Nucleon-antinucleon interaction from the modified Skyrme model

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We calculate the static nucleon-antinucleon interaction potential from the modified Skyrme model with an additional $B^\mu B_\mu$ term using the product ansatz. The static properties of the single baryon are improved in the modified Skyrme model. State mixing is taken into account by perturbation theory, which substantially increases the strength of the central attraction. We obtain a long- and mid-range potential which is in qualitative agreement with some phenomenological potentials.

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I. INTRODUCTION

The Skyrme model, which is considered to be the low energy limit of quantum chromodynamics (QCD), models QCD in the classical or large number of colors (N_C) limit and regards baryon as the soliton in the pion field [1–4]. Upon quantizing a slowly rotating Skyrmion, workers calculated the static property of nucleons and Δ, and their results agreed with experimental data within 30% [3,4]. Recently, the model has been widely used to discuss exotic hadrons [5–7]. The minimal version of the model consists of the following Lagrangian terms: the nonlinear sigma term with chiral order $\mathcal{O}(p^2)$ and the Skyrme term with $\mathcal{O}(p^4)$. Even though the minimal version of the Skyrme model (Min-SKM) can be regarded as a successful phenomenological model in spite of its simplicity, it cannot be used to study the problem of quark spin contents of proton or EMC (The European Muon Collaboration) effects [8–10] which are important QCD effects in baryon physics. This is a very unsatisfactory defect for Min-SKM. To remove this shortcoming, additional terms with $\mathcal{O}(p^6)$ or higher orders have to be added into the model's Lagrangian to construct modified Skyrme models. Among them, the simplest one is the model with the Min-SKM Lagrangian plus only one additional $B^\mu B_\mu$ term [9], where B_μ is the baryon current (or Goldstone-Wilczek current). Hereafter, we will call this simplest modified Skyrme model as Mod-SKM. It is expected that Mod-SKM will be more realistic than Min-SKM. To discuss this issue and to fix the parameters in Mod-SKM are two of the aims of this paper. Moreover, the Mod-SKM can be obtained by considering the infinite ω mass limit of the vector meson ω term of the chiral Lagrangian studied in Ref. [11].

An interesting application of the Skyrme model is the investigation of the baryon-baryon interaction, especially the nucleon-nucleon (*NN*) interaction [12–17]. The Skyrme picture gives us a qualitative understanding of the principal features of the *NN* interaction: it has the correct long-range one-pion-exchange potential which dominates the tensor force, there is a strong short-range repulsion, and there is a pronounced central attraction at intermediate range, albeit weakly attractive compared with the phenomenological potential. However, the recent development of obtaining the *NN* interaction from the Skyrme model has shown that the combined effect of the careful treatment of the nonlinear equations and the configuration mixing is to give substantial central mid-range attraction for the *NN* system which is in qualitative agreement with the data [16].

The *NN* and $N\bar{N}$ potentials have been investigated by means of the Min-SKM and the algebraic methods in Refs. [18–20]. The phenomenon and puzzles in the baryon-antibaryon physics have attracted much attention recently thanks to the remarkable discovery of baryon-antibaryon enhancements in the J/ψ and B decays [21–26]. The $N\bar{N}$ interaction and the possible nucleon-antinucleon bound states have been investigated from the constituent quark model [27–29]. In the Skyrme model, the interactions between classical Skyrmion and anti-Skyrmion, i.e., $S\bar{S}$, were explored in Refs. [6,7]. In the present paper, we will study the $N\bar{N}$ potential using the Mod-SKM and following the methods developed in Refs. [16,18,19].

It is well known that phenomenologically the $N\bar{N}$ potential is not as well established as the *NN* potential. At a distance of less than about 1 fm, the interaction is dominated by annihilation. However, at larger distances, a meaningful potential can be defined and studied either by G-parity transformation on the *NN* meson-exchange potential or phenomenologically.

We will compare our Mod-SKM results to some phenomenological potentials. The $B^\mu B_\mu$ term in Mod-SKM reflects the effect of ω meson exchange [11,30,31]. We will see that at large distances, where the product ansatz makes the best sense, the potentials based on the Skyrme model agree qualitatively and, in most cases, quantitatively with the phenomenological interactions. At intermediate and short distances, the model does less well, but at these distances the product ansatz is not valid. However, the results are still suggestive.

In the following section, we give the Mod-SKM Lagrangian, then reproduce a number of static properties of the single baryon which are both qualitatively appealing and quantitatively satisfactory. In Sec. III, we study the Skyrmion-anti-Skyrmion interactions in Mod-SKM and project them to the nucleon space by algebraic methods [18–20]. We also consider the effects of rotational excitations by including the intermediate states Δ and $\bar{\Delta}$, and evaluate the corrections to

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the $N\bar{N}$ potential in perturbation theory. Sec. IV closes this paper with some discussions related to the present study.

II. MODIFIED SKYRME MODEL AND STATIC PROPERTIES OF SINGLE BARYON

The Skyrme model Lagrangian is generalized to include an additional $B^\mu B_\mu$ term which simulates the effects of ω meson. This modified Skyrme model Lagrangian provides a better description of both the single-baryon static properties and the low-energy NN interaction [30,31]. This Lagrangian has the form

$$\mathcal{L} = \frac{F_\pi^2}{16}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2}\mathrm{Tr}([U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2) + \frac{1}{8}m_\pi^2 F_\pi^2 \mathrm{Tr}(U-1) - \frac{3\pi^2 N_C}{5m^2}B^\mu B_\mu, \quad (1)$$

where U is an SU(2) valued field $U = \exp[2i\tau_a\pi_a/F_\pi]$, B^μ is the topological current $B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}[(U^\dagger\partial_\nu U)(U^\dagger\partial_\alpha U)(U^\dagger\partial_\beta U)]$, and e, F_π, and m are parameters to be determined. The first term is the Lagrangian of meson fields in the nonlinear sigma model, and the second term is the so-called Skyrme term which stabilizes the soliton. The third term is the pion mass term, and the fourth term is the additional $B^\mu B_\mu$ term. U transforms as $U \to U' = LUR^\dagger$ under the chiral group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, where both L and R are SU(2) matrices.

The chiral soliton model [11], where, as an alternative to the Skyrme term, the vector meson ω term $\beta\omega^\mu B_\mu$ stabilizes the soliton, provides support for the interpretation of the $B_\mu B^\mu$ term which emerges in the limit $m_\omega \to \infty$. Traditional nuclear interaction theories within the potential framework, which are mainly based on the single meson exchange, show that the $N\bar{N}$ system is more attractive than the NN system, because in those theories there is a strong ω exchange, so inclusion of this term which models the effect of the ω meson could help provide a better description of the $N\bar{N}$ system. Furthermore, the study of the quark spin content also supports the notion that we should add this six-derivative term in order to yield a spin content consistent with the present experiment [8,32]. Generally, terms in $\mathcal{L}$ with more than two time derivatives can lead to pathological runaway solutions when the adiabatic approximation is relaxed, and they present obvious difficulties in quantizing the theory. But the Lagrangian of Mod-SKM has, at most, two time derivatives; hence, there is no such difficulty.

For the case with the single static Skyrmion, we use the so-called hedgehog ansatz,

$$U_0(\mathbf{r}) = \exp[i\tau_a\hat{r}_a F(r)], \quad (2)$$

where $F(r)$ is the chiral angle which minimizes the static soliton energy subject to the boundary condition $F(0) = \pi$ and $F(\infty) = 0$. From Eqs. (1) and (2), the mass of the classical soliton is obtained as

$$M_s = \frac{\pi}{2}\frac{F_\pi}{e}\int_0^\infty \left\{x^2F'^2 + 2S^2 + 4S^2\left(2F'^2 + \frac{S^2}{x^2}\right) + 2\mu^2x^2(1-C) + \nu^2\frac{S^4}{x^2}F'^2\right\}, \quad (3)$$

with

$$x = eF_\pi r, \quad \mu^2 = \frac{m_\pi^2}{e^2F_\pi^2}, \quad F' = \frac{dF}{dx}, \quad \nu^2 = \frac{18e^4F_\pi^2}{5\pi^2m^2}, \quad S = \sin F, \quad C = \cos F. \quad (4)$$

In Eq. (3), the term proportional to ν^2 comes from the $B^\mu B_\mu$ term and is absent in the conventional Skyrme model. Minimizing M_s with respect to F, $\delta M_s = 0$, we have the following equation for F:

$$\left(\frac{x^2}{4} + 2S^2 + \frac{\nu^2}{4}\frac{S^4}{x^2}\right)F'' + \left(\frac{\nu^2}{2}\frac{S^3C}{x^2} + 2SC\right)F'^2 + \left(\frac{x}{2} - \frac{\nu^2}{2}\frac{S^4}{x^3}\right)F' - \left(\frac{1}{2}SC + \frac{2S^3C}{x^2} + \frac{1}{4}\mu^2x^2S\right) = 0. \quad (5)$$

From this equation, we can see that the chiral angle F asymptotically tends to the following expression when r goes to infinity:

$$F(r) \to \mathcal{A}\left(\frac{1}{m_\pi eF_\pi r^2} + \frac{1}{eF_\pi r}\right)e^{-m_\pi r}, \quad r \to \infty. \quad (6)$$

The coefficient $\mathcal{A}$ is related to the pion-nucleon coupling constant $g_{\pi NN}$ through [3]

$$g_{\pi NN} = \frac{4\pi M_N \mathcal{A}}{3em_\pi}. \quad (7)$$

Associated with the chiral symmetry, the vector current $J_V^{\mu a}$ and axial vector current $J_A^{\mu a}$ can be obtained from the Skyrme Lagrangian Eq. (1) following the standard procedure,

$$J_V^{\mu a} = \frac{iF_\pi^2}{8}\mathrm{Tr}\left[\frac{\tau_a}{2}(\partial^\mu UU^\dagger + \partial^\mu U^\dagger U)\right] - \frac{i}{8e^2}\mathrm{Tr}\left\{\left[\frac{\tau_a}{2}, \partial_\nu UU^\dagger\right][\partial^\mu UU^\dagger, \partial^\nu UU^\dagger] + \left[\frac{\tau_a}{2}, \partial_\nu U^\dagger U\right]\left[\partial^\mu U^\dagger U, \partial^\nu U^\dagger U\right]\right\} - \frac{3N_C i}{20m^2}\varepsilon^{\mu\nu\alpha\beta}B_\nu\mathrm{Tr}\left[\frac{\tau_a}{2}(\partial_\alpha UU^\dagger\partial_\beta UU^\dagger - \partial_\alpha U^\dagger U\partial_\beta U^\dagger U)\right], \quad (8)$$

$$\begin{aligned} J_A^{\mu a} = & \frac{iF_\pi^2}{8}\mathrm{Tr}\left[\frac{\tau_a}{2}(\partial^\mu UU^\dagger - \partial^\mu U^\dagger U)\right] \\ & - \frac{i}{8e^2}\mathrm{Tr}\left\{\left[\frac{\tau_a}{2}, \partial_\nu UU^\dagger\right][\partial^\mu UU^\dagger, \partial^\nu UU^\dagger] \right.\\ & \left. - \left[\frac{\tau_a}{2}, \partial_\nu U^\dagger U\right][\partial^\mu U^\dagger U, \partial^\nu U^\dagger U]\right\} \\ & - \frac{3N_C i}{20m^2}\varepsilon^{\mu\nu\alpha\beta}B_\nu \mathrm{Tr}\left[\frac{\tau_a}{2}(\partial_\alpha UU^\dagger\partial_\beta UU^\dagger \right.\\ & \left. + \partial_\alpha U^\dagger U\partial_\beta U^\dagger U)\right]. \end{aligned} \tag{9}$$

The classical field configuration of the hedgehog form does not have definite spin and isospin. However, nucleons carry both spin and isospin, and in any reasonable model of nucleons the appropriate spin and isospin states must appear. Following the conventional way, we perform the collective coordinate quantization. We make a time-dependent SU(2) rotation of our static soliton solution,

$$U(x) = A(t)U_0(x)A^\dagger(t), \tag{10}$$

then

$$L = -M_s + I\,\mathrm{Tr}[\partial_0 A^\dagger(t)\partial_0 A(t)], \tag{11}$$

where $A(t) \in$ SU(2) matrix is the collective coordinate, and I is the moment of inertia, which is given by

$$I = \frac{1}{F_\pi e^3}\frac{2\pi}{3}\int_0^\infty dx\{S^2[x^2 + 4(x^2F'^2 + S^2)] + \nu^2 S^4 F'^2\}. \tag{12}$$

If the SU(2) matrix $A(t)$ is parametrized by $A(t) = a_0 + i\tau_n a_n$, with $a_0^2 + \sum_{n=1}^3 a_n^2 = 1$, the Hamiltonian is

$$H = M_s - \frac{1}{8I}\sum_{n=0}^3\left(\frac{\partial}{\partial a_n}\right)^2 = M_s + \frac{\mathbf{S}^2}{2I} = M_s + \frac{\mathbf{I}^2}{2I}. \tag{13}$$

Noting that the I in the denominator is the moment of inertia, **S** and **I** are the spin and isospin operators, respectively. As in Ref. [3], we can calculate the static properties of the single baryon. In going from the classical results to the quantum results for rotation operators, we must symmetrize them [32], i.e., we perform the Weyl order of these operators.

From Eq. (13) the masses of the nucleon and Δ, respectively, are

$$M_N = M_s + \frac{3}{8I}, \quad M_\Delta = M_s + \frac{15}{8I}. \tag{14}$$

The isoscalar and isovector mean square electric radii are

$$\langle r^2\rangle_{E,I=0} = \frac{1}{(eF_\pi)^2}\int_0^\infty dx\frac{-2}{\pi}x^2S^2F', \tag{15}$$

$$\begin{aligned}\langle r^2\rangle_{E,I=1} = & \frac{1}{(eF_\pi)^2}\frac{1}{Ie^3F_\pi}\int_0^\infty dx\left\{\frac{2\pi}{3}x^4S^2\right.\\ & \left.\times\left[1 + 4\left(F'^2 + \frac{S^2}{x^2}\right)\right] + \frac{2\pi}{3}\nu^2x^2S^4F'^2\right\}.\end{aligned} \tag{16}$$

The corresponding proton and neutron mean square charge radii are

$$\begin{aligned}\langle r^2\rangle_{E,p} &= \tfrac{1}{2}(\langle r^2\rangle_{E,I=0} + \langle r^2\rangle_{E,I=1}),\\ \langle r^2\rangle_{E,n} &= \tfrac{1}{2}(\langle r^2\rangle_{E,I=0} - \langle r^2\rangle_{E,I=1}).\end{aligned} \tag{17}$$

After somewhat tedious but rather straightforward calculations, we can obtain the proton and neutron magnetic moments, which are, respectively,

$$\begin{aligned}\mu_p &= 2M_N\left(\frac{1}{12I}\langle r^2\rangle_{E,I=0} + \frac{I}{6}\right),\\ \mu_n &= 2M_N\left(\frac{1}{12I}\langle r^2\rangle_{E,I=0} - \frac{I}{6}\right).\end{aligned} \tag{18}$$

In the above, we have symmetrized the rotation operators, and the proton and neutron magnetic momenta are defined through

$$\begin{aligned}\langle p, 1/2|\mu^3|p, 1/2\rangle &= \frac{1}{2M_N}\mu_p,\\ \langle n, 1/2|\mu^3|n, 1/2\rangle &= \frac{1}{2M_N}\mu_n.\end{aligned} \tag{19}$$

After some lengthy calculations, we can also get the axial coupling constant [3]

$$\begin{aligned} g_A = & -\frac{2\pi}{9e^2}\int_0^\infty dx x^2\left\{\frac{2CS}{x}\left[1 + 4\left(F'^2 + \frac{S^2}{x^2}\right)\right]\right.\\ & \left. + F'\left(1 + \frac{8S^2}{x^2}\right)\right\} - \frac{12}{5\pi m^2}\int_0^\infty dx S^2F'\left\{(eF_\pi)^2\right.\\ & \left.\times\left(\frac{2CS}{3x}F' + \frac{S^2}{3x^2}\right) - (4xCSF' + S^2)\frac{1}{18I^2}\right\}. \end{aligned} \tag{20}$$

There are three parameters in the modified Skyrme model, i.e., e, F_π, and m; the pion mass is $m_\pi = 138$ MeV, and the number of color $N_C = 3$. In the conventional Skyrme model, there is always a conflict between the e and F_π data input settings used to give the correct nucleon and Δ masses and those used to give the correct strength of the pion tail [14,16,19]. But a satisfactorily simultaneous description of the nucleon, Δ mass, and strength of the pion tail is possible by properly choosing the parameters e, F_π, and m in Mod-SKM. Throughout our calculation, we choose the three parameters as $e = 19.48$, $F_\pi = 129.11$ MeV, and $m = 420$ MeV; this parameter setting gives $g_{\pi NN} \approx 13.5$ through Eq. (7), which leads to the correct one-pion-exchange potential of $N\bar{N}$ interaction as the distance tends to infinity. The connection between the Mod-SKM and the chiral soliton model including ω meson [11] allows us to relate the parameter m to the coupling β, i.e., $m = \sqrt{\frac{2}{5}\frac{3\pi m_\omega}{\beta}}$; and the best fit of the parameters in Ref. [11] gives $m \approx 298.8$ MeV, which is not too far from the value of m in this work. The static single-baryon properties are summarized in the Table I, along with the results of the conventional Skyrme model [4] and experimental values.

In Table I, we can see that the Mod-SKM predictions are closer to the experimental values than those of the conventional Skyrme model [4], so we expect that Mod-SKM provides a better description of other static properties of baryons including the low-energy $N\bar{N}$ interaction.

TABLE I. Static properties of single baryon in Mod-SKM model compared with those in the conventional one [4] and experimental results.

Physical quantity	Mod-SKM	Conventional Skyrme	Experiment
M_N	938.9 MeV(input)	938.9 MeV(input)	938.9 MeV
M_Δ	1232 MeV(input)	1232 MeV(input)	1232 MeV
M_π	138 MeV(input)	138 MeV(input)	138 MeV
e	19.48	4.84	–
F_π	129.11 MeV	108 MeV	186 MeV
$\langle r^2\rangle^{1/2}_{E,I=0}$	0.71 fm	0.68 fm	0.72 fm
$\langle r^2\rangle^{1/2}_{E,I=1}$	1.04 fm	1.04 fm	0.88 fm
μ_p	2.01	1.97	2.79
μ_n	−1.20	−1.23	−1.91
g_A	0.82	0.65	1.24

III. ADIABATIC $N\bar{N}$ INTERACTION

A. Formulation

We now study, in the product ansatz, the interaction energy of the Skyrmion-anti-Skyrmion ($S\bar{S}$) system, which is a function of the separation between S and $\bar{S}$ and the relative orientation. This interaction energy can be calculated numerically. We rotate the two solitons independently in SU(2) space,

$$\begin{aligned} U_0(\mathbf{r}-\mathbf{R}/2) &\to \mathrm{A}U_0(\mathbf{r}-\mathbf{R}/2)A^\dagger, \\ U_0^\dagger(\mathbf{r}+\mathbf{R}/2) &\to \mathrm{B}U_0^\dagger(\mathbf{r}+\mathbf{R}/2)B^\dagger, \end{aligned} \tag{21}$$

where both A and B are SU(2) matrices. To obtain the static $N\bar{N}$ interaction, we describe the $N\bar{N}$ configuration with the product ansatz (exact in the large R limit) as

$$U(\mathbf{r}) = \mathrm{A}U_0(\mathbf{r}-\mathbf{R}/2)A^\dagger \mathrm{B}U_0^\dagger(\mathbf{r}+\mathbf{R}/2)B^\dagger, \tag{22}$$

where one baryon is located at $\mathbf{R}/2$ and the antibaryon at $-\mathbf{R}/2$. Retaining only the potential energy density in the modified Skyrme Lagrangian (1), the energy in the field of Eq. (22) is the same as in

$$U(\mathbf{r}) = U_0(\mathbf{r}-\mathbf{R}/2)\mathrm{C}U_0^\dagger(\mathbf{r}+\mathbf{R}/2)C^\dagger, \tag{23}$$

where $C = A^\dagger B = c_4 + i\tau\cdot\mathbf{c}$ is an SU(2) matrix, too. Discarding nonstatic terms containing time derivatives, the static $N\bar{N}$ potential is defined by

$$V(\mathbf{R}, C) = -\int d^3x\mathcal{L}[U(\mathbf{r})] - 2M_s. \tag{24}$$

$V(\mathbf{R}, C)$ can be written in the notation of Vinh Mau *et al.* [13] as

$$\begin{aligned} V(\mathbf{R}, C) &= V_1(R) + V_2(R)c_4^2 + V_3(R)(\mathbf{c}\cdot\hat{\mathbf{R}})^2 + V_4(R)c_4^4 \\ &\quad + V_5(R)c_4^2(\mathbf{c}\cdot\hat{\mathbf{R}})^2 + V_6(R)(\mathbf{c}\cdot\hat{\mathbf{R}})^4, \end{aligned} \tag{25}$$

where $V_i(i = 1-6)$ are functions of R. Generally, for $S\bar{S}$, the symmetry $\mathbf{R}\to-\mathbf{R}$ is broken by the product ansatz, and we need three additional terms for a consistent expansion,

$$\begin{aligned} V(\mathbf{R}, C) &= V_1(R) + V_2(R)c_4^2 + V_3(R)(\mathbf{c}\cdot\hat{\mathbf{R}})^2 + V_4(R)c_4^4 \\ &\quad + V_5(R)c_4^2(\mathbf{c}\cdot\hat{\mathbf{R}})^2 + V_6(R)(\mathbf{c}\cdot\hat{\mathbf{R}})^4 \\ &\quad + V_7(R)c_4(\mathbf{c}\cdot\hat{\mathbf{R}}) + V_8(R)c_4^3(\mathbf{c}\cdot\hat{\mathbf{R}}) \\ &\quad + V_9(R)c_4(\mathbf{c}\cdot\hat{\mathbf{R}})^3. \end{aligned} \tag{26}$$

These terms odd in $\mathbf{R}$ are artifacts of the symmetry of the product ansatz and should be discarded. One can use the symmetrized energy $\frac{V(\mathbf{R},C)+V(-\mathbf{R},C)}{2}$ to extract $V_1(R)$ to $V_6(R)$, since the $V_7(R)$ to $V_9(R)$ terms drop out in this combination.

Next, we have to map the Skyrmion-anti-Skyrmion ($S\bar{S}$) interaction to the nucleon-antinucleon ($N\bar{N}$) interaction. This problem has been tackled in various ways by various groups for the NN case [12,13,18]. Each of the forms used in these works can always be cast in the form of the algebraic model [18]. So we will also use the algebraic method for mapping the $S\bar{S}$ interaction to the $N\bar{N}$ interaction [16,18,19]. This method allows us to both study both the large N_C limit and include the finite N_C effects explicitly in a systematic way. Most of the formulas given below can be found in Refs. [16,18,19]; however, for the sake of completeness, we recall here the important ones.

The algebraic model consists of two sets of $U(4)$ algebras, one for each Skyrmion (or anti-Skyrmion), as well as the radial coordinate $\mathbf{R}$. This method was developed in Refs. [16,18] for the NN system and also generalized to the $N\bar{N}$ system in Refs. [19,20]. In large N_C limit, the $S\bar{S}$ interaction can be expanded in terms of three operators: the identity, operator W, and operator Z,

$$\begin{aligned} W &= T^\alpha_{pi}T^\beta_{pi}/N_C^2, \\ Z &= T^\alpha_{pi}T^\beta_{pj}[3\hat{R}_i\hat{R}_j - \delta_{ij}]/N_C^2. \end{aligned} \tag{27}$$

Here α and β label two different sets of bosons, used to realize the U(4) algebra, and T is a one-body operator with spin and isospin 1. The semiclassical (large N_C) limit of these operators can be given in terms of $\hat{\mathbf{R}}$ and $C = c_4 + i\tau\cdot\mathbf{c}$ as [18]

$$\begin{aligned} W_{\rm cl}(A, B) &= 3c_4^2 - \mathbf{c}^2 = 4c_4^2 - 1, \\ Z_{\rm cl}(A, B, \hat{\mathbf{R}}) &= 6(\mathbf{c}\cdot\hat{\mathbf{R}})^2 - 2\mathbf{c}^2 = 2c_4^2 - 2 + 6(\mathbf{c}\cdot\hat{\mathbf{R}})^2. \end{aligned} \tag{28}$$

The $S\bar{S}$ interaction can be expressed as

$$\begin{aligned} V(\mathbf{R}, C) &= \upsilon_1(R) + \upsilon_2(R)W_{\rm cl} + \upsilon_3(R)Z_{cl} + \upsilon_4(R)W_{\rm cl}^2 \\ &\quad + \upsilon_5(R)W_{\rm cl}Z_{\rm cl} + \upsilon_6(R)Z_{\rm cl}^2. \end{aligned} \tag{29}$$

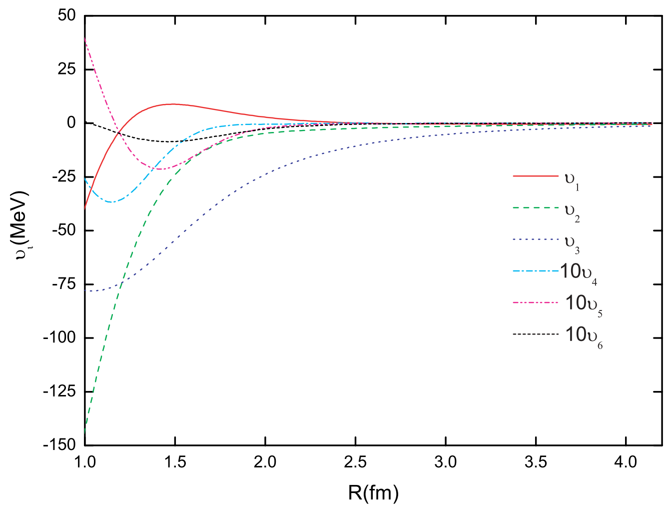

FIG. 1. (Color online) Various terms of the Skyrmion-anti-Skyrmion potential, Eq. (29).

in the semiclassical limit. We can obtain the relations between V_i and $\upsilon_i (i = 1-6)$ by comparing Eqs. (25) and (29), that is,

$$\begin{aligned} V_1(R) &= \upsilon_1(R) - \upsilon_2(R) - 2\upsilon_3(R) + \upsilon_4(R) + 2\upsilon_5(R) \\ &\quad + 4\upsilon_6(R), \\ V_2(R) &= 4\upsilon_2(R) + 2\upsilon_3(R) - 8\upsilon_4(R) - 10\upsilon_5(R) - 8\upsilon_6(R), \\ V_3(R) &= 6\upsilon_3(R) - 6\upsilon_5(R) - 24\upsilon_6(R), \\ V_4(R) &= 16\upsilon_4(R) + 8\upsilon_5(R) + 4\upsilon_6(R), \\ V_5(R) &= 24\upsilon_5(R) + 24\upsilon_6(R), \\ V_6(R) &= 36\upsilon_6(R). \end{aligned} \tag{30}$$

Six independent choices of the matrix C can yield enough independent linear equations to determine $\upsilon_i(R)(i = 1\text{–}6)$ or equivalently $V_i(R)(i = 1\text{–}6)$ through Eq. (30); the numerical results for $\upsilon_i(R)(i = 1\text{–}6)$ coming from the Mod-SKM are shown in Fig. 1. There we can see that the first three terms $\upsilon_1(R), \upsilon_2(R)$, and $\upsilon_3(R)$ are dominant. It seems a good approximation to neglect the interaction terms which are nonlinear in the expansion of operators W and Z. In the following discussion, we will mostly concentrate on the first three terms, then the leading term in this expansion is given by

$$V(\mathbf{R}, C) = \upsilon_1(R) + \upsilon_2(R)W + \upsilon_3(R)Z. \tag{31}$$

The algebraic operators W and Z have simple expectation values for the nucleons [18]

$$\begin{aligned} \langle N|T^{\alpha}_{pi}|N\rangle &= -\frac{N_C}{3} P_N \langle N|\tau^{\alpha}_p \sigma^{\alpha}_i|N\rangle, \\ \langle N\bar{N}|W|N\bar{N}\rangle &= \tfrac{1}{9} P_N^2 \langle N\bar{N}|\boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2|N\bar{N}\rangle, \\ \langle N\bar{N}|Z|N\bar{N}\rangle &= \tfrac{1}{9} P_N^2 \langle N\bar{N}|(3\boldsymbol{\sigma}^1 \cdot \hat{\mathbf{R}}\boldsymbol{\sigma}^2 \cdot \hat{\mathbf{R}} - \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2) \\ &\quad \times \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2|N\bar{N}\rangle. \end{aligned} \tag{32}$$

Here P_N is the finite N_C correction factor $P_N = 1 + \frac{2}{N_C}$. By using Eq. (32), we take the $N\bar{N}$ matrix element of the interaction and evaluate the $N\bar{N}$ potential, which only contains three independent multipole components, i.e., the central part V_c, the spin-spin part V_s, and the tensor term V_t:

$$\begin{aligned} V^{(0)}(\mathbf{R}) &= V_c(R) + V_s(R)\boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2 \boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2 + V_t(R) \\ &\quad \times (3\boldsymbol{\sigma}^1 \cdot \hat{\mathbf{R}}\boldsymbol{\sigma}^2 \cdot \hat{\mathbf{R}} - \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2)\boldsymbol{\tau}^1 \cdot \boldsymbol{\tau}^2, \end{aligned} \tag{33}$$

with

$$V_c = \upsilon_1, \quad V_s = \frac{\upsilon_2 P_N^2}{9}, \quad V_t = \frac{\upsilon_3 P_N^2}{9}. \tag{34}$$

The $N\bar{N}$ potential in the above is calculated by projecting Eq. (31) to the nucleon degrees of freedom only, and this is the correct procedure for large separation. However, at short distances, the nucleons may deform or excite as they interact, and they can be virtually whatever the dynamics requires, for example, $\Delta(\text{or}\bar{\Delta})$. This means that we need to consider the state mixing effect. For the NN interaction, we saw that state mixing plays an important role in obtaining the phenomenologically correct potential. We expect the state mixing effect to be very important in the $N\bar{N}$ interaction as well. State mixing comes into effect at the distance where the product ansatz no longer makes sense, so our results at short and intermediate distances should be suggestive, although we include state mixing. As a guide, we study the effects of the intermediate states $N\bar{\Delta}$, $\Delta\bar{N}$, and $\Delta\bar{\Delta}$ perturbatively; then to second order, the $N\bar{N}$ interaction is given by

$$\begin{aligned} V(\mathbf{R}) &= \langle N\bar{N}|V(\mathbf{R}, C)|N\bar{N}\rangle \\ &\quad + {\sum_s}' \frac{\langle N\bar{N}|V(\mathbf{R}, C)|s\rangle\langle s|V(\mathbf{R}, C)|N\bar{N}\rangle}{E_{N\bar{N}} - E_s}. \end{aligned} \tag{35}$$

Here $E_{N\bar{N}}$ is the two-nucleon energy, and E_s is the energy of the relevant excited state. The first term on the right is the direct nucleon-antinucleon projection of $V(\mathbf{R}, C)$, and it is exactly the expression $V^{(0)}(\mathbf{R})$. The second term is the correction due to rotational or excited states. It is clear from the energy denominator that the second term is attractive. We need to evaluate the three sets of matrix elements $\langle N\bar{N}|V(\mathbf{R}, C)|N\bar{\Delta}\rangle\langle N\bar{\Delta}|V(\mathbf{R}, C)|N\bar{N}\rangle$, $\langle N\bar{N}|V(\mathbf{R}, C)|\Delta\bar{N}\rangle$ $\langle\Delta\bar{N}|V(\mathbf{R}, C)|N\bar{N}\rangle$, and $\langle N\bar{N}|V(\mathbf{R}, C)|\Delta\bar{\Delta}\rangle\langle\Delta\bar{\Delta}|V(\mathbf{R}, C)$ $|N\bar{N}\rangle$; and the final result for the first-order correction to the $N\bar{N}$ interaction is [16,19]

$$\begin{aligned} V^{(1)}_{\rm PT}(\mathbf{R}) &= -\frac{Q_N^2}{\delta}\Bigg\{\left[\frac{1}{3}Q_N^2 P_0^\tau + \left(\frac{16}{27}P_N^2 + \frac{5}{27}Q_N^2\right)P_1^\tau\right] \\ &\quad \times \left[\upsilon_2^2(R) + 2\upsilon_3^2(R)\right] + (\boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2)\Bigg[-\frac{1}{18}Q_N^2 P_0^\tau \\ &\quad + \left(\frac{16}{81}P_N^2 - \frac{5}{162}Q_N^2\right)P_1^\tau\Bigg]\left[\upsilon_2^2(R) - \upsilon_3^2(R)\right] \\ &\quad + (3\boldsymbol{\sigma}^1 \cdot \hat{\mathbf{R}}\sigma^2 \cdot \hat{\mathbf{R}} - \boldsymbol{\sigma}^1 \cdot \boldsymbol{\sigma}^2)\Bigg[-\frac{1}{18}Q_N^2 P_0^\tau \\ &\quad + \left(\frac{16}{81}P_N^2 - \frac{5}{162}Q_N^2\right)P_1^\tau\Bigg] \\ &\quad \times \left[\upsilon_3^2(R) - \upsilon_2(R)\upsilon_3(R)\right]\Bigg\}. \end{aligned} \tag{36}$$

Here Q_N is another finite N_C correction factor, $Q_N = \sqrt{(1-1/N_C)(1+5/N_C)}$. δ is the $N-\Delta$ energy difference, which is about 300 MeV, and $P_T^\tau (T=0,1)$ is a projection operator onto the isospin T, $P_0^\tau = \frac{1}{4}(1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$, $P_1^\tau = \frac{1}{4}(3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$.

B. Results

For each total isospin $T=0,1$ we parametrize the $N\bar{N}$ interaction potential by

$$V_{N\bar{N}}^T = V_c^T + V_s^T \boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2 + V_t^T(3\boldsymbol{\sigma}^1\cdot\hat{\mathbf{R}}\boldsymbol{\sigma}^2\cdot\hat{\mathbf{R}} - \boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2). \tag{37}$$

We now calculate V_c^T, V_s^T, and V_t^T for each isospin $T(T=0,1)$ following the methods outlined above. Such a calculation requires considerable computing time. We would like to compare the Skyrmion model potentials with the realistic nucleon-antinucleon interaction potentials. However, we cannot relate our results to the modern $N\bar{N}$ interaction potential such as the Paris potential [33] and the Julich potential [34], since their central parts contain explicit momentum-dependent terms. For that reason, we compare our results with the phenomenological potentials of Bryan and Phillips [35] and the Nijmegen group [36]. These potentials provide successful descriptions of both the $N\bar{N}$ scattering experiments data and the spectrum of resonances, and they are not qualitatively different from each other. At large distances, all these potentials can be correctly described by the one-boson-exchange mechanism, and the $N\bar{N}$ potential can be obtained by G-parity transformation of the corresponding parts of the NN interaction potential. Using the equation of motion and the asymptotic form Eq. (6) of the chiral angle $F(r)$, we see that the $N\bar{N}$ interaction based on the Mod-SKM tends to one-pion-exchange potential in the long-distance region [17],

$$V^{N\bar{N}}(r) \to \frac{-1}{4\pi}\left(\frac{g_{\pi NN}}{2M_N}\right)^2(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2)(\boldsymbol{\sigma}^1\cdot\nabla) \times(\boldsymbol{\sigma}^2\cdot\nabla)\frac{e^{-m_\pi r}}{r}, \quad r\to\infty. \tag{38}$$

The parameters e, F_π, amd m are properly chosen to guarantee that the long-distance tail of the $N\bar{N}$ interaction agrees with the phenomenology. To model the annihilation effect at short distances, various cutoffs have been used in the Bryan-Phillips, Nijmegen, and other similar potentials. At short distances, the interaction is dominated by the strong absorptive potential of order of 1 GeV, and it is significantly different from the meson-exchange potential. Furthermore, the Skyrme model at short distances is no longer meaningful. So we should not take seriously the comparison of our results with the phenomenological potentials at 1 fm and less; however, the results are still indicative at short distances. We find that the principal feature of the phenomenological $N\bar{N}$ interaction emerges from the careful calculation of that interaction based on the Mod-SKM, i.e., the strong central attraction.

Figures 2 and 3 show the central potential V_C^T calculated from Eq. (35) and from only the first term of the right-hand side of Eq. (35). To keep the figures clear, we plot the potential curves of $T=0$ and $T=1$ separately. For the case with the nucleon only, the results of V_C^T are independent of the isospin T and less attractive than the phenomenological potentials. When the perturbation corrections due to the effects of the intermediate states $N\bar{\Delta}$, $\Delta\bar{N}$, and $\Delta\bar{\Delta}$ (i.e., Δ mixing effects) are taken into account, the results of V_C^T show significant attraction effects explicitly and are closer to the Bryan-Phillips and Nijmegen potential. These perturbation results are rather realistic. The effects of $\Delta(\bar{\Delta})$ mixing are so striking in the case of $T=1$ that the perturbation result is more attractive than the phenomenological potential for $T=1$. Furthermore, we would like to mention that because of isospin conservation, the $N\Delta$ transition is missing in the $T=0$ channel, which differentiates then the effect of the perturbation result between the $T=0$ and the $T=1$ channels. As a cross-check of our numerical calculation, we reproduce the results of Ref. [19];

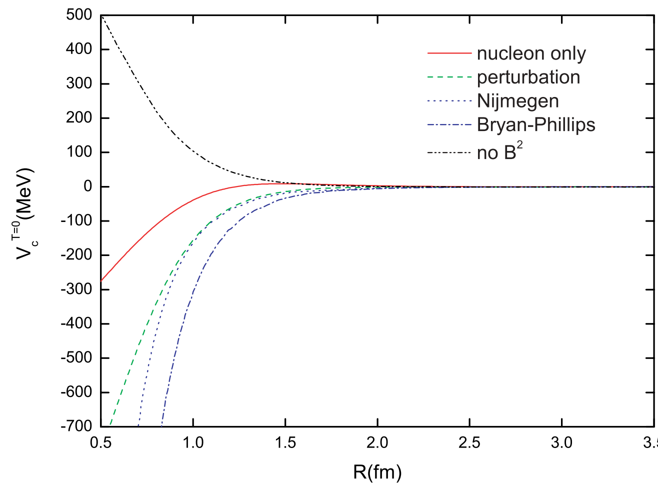

FIG. 2. (Color online) Central potential V_c^T as a function of distance R for the $T=0$ channel, showing the nucleon-only result, the result of the states mixing by perturbation theory, the nucleon-only potential (no B^2) in the conventional Skyrme model [19], and the Bryan-Phillips and Nijmegen phenomenological potentials based on meson exchange.

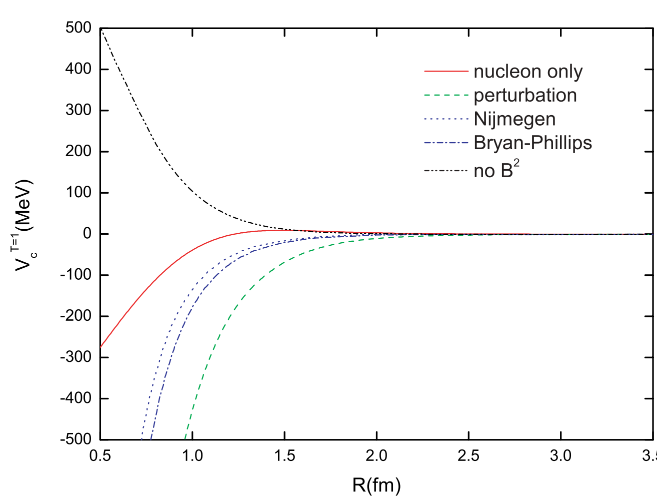

FIG. 3. (Color online) Same as Fig. 2, but for the $T=1$ channel.

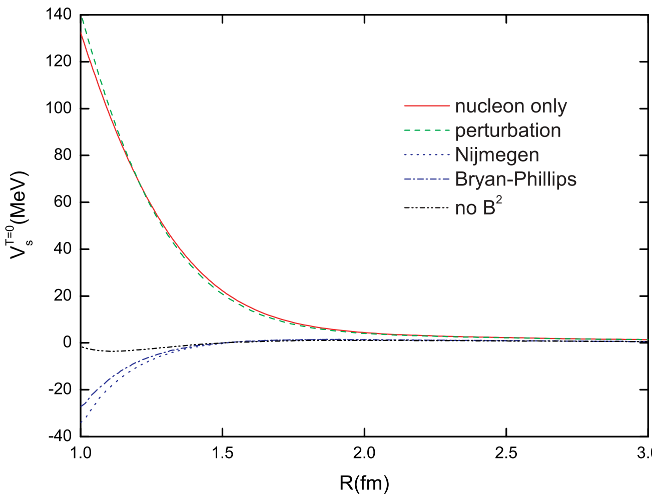

FIG. 4. (Color online) Spin-dependent potential V_s^T as a function of distance R for the $T = 0$ channel.

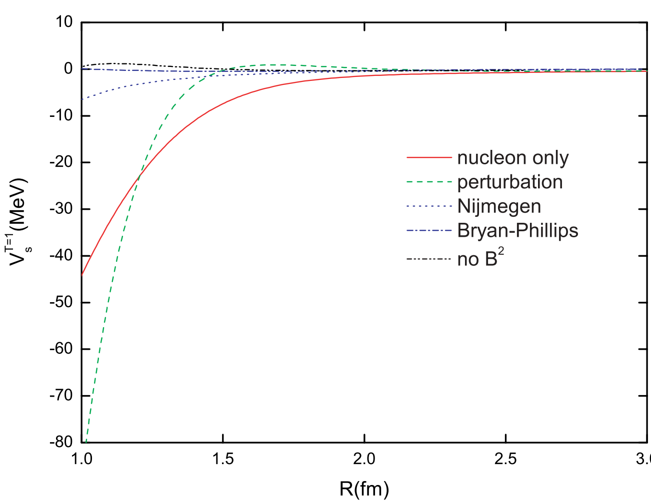

FIG. 5. (Color online) Same as Fig. 4, but for $T = 1$.

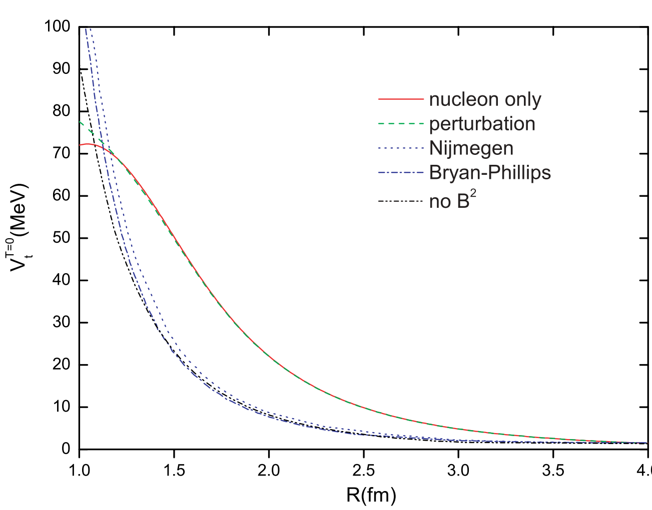

FIG. 6. (Color online) Tensor potential V_t^T as a function of distance R for the $T = 0$ channel.

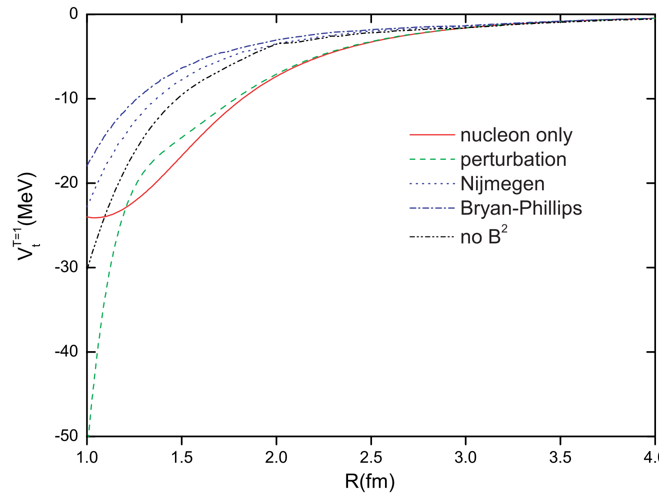

FIG. 7. (Color online) Same as Fig. 6, but for $T = 1$.

the nucleon-only results of Ref. [19] are also shown in Figs. 2 and 3 to illustrate the role of the $B^\mu B_\mu$ term. From these figures, we can see that the central potentials from the Mod-SKM are in better agreement with the phenomenology potentials.

In Figs. 4 and 5, we show the $T = 0$ and $T = 1$ spin-dependent potentials. In these cases, the nucleon-only potential and the perturbative results are quite similar. From 1 fm to about 1.5 fm, the potentials from the modified Skyrme model are not so close to the phenomenological potentials. Especially in the $T = 0$ case, both the nucleon-only and perturbative analysis give a positive spin-spin potential, in contrast to the negative values of the phenomenological potentials. It is important to see if the more complete Skyrme calculations can repair this disagreement. However, the smallness of the potential is reproduced. In our calculation, that small value arises from the cancelations of large terms.

Figures 6 and 7 show the tensor potential V_t^T. Being similar to case of the spin-dependent potential, the nucleon-only potential and the perturbative results are also quite similar. Particularly at large distances, these results agree with the phenomenological potential, but the agreement is not so good at intermediate distances. However, the difference between the theoretical and the phenomenological results is of the order of 10 MeV, compared to the static soliton mass or the nucleon mass which is about 1 GeV, the difference is small enough. Here again, an improved Skyrme model dynamical calculation that goes beyond the product ansatz, uses diagonalization for state mixing, and includes explicitly the vector meson (ρ, ω) and some high derivative terms in the Lagrangian might lead to a better agreement.

IV. CONCLUSION AND DISCUSSION

We have shown that the modified Skyrme model with product ansatz can give an $N\bar{N}$ interaction that is in qualitative agreement with the phenomenological potential, and it provides a description of the static properties of the single baryon that is better than that given by the minimal version Skyrme model. We see that it is very important to include configuration mixing, and we roughly estimate this effect by perturbation theory. A more sophisticated method of considering the state mixing effect is the Born-Oppenheimer approximation. The potential curves in the Born-Oppenheimer approximation are similar to the perturbative results, especially for the spin-dependent and the tensor potential [16,19].

To go from this work to a theory that can be confronted with experiment in detail is a difficult challenge, i.e, predicting the nucleon-antinucleon scattering cross section, the polarization, the spectra of the nucleon-antinucleon system, etc. There are nonadiabatic effects that are particularly important at small R, and there are other mesons which should be included in the Skyrme Lagrangian. The effects due to vector mesons may be particularly important at small distances. Obtaining the static nucleon-antinucleon interaction from a Skyrme model based on large N_C QCD can be a promising approach. We expect that we can further discuss whether there exists a nucleon-antinucleon bound state (baryonium) in this framework.

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