for Ti and 22 out the exact 24 for Ca.

We also tested the validity of the quenching factor of 0.77 by comparing the β-decay probabilities for the ^{48} Sc$(6^+) \rightarrow$ ^{48}Ti(6^+) transitions with the experimental data [27]. The results are presented in Table I and confirm that this value reasonably describes the B(GT) quenching for this mass region.

The running NME, $M_{\rm GT}^{2\nu}$ of Eqs. (1) and (2), as a function of the excitation energy of the 1^+ states in ^{48}Sc are presented in Figs. 1 and 2, respectively. The convergence trends are similar to the ones found in Refs. [11,28], and it is also supported by the nearly exhausted sum rules. It is also clear that the phases of the intermediate states in the double sum play an essential role: the contribution of the intermediate states is not coherent and the sum is not continuously increasing [29]. This is illustrated more clearly in Fig. 3, where the coherent sum (signed amplitudes) is compared to the incoherent sum (absolute GT amplitudes). Figure 4 presents the relevant B(GT) transition probabilities. We note that most of the positive contribution to the double-β matrix element to the ^{48}Ti g.s. shown in Fig. 1 comes from two of the five intermediate 1^+ states below 5 MeV excitation in ^{48}Sc. For the product $[\langle 0_f\|\sigma\tau^-\|1_k^+\rangle][\langle 1_k^+\|\sigma\tau^-\|0_i\rangle]$ the lowest 1^+ (at 2.5 MeV) gives [0.185][1.15] = 0.122 and the third 1^+ (at 3.8 MeV) gives [0.42][0.35] = 0.147.

TABLE I. Theoretical end experimental log(ft) for the ^{48}Sc $\rightarrow$ ^{48}Ti β-decay transitions.

$E_x(6^{+48}\text{Ti})$	$\log(\text{ft})_{\rm exp}$	$\log(\text{ft})_{\rm GXPF1A}$
3.333	5.247	5.532
3.508	6.083	6.010

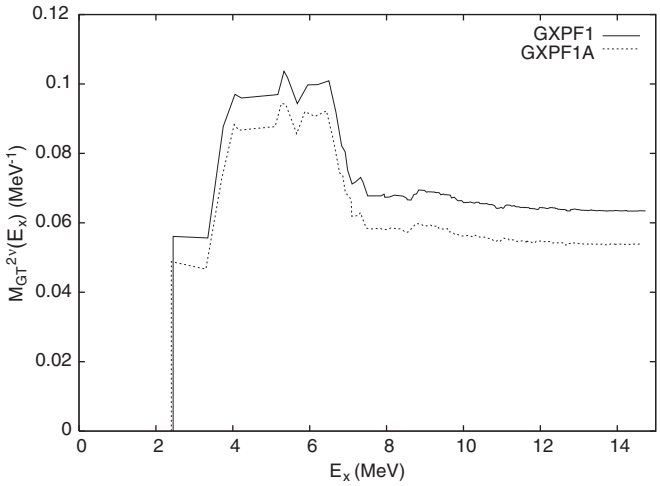

FIG. 1. The $2\nu\beta\beta$ decay running matrix element of Eq. (1) as a function of the excitation energy of the intermediate 1^+ states in ^{48}Sc for the GXPF1 and GXPF1A interactions.

Recent experiments have attempted to extract B(GT) values from ^{48}Ca(^{3}He,t)^{48}Sc and ^{48}Ti(d,^{2}He)^{48}Sc reaction cross sections (see, e.g., Fig. 4 of a recent review, Ref. [30]). The results obtained for the lowest strong 1^+ observed in ^{48}Ca(^{3}He,t) at 2.5 MeV [B(GT)$^{1/2}$ for the above product] is $|0.12(3)| \times |0.95(5)| = |0.11(3)|$, in good agreement with theory given the uncertainties that exist in extracting B(GT) from charge-exchange cross sections [31] (the state at 2.2 MeV in ^{48}Ti(d,^{2}He) associated with 1^+ does not have a correspondence in the theory—it is near a state previously assigned 3^+ in the literature and its J^π value should be confirmed). The double-β strength associated with the theoretical state at 3.8 MeV appears to be spread over several states near 3 MeV in experiment.

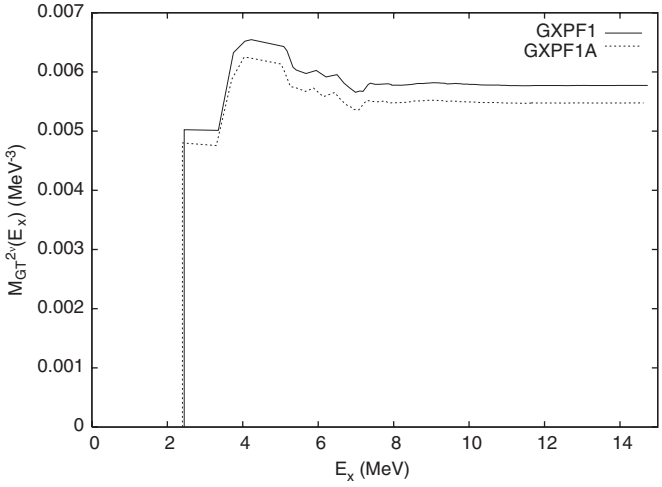

FIG. 2. The $2\nu\beta\beta$ decay running matrix element of Eq. (2) (up to a factor of $\sqrt{5}$) as a function of the excitation energy of the intermediate 1^+ states in ^{48}Sc for the GXPF1 and GXPF1A interactions.

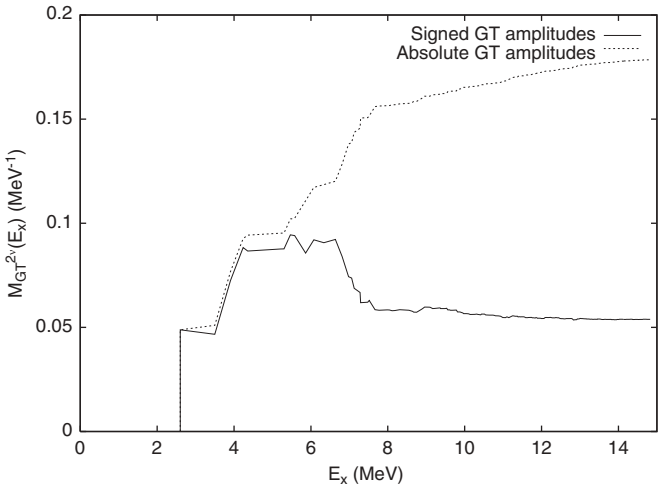

FIG. 3. The $2\nu\beta\beta$ decay running matrix element of Eq. (1) as a function of the excitation energy of the intermediate 1^+ states in ^{48}Sc compared with the similar sum where the absolute values of the GT matrix elements are used.

Using the results from Figs. 1 and 2 one gets the following converged results for the $2\nu\beta\beta$ matrix elements:

(i) $|M(0^+ \to 0^+)| = 0.0539$ MeV^{-1} for GXPF1A, and $|M(0^+ \to 0^+)| = 0.0635$ MeV^{-1} for GXPF1;
(ii) $|M(0^+ \to 2^+)| = 0.0122$ MeV^{-3} for GXPF1A, and $|M(0^+ \to 2^+)| = 0.0129$ MeV^{-3} for GXPF1.

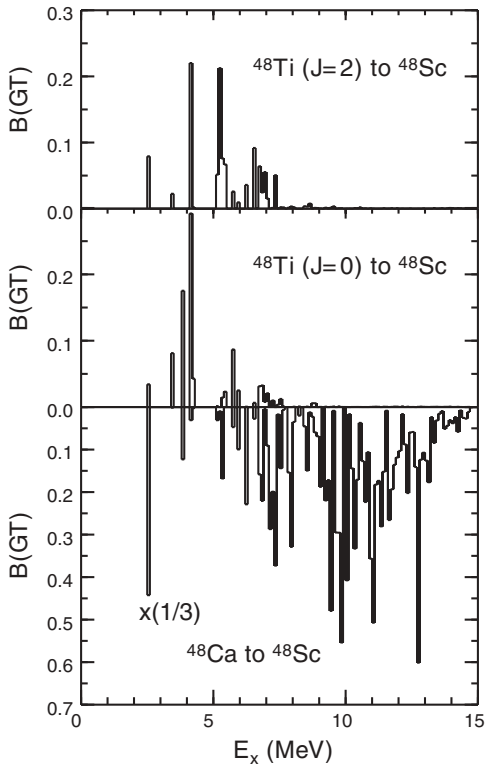

FIG. 4. The relevant B(GT) probabilities for transitions to the 1^+ states in ^{48}Sc.

Using these matrix elements and the phase factors of Ref. [1] in Eqs. (3) one gets for the $2\nu\beta\beta$ decay half-lives:

(i) $T_{1/2}(0^+ \to 0^+) = 3.3 \times 10^{19}$ yr for GXPF1A and $T_{1/2}(0^+ \to 0^+) = 2.4 \times 10^{19}$ yr for GXPF1;
(ii) $T_{1/2}(0^+ \to 2^+) = 8.5 \times 10^{23}$ yr for GXPF1A and $T_{1/2}(0^+ \to 2^+) = 7.5 \times 10^{23}$ yr GXPF1.

Our value for the $T_{1/2}(0^+ \to 0^+)$, corresponding to the $|M(0^+ \to 0^+)| = 0.0539$ MeV^{-1} NME calculated with GXPF1A interaction, is within the experimental range [32]: $(4.3^{-1.3}_{+3.3}) \times 10^{19}$ yr. The calculations performed with the GXPF1 interaction seem to give a larger value for the NME that leads to a half-life value that is just below the present experimental range. Comparing our results to the previous similar ones of Refs. [11] and [14] we note that the calculations of Zhao, Brown, and Richter are performed in a restricted *pf* model space, and they found an NME of 0.07 MeV^{-1} as their best value. Furthermore, using a phase-space factor slightly different from ours, they obtain $T_{1/2}(0^+ \to 0^+) = 1.9 \times 10^{19}$ yr. This half-life is about half of our value and is significantly below the experimental range.

Caurier, Poves, and Zuker found $|M(0^+ \to 0^+)| = 0.0402$ MeV^{-1} in their work [14]. We repeated their calculations as described in Ref. [14] and obtained 0.047 MeV^{-1} for the same NME, a value that is in agreement with the NME reported by Nowacki in Ref. [15] and that differs by about 13% from ours. This value is also within the present experimental range.

For the g.s.-to-2_1^+ transition we obtained a NME value, which is about half the numerical value reported by Caurier, Poves, and Zuker in Ref. [14]. However, Ref. [14] used the same Eq. (1) for the $|M(0^+ \to 2^+)|$ NME instead of our Eq. (2), which is recommended in the literature [1,10]. Based on Eq. (1), Ref. [14] suggests that the $0^+ \to 2^+$ decay rate is about 3% of the $0^+ \to 0^+$ decay rate. Our value obtained for the $2\nu\beta\beta$ half-life corresponding to the transition to the 2_1^+ excited state of ^{48}Ti from Eq. (2) is about four orders of magnitude larger than that for the g.s.-to-g.s transition.

In conclusion, we calculated the NME and half-lives for $2\nu\beta\beta$ decay of ^{48}Ca within an SM approach in the full *pf* model space. We calculated both the g.s.-to-g.s. and g.s.-to-2_1^+ excited-state transitions. We use for the first time [33] in such calculations the two versions of GXPF1 two-body interaction, which were recently proposed and successfully used to reproduce the spectroscopic properties of many nuclei in the nuclear mass range $A = 47$–66. Our results are based on 250 1^+ intermediate states in ^{48}Sc nucleus that are enough to exhaust almost the entire B(GT) sum rules for the transitions from ^{48}Ti and ^{48}Ca. We also checked the validity of 0.77 quenching factor for the Gamow-Teller operator used in the SM calculations by comparing the calculated β transitions ^{48}Sc $\to$ ^{48}Ti with the experimental ones. For NME we propose $|M(0^+ \to 0^+)| = 0.0539$ MeV^{-1} and $|M(0^+ \to 2^+)| = 0.0122$ MeV^{-3} as the best values, which were obtained using the GXPF1A interaction. They correspond to $T_{1/2}(0^+ \to 0^+) = 3.3 \times 10^{19}$ yr and $T_{1/2}(0^+ \to 2^+) = 8.5 \times 10^{23}$ yr, respectively. Future experiments on $\beta\beta$ decay of ^{48}Ca, CANDLES [34], and CARVEL [35], may reach the required sensitivity of measuring such transitions and our results could be useful for the planning of these experiments.

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