Target-mass corrections to the truncated moments of nucleon spin structure functions and quark-hadron duality

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The Cornell-Norton and Nachtmann moments of nucleon spin structure functions are discussed with a nonvanishing nucleon mass. Target-mass corrections to the truncated moments of the structure functions are calculated. Moreover, the corrections to the Bloom-Gilman quark-hadron dualities of g_1^p and g_2^p in both the inelastic resonance production region and the elastic one are analyzed.

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I. INTRODUCTION

We know that the study of the Bloom-Gilman quark-hadron duality is essential to understand the physics behind the connection between perturbative QCD (pQCD) and nonperturbative QCD [1]. In 2000, new evidence of the valence-like quark-hadron duality in the nucleon unpolarized structure function F_2 was reported by Jefferson Lab. [2]. The new data can revisit the quark-hadron duality and show that the duality works even in a rather low momentum transfer region of $Q^2 \sim 1$ GeV^2. The well-known Bloom-Gilman quark-hadron duality [3] tells us that prominent resonances do not disappear relative to the background even at a large Q^2. It also means that the average of the oscillating resonance peaks in a resonance region is the same as the scaling structure function at a large Q^2 value. The origin of the Bloom-Gilman quark-hadron duality has been discussed by De Rujula, Georgi, and Politzer [4] with a QCD explanation. According to operator production expansion (OPE), it is argued that higher twist effects turn out to be small in the integral of the structure functions [4]. The empirical duality was also extensively studied [5] through a consideration of the asymptotic perturbative QCD behaviors of the resonance electromagnetic transition amplitudes at a large momentum transfer. Recently, many interesting studies of quark-hadron duality have been published [6–12]. Particularly, Close and Isgur [1] discussed the evolution of the nucleon structure functions from a coherent resonance region to an incoherent inelastic scattering one. For a review paper about quark-hadron duality see Ref. [13].

So far, there is no definitely experimental evidence of the Bloom-Gilman quark-hadron duality of the nucleon spin structure functions, such as g_1^p and g_2^p. It is natural to expect that the Q^2 value of the occurrence of the quark-hadron duality of g_1^p is larger than that of the unpolarized structure function F_2^p [5]. This is because a very strong Q^2 dependence of g_1^p in the low-Q^2 region is required by the well-known Gerasimov-Drell-Hearn (GDH) sum rule [14]. Experimental studies of the quark-hadron duality of g_1^p were performed by HERMES and JLab recently [15]. The limited data indicate that the onset of the duality of g_1^p is likely at a Q^2 value larger than 1.7 GeV^2. Theoretical analyses also gave a similar conclusion that the occurrence of the duality of g_1^p is about $Q^2 \sim 2$ GeV^2 [10].

We know that the target-mass corrections to the nucleon structure functions is of pure kinematical origin. They are different from the other higher twist effects from dynamical multigluon exchanges or parton correlations. There are several works about the target-mass corrections to $g_{1,2}$ in the literature [16,17]. The corrections to the Bjorken sum rule were studied in Ref. [18] by considering the Nachtmann moments. Recently, the explicit expressions of all the nucleon spin structure functions with the target-mass corrections have been given in Refs. [19–23]. In this work, the target-mass corrections to the truncated moments of the proton spin structure functions are calculated. Moreover, the corrections to the quark-hadron dualities of the proton spin structure functions are analyzed.

This paper is organized as follows. In Sec. II, the target-mass corrections to the Cornell-Norton and Nachtmann moments of the nucleon spin structure functions are discussed. Section III is devoted to the numerical results of the target-mass corrections to the truncated moments of g_1^p and g_2^p and to the quark-hadron dualities in the inelastic resonance production region. The corrections to the local dualities of the proton spin structure functions in the elastic region are displayed in Sec. IV. Finally, concluding remarks are given in the last section.

II. THE CORNELL-NORTON AND NACHTMANN MOMENTS OF THE NUCLEON SPIN STRUCTURE FUNCTIONS AND THE TARGET-MASS CORRECTIONS

According to the method of Georgi and Politzer [24], one can get the target-mass corrections (TMCs) to the spin structure functions. Recent calculations show that [19,20]

$$g_1^{\mathrm{TMCs}}(x, Q^2) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn x^{-n} \sum_{j=0}^{\infty}\left(\frac{M^2}{Q^2}\right)^j \times \frac{n(n+j)!}{j!(n-1)!(n+2j)^2} a_1^{(n+2j)}(Q^2), \quad (1)$$

where $a_1^{(n)}(Q^2)$ stands for the nth Cornwall-Norton (CN) moment of $g_1(x, Q^2; M = 0)$ with

$$a_1^{(n)}(Q^2) = \int_0^1 dx x^{n-1} g_1(x, Q^2; M = 0). \quad (2)$$

$a_1^{(n)}(Q^2)$ can be calculated in pQCD with all the mass terms $\mathcal{O}(M^n/Q^n)$ being neglected. When the target-mass corrections are considered, g_1 is [19,20]

$$\begin{aligned} g_1^{\rm TMCs}(x, Q^2) = & \frac{x g_1(\xi, Q^2; M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} \\ & + \frac{4M^2x^2}{Q^2}\frac{x+\xi}{\xi(1+4M^2x^2/Q^2)^2} \\ & \times \int_\xi^1 \frac{d\xi'}{\xi'} g_1(\xi', Q^2; M=0) \\ & - \frac{4M^2x^2}{Q^2}\frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}} \\ & \times \int_\xi^1 \frac{d\xi'}{\xi'} \int_{\xi'}^1 \frac{d\xi''}{\xi''} g_1(\xi'', Q^2, M=0), \end{aligned} \quad (3)$$

where only the contributions of twist-2 operator matrix elements are taken into account, and the Nachtmann variable [16] ξ in Eq. (3) is

$$\xi = \frac{2x}{1+\sqrt{1+\frac{4M^2x^2}{Q^2}}}, \quad (4)$$

with the Bjorken variable $x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2+W^2-M^2}$ (W being the center-of-mass energy). Moreover, we have

$$\begin{aligned} g_2^{\rm TMCs}(x, Q^2) = & -\frac{x g_1(\xi, Q^2; M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} \\ & + \frac{x(1-4M^2x\xi/Q^2)}{\xi(1+4M^2x^2/Q^2)^2} \\ & \times \int_\xi^1 \frac{d\xi'}{\xi'} g_1(\xi', Q^2; M=0) \\ & + \frac{3}{2}\frac{4M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}} \int_\xi^1 \frac{d\xi'}{\xi'} \\ & \times \int_{\xi'}^1 \frac{d\xi''}{\xi''} g_1(\xi'', Q^2, M=0). \end{aligned} \quad (5)$$

It should be mentioned that this above $g_2^{\rm TMCs}$, with the contribution of the twist-2 operator matrix elements, satisfies the well-known Wandzur-Wilczek (WW) relation [25]

$$g_2^{\rm TMCs}(x) = -g_1^{\rm TMCs}(x) + \int_x^1 \frac{dy}{y} g_1^{\rm TMCs}(y), \quad (6)$$

which means that the WW relation is not broken by TMCs.

Generally speaking, when a photon is absorbed by a parton k, we have

$$(\xi P + q)^2 = m_k^2,$$

where P is the total momentum of the initial nucleon, m_k is the final parton mass, and ξ represents the fraction of the total momentum carried by the parton k. If the momentum transfer $Q^2 = -q^2$ is very large and m_k is very small, we have $\xi = x$. However, if the momentum transfer is not large enough, we get the Nachtmann variable defined by Eq. (4). In a general case, we have

$$\xi = \frac{2x}{1+\sqrt{1+\frac{4M^2x^2}{Q^2}(1+\frac{m_k^2}{Q^2})}}\left(1+\frac{m_k^2}{Q^2}\right).$$

Note that the physical meaning of the Nachtmann variable ξ is clear and is more general than that of the Bjorken variable x, since ξ takes the target-mass corrections into account.

The CN moments of the nucleon spin structure functions with TMCs are

$$M_{1,2}^{(n)}(Q^2; {\rm TMCs}) = \int_0^1 dx x^{n-1} g_{1,2}^{\rm TMCs}(x, Q^2). \quad (7)$$

They can be expanded to the third order of M^2/Q^2 as

$$\begin{aligned} M_1^{(n)}(Q^2; {\rm TMCs}) = & a_1^{(n)} + \frac{M^2}{Q^2}\frac{n^2(n+1)}{(n+2)^2} a_1^{(n+2)} \\ & + \left(\frac{M^2}{Q^2}\right)^2 \frac{n^2(n+1)(n+2)}{2(n+4)^2} a_1^{(n+4)} \\ & + \left(\frac{M^2}{Q^2}\right)^3 \frac{n^2(n+1)(n+2)(n+3)}{6(n+6)^2} \\ & \times a_1^{(n+6)} + \mathcal{O}\left(\frac{M^8}{Q^8}\right) \end{aligned} \quad (8)$$

and

$$\begin{aligned} M_2^{(n)}(Q^2; {\rm TMCs}) = & -\frac{n-1}{n} a_1^{(n)} - \frac{M^2}{Q^2}\frac{n(n-1)(n+1)}{(n+2)^2} a_1^{(n+2)} \\ & - \left(\frac{M^2}{Q^2}\right)^2 \frac{n(n-1)(n+1)(n+2)}{2(n+4)^2} a_1^{(n+4)} \\ & - \left(\frac{M^2}{Q^2}\right)^3 \frac{n(n-1)(n+1)(n+2)(n+3)}{6(n+6)^2} \\ & \times a_1^{(n+6)} + \mathcal{O}\left(\frac{M^8}{Q^8}\right). \end{aligned} \quad (9)$$

Clearly, when the nucleon mass vanishes, we have

$$M_1^{(n)}(Q^2; M=0) = a_1^{(n)}, \quad M_2^{(n)}(Q^2; M=0) = -\frac{n-1}{n} a_1^{(n)}. \quad (10)$$

Here, we reiterate that Eq. (9) indicates that the WW relation of Eq. (6) and the Burkhardt-Cottingham (BC) sum rule [26] for $M_2^{(1)}(Q^2; {\rm TMCs})$ are satisfied.

We know that the CN moment of g_1 can be generally expanded in inverse powers of Q^2 in OPE. A typical example is

$$\Gamma_1 = \int_0^1 dx g_1(x, Q^2; M \neq 0) = \sum_{\tau=2,{\rm even}}^{\infty} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}. \quad (11)$$

In Eq. (11) the leading-twist (twist-2) component μ_2 is determined by the matrix elements of the axial vector operator $\bar\psi\gamma_\mu\gamma_5\psi$, summed over various quark flavors. The coefficient of the $1/Q^2$ term contains the contributions from the terms of twist-2 $\tilde{a}_2$, twist-3 $\tilde{d}_2$, and twist-4 $\tilde{f}_2$, respectively. Thus,

$$\mu_4 = \tfrac{1}{9}M^2(\tilde{a}_2 + 4\tilde{d}_2 + 4\tilde{f}_2). \tag{12}$$

In Eq. (12) $\tilde{a}_2$ arises from TMCs and is of purely kinematical origin. It relates to the second moment of the twist-2 part of $g_1(x, Q^2; M = 0)$. The other higher twist terms, such as $\tilde{d}_2$ and $\tilde{f}_2$, result from the reduced matrix elements, which are of dynamical origin since they show correlations among the partons.

At variance with the CN moments $M_{1,2}^{(n)}(Q^2)$, the Nachtmann moments of the nucleon spin structure functions are [16,17]

$$M_1^{(n)}(Q^2; N) = \int_0^1 dx\frac{\xi^{n+1}}{x^2}\left\{\left[\frac{x}{\xi} - \frac{n^2}{(n+2)^2}\frac{M^2x^2}{Q^2}\frac{\xi}{x}\right]g_1(x, Q^2; M \neq 0) - \frac{M^2x^2}{Q^2}\frac{4n}{n+2}g_2(x, Q^2; M \neq 0)\right\}(n = 1, 3, 5, \ldots) \tag{13}$$

and

$$M_2^{(n)}(Q^2; N) = \int_0^1 dx\frac{\xi^{n+1}}{x^2}\left\{\frac{x}{\xi}g_1(x, Q^2; M \neq 0) + \left[\frac{n}{n-1}\frac{x^2}{\xi^2} - \frac{n}{n+1}\frac{M^2x^2}{Q^2}\right] \times g_2(x, Q^2; M \neq 0)\right\}(n = 3, 5, \ldots). \tag{14}$$

The difference between the CN and the Nachtmann moments comes from the trace terms appearing in the matrix elements of the operators of a definite spin. Those trace terms are disregarded in the CN moments, but they are kept in the Nachtmann moments [23]. The $M_{1,2}^{(n)}(Q^2; N)$ in these equations contain the contributions from the spin-n and twist-2 operators. Moreover, they are constructed to protect themselves from target-mass corrections of kinematical origin and they contain only the dynamical higher twists. Clearly, the two Nachtmann moments are simultaneously constructed by using the two moments of the spin structure functions $g_{1,2}$. They can be expanded to the third order of M^2/Q^2 as

$$\begin{aligned} M_1^{(n)}(Q^2; N) = {} & M_1^{(n)}(Q^2) - \left(\frac{M^2}{Q^2}\right)\frac{n}{n+2} \times \left[\frac{(n+1)(n+4)}{n+2}M_1^{(n+2)} + 4M_2^{(n+2)}\right] \\ & + \left(\frac{M^2}{Q^2}\right)^2\frac{n(n+1)}{n+2}\left[\frac{n+6}{2}M_1^{(n+4)} + 4M_2^{(n+4)}\right] \\ & - \left(\frac{M^2}{Q^2}\right)^3\frac{n(n+1)}{n+2}\left[\frac{(n+5)(n+8)}{6} \times M_1^{(n+6)} + 2(n+4)M_2^{(n+6)}\right] + \mathcal{O}\left(\frac{M^8}{Q^8}\right) \end{aligned} \tag{15}$$

and

$$\begin{aligned} M_2^{(n)}(Q^2; N) = {} & M_1^{(n)} + \frac{n}{n-1}M_2^{(n)} - \left(\frac{M^2}{Q^2}\right)\frac{n}{n+1} \times \left[(n+1)M_1^{(n+2)} + (n+2)M_2^{(n+2)}\right] \\ & + \left(\frac{M^2}{Q^2}\right)^2\frac{n}{2}\left[(n+3)M_1^{(n+4)} + (n+4)M_2^{(n+4)}\right] \\ & - \left(\frac{M^2}{Q^2}\right)^3\frac{n(n+4)}{6}\left[(n+5)M_1^{(n+6)} + (n+6)M_2^{(n+6)}\right] + \mathcal{O}\left(\frac{M^8}{Q^8}\right), \end{aligned} \tag{16}$$

with

$$M_{1,2}^{(n)}(Q^2) = \int_0^1 x^{n-1}dx\, g_{1,2}(x, Q^2; M \neq 0). \tag{17}$$

Furthermore, if we consider only the twist-2 contribution, we get, from Eqs. (8)–(10),

$$M_1^{(n)}(Q^2; N) \sim a_1^{(n)}; \quad M_2^{(n)}(Q^2; N) \sim 0. \tag{18}$$

Thus, $M_1^{(1)}(Q^2; N) \sim \mu_2$ and $M_1^{(2)}(Q^2; N) \sim \mu_4$ [see Eq. (12)]. Here, it is clear that the Nachtmann moments relate to the dynamical matrix elements of the operators with a definite spin [23].

Piccone and Ridolfi [19] have compared the Nachtmann moments with the CN moments. They argued that the Nachtmann moments of Eqs. (13) and (14) are not directly applicable in a full analysis of the polarized deep inelastic scattering (DIS) data because the target-mass-corrected reduced matrix elements of the relevant operators in OPE, such as $\tilde{a}_n$ and $\tilde{d}_n$, are expressed in terms of the spin structure functions $g_{1,2}$. If the target-mass corrections are taken into account, those expressions reduce to the moments of the structure functions in the massless limit, but they do not have a simple parton model interpretation in this limit. Thus, it is claimed that Eqs. (3), (5), (8), and (9) have the advantage that the moments of the spin structure functions can be expressed as the functions of the reduced operator matrix elements and the target-mass corrections to the spin structure functions can be explicitly seen.

III. NUMERICAL RESULTS OF THE TARGET-MASS CORRECTIONS

To check the target-mass corrections, we explicitly compare $g_1(x, Q^2)$ with $g_1^{\mathrm{TMCs}}(x, Q^2)$ and $g_2(x, Q^2)$ with $g_2^{\mathrm{TMCs}}(x, Q^2)$ in Fig. 1 by calculating the following two ratios:

$$\Delta R_{1,2}(x, Q^2) = \frac{g_{1,2}^{\mathrm{TMCs}}(x, Q^2) - g_{1,2}(x, Q^2; M = 0)}{g_{1,2}(x, Q^2; M = 0)}. \tag{19}$$

Here, we employ a next-to-leading order pQCD calculation for the spin structure functions in the deep inelastic scattering region with $M = 0$. There are several known calculations in the literature, such as those by Leader, Sidorov, and Stamenov (LSS) [27], by Glück, Reya, Stratmann, and Vogelsang

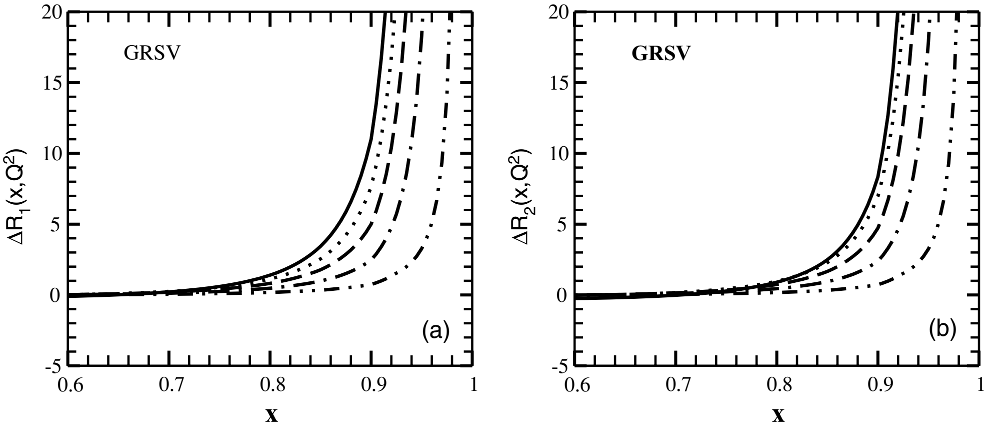

FIG. 1. Comparisons of the spin structure functions with and without TMCs: (a) $\Delta R_1(x, Q^2)$ and (b) $\Delta R_2(x, Q^2)$. The solid, dotted, dashed, dotted-dashed, and double-dotted-dashed curves represent the results with $Q^2 = 1, 3, 5, 10$, and 30 GeV^2, respectively.

(GRSV) [28], by the Asymmetry Analysis Collaboration (AAC) [29], and by others [30]. In this work, we simply use the predictions of GRSV to analyze the target-mass corrections to the spin structure functions.

From Fig. 1 we clearly see that TMCs play a remarkable role. They enlarge the amplitudes of the spin structure functions, particularly in the large-x region. Figures 1(a) and 1(b) reasonably show that the smaller the momentum transfer Q^2 is, the larger the effects of TMCs are. Moreover, it is found that $g_{1,2}^{\rm TMCs}(x, Q^2)$ do not vanish in the limit of $x \to 1$, although $g_{1,2}(x, Q^2; M = 0) \to 0$. This phenomenon can be easily understood from Eqs. (3) and (5), since we always have $\xi < 1$. For example, $\xi(x = 1, Q^2) \sim 0.64$, 0.85, and 0.92 for $Q^2 = 1$, 5, and 10 GeV^2, respectively.

In Figs. 2 and 3, we respectively display $g_{1,2}(\xi, Q^2)$ and $g_{1,2}^{\rm TMCs}(\xi, Q^2)$ for six different Q^2 values. We find that, although the ξ dependences of $g_{1,2}$ with $M = 0$ are obviously different at different values of Q^2, the ξ shapes of the target-mass-corrected scaling structure functions $g_{1,2}^{\rm TMCs}$ with $Q^2 \geqslant 3$ GeV^2 are similar to the ones of the scaling spin structure functions $g_1(\xi, Q^2_{\rm high}; M = 0)$ seen at high $Q^2_{\rm high}$ (say $Q^2_{\rm high} = 30$ GeV^2, for example). Thus, we may approximately get a dual relation

$$g_{1,2}^{\rm TMCs}(\xi, Q^2) \sim g_{1,2}^{\rm TMCs}\big(\xi, Q^2_{\rm high}\big) \sim g_{1,2}\big(\xi, Q^2_{\rm high}; M = 0\big). \quad (20)$$

This relation tells us that TMCs partly compensate the effects of the pQCD evolution. To explicitly show this approximate relation, we respectively plot the relative discrepancies between $g_{1,2}^{\rm TMCs}(x, Q^2)$ and $g_{1,2}(x, Q^2_{\rm high}; M = 0)$ (select $Q^2_{\rm high} = 30$ GeV^2) in Fig. 4 by calculating the two ratios of

$$\Delta R_{1,2}^{(30)}(\xi, Q^2) = \frac{g_{1,2}^{\rm TMCs}(\xi, Q^2) - g_{1,2}\big(\xi, Q^2_{\rm high} = 30\ {\rm GeV}^2; M = 0\big)}{g_{1,2}\big(\xi, Q^2_{\rm high} = 30\ {\rm GeV}^2; M = 0\big)}. \quad (21)$$

Figure 4 obviously tells us that in the regions of $0.2 \leqslant \xi \leqslant 1$ and $Q^2 \geqslant 3$ GeV^2 this dual relation is approximately valid. The divergences from this relation are about 20%. This dual relation is similar to the dual relation obtained for the unpolarized structure function F_2^p in Ref. [12].

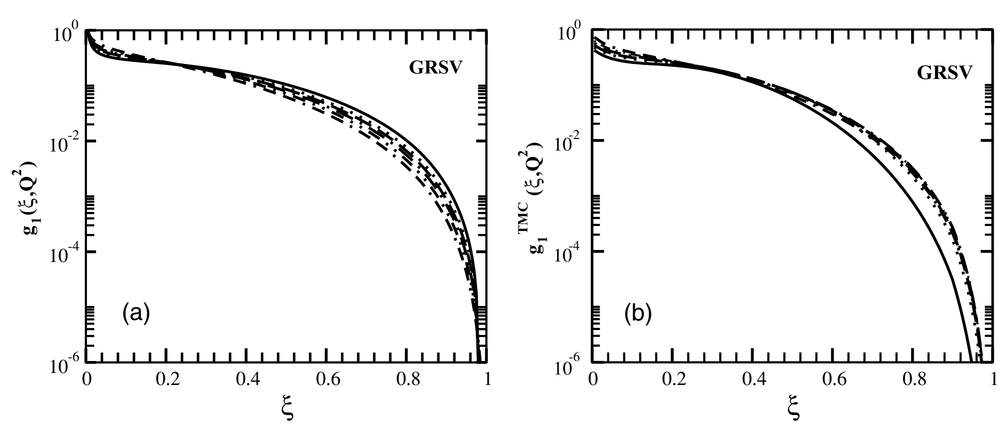

FIG. 2. Proton spin structure function $g_1(\xi, Q^2)$ (a) without and (b) with the target-mass corrections. The solid, dotted, dashed, dotted-dashed, double-dotted-dashed, and dotted-double-dashed curves stand for the cases of $Q^2 = 1, 2, 3, 5, 10$, and 30 GeV^2, respectively.

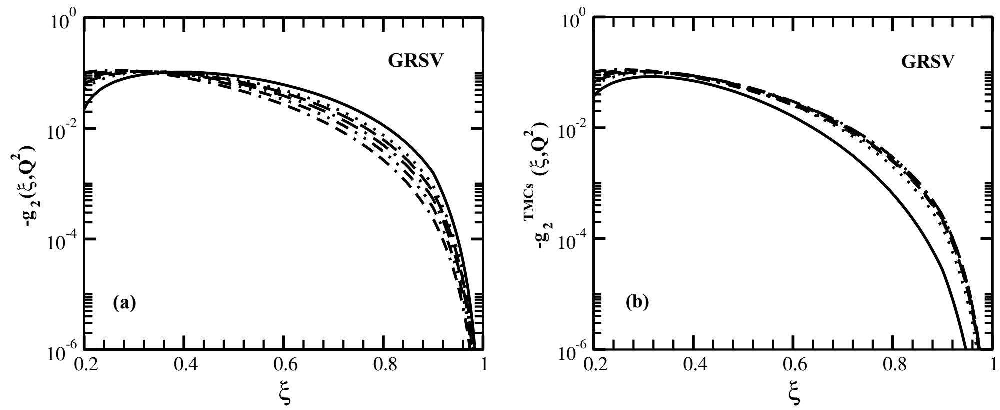

FIG. 3. Proton spin structure function $-g_2(\xi, Q^2)$ (a) without and (b) with the target-mass corrections. Notation as in Fig. 2.

It should be reiterated that the x (or ξ) dependences of the target-mass-corrected structure functions $g_{1,2}^{\rm TMCs}$ are different from those without TMCs, particularly in the large-x (or large-ξ) region. We know that $\xi \leqslant x$, and the ξ range extends up to $\xi_{\max} = 1$, corresponding to unphysical, but finite values of $x > 1$ in the case of $Q > M$. Clearly, the larger Q^2 is, the smaller the difference between $g_{1,2}$ and $g_{1,2}^{\rm TMCs}$ becomes (see Figs. 2–4). For example, when $Q^2 = 30\ {\rm GeV}^2$, the discrepancies between $g_{1,2}^{\rm TMCs}(x, Q^2)$ and $g_{1,2}(x, Q^2; M = 0)$ are only less than 15% in the region of $0.2 \leqslant \xi \leqslant 1$ as shown by Fig. 4.

We know that the Bloom-Gilman quark-hadron duality means that the smooth scaling curve seen in the high-Q^2 region is an average over the resonance bumps seen in the low-Q^2 region. To check TMCs to the quark-hadron duality numerically, we calculate three truncated CN moments. They are defined in the inelastic resonance production region as

$$\bar{M}_{1,2}^{(n)}(Q^2) = \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}^{\rm Res}(\xi, Q^2), \qquad (22)$$

$$\bar{A}_{1,2}^{(n)}(Q^2) = \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}^{\rm TMCs}(\xi, Q^2), \qquad (23)$$

and

$$\bar{A}_{1,2}^{(n)}(Q^2; M = 0) = \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}(\xi, Q^2; M = 0), \qquad (24)$$

where ξ_π (or ξ^*) stands for the Nachtmann variable with the minimum of the center-of-mass energy $W = M + m_\pi$ (or the maximum of $W_{\max} = 2.5$ GeV). Here, the moments are truncated ones and are very sensitive to the Bloom-Gilman quark-hadron duality with respect to the untruncated moments because most of the contributions of the resonance bumps are considered. In Eq. (22) $g_{1,2}^{\rm res}$ means the structure functions estimated by a resonance language. They can be calculated by taking into account the parametrization forms of the proton spin structure functions in the resonance region proposed by Simula *et al.* [31]. Those empirical forms contain the contributions of the nucleon elastic peak, of the nucleon resonances, and of the nonresonance background. They have altogether 14 parameters fixed by fitting to the available data in the resonance region. A simple Breit-Wigner shape was used to describe the W dependence of the contribution of an isolated resonance. All four-star resonances, having a total transverse photo-amplitude $\sqrt{|A_{1/2}|^2 + |A_{3/2}|^2}$ larger than $0.05\ {\rm GeV}^{-1/2}$, are considered. It should be mentioned

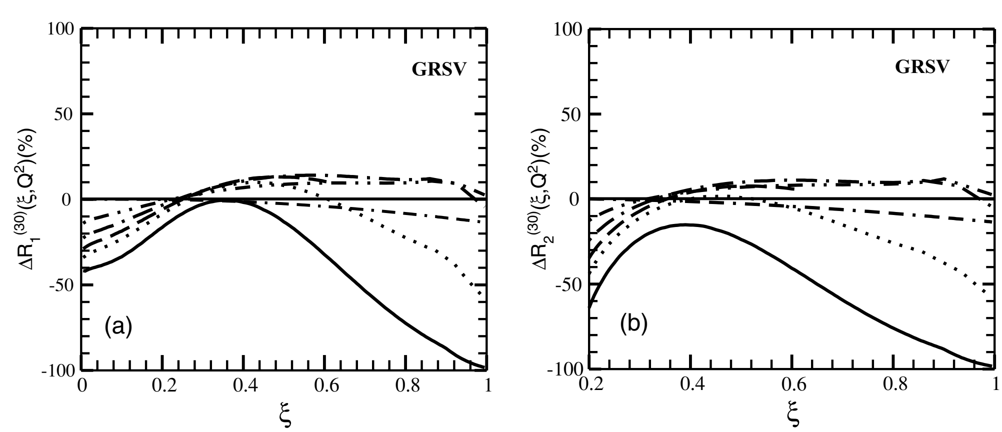

FIG. 4. (a) $\Delta R_1^{(30)}$ and (b) $\Delta R_2^{(30)}$. The solid, dotted, dashed, dotted-dashed, double-dotted-dashed, and dotted-double-dashed curves stand for the cases of $Q^2 = 1, 2, 3, 5, 10$, and $30\ {\rm GeV}^2$, respectively.

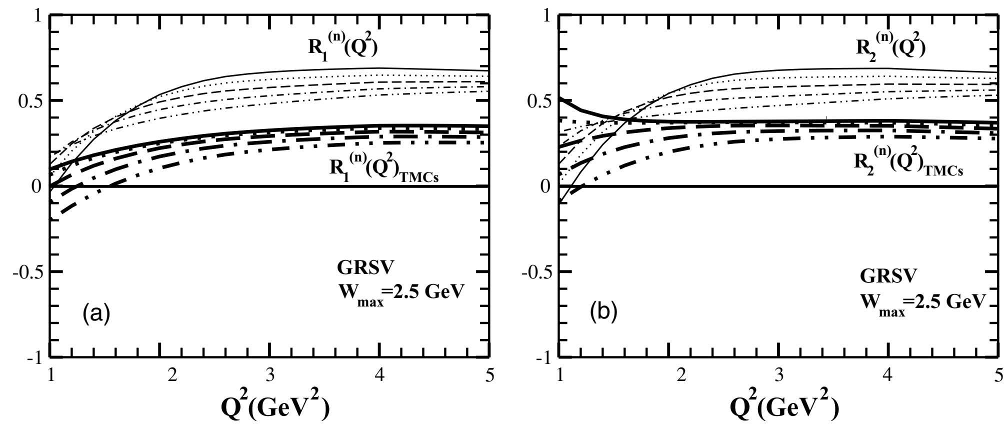

FIG. 5. Comparisons of $R^n_{1,2}(Q^2)_{\rm TMCs}$ (thick curves) with TMCs to $R^n_{1,2}(Q^2)$ (thin curves) without TMCs, for (a) g_1^p and (b) g_2^p. The solid, dotted, dashed, dotted-dashed, and double-dotted-dashed curves are the results with $n = 1, 2, 3, 4$, and 5, respectively.

that we take ξ as the integrated variable in the integrals of Eqs. (22)–(24) since the Nachtmann variable is the correct one for studying QCD scaling violations in the nucleon. Besides the integrands $g_{1,2}^{\rm TMCs}(x, Q^2)$, the integrals with respect to ξ, instead of x, also can partially take the target-mass corrections into account [32]. Here the differences between Eqs. (23) and (24) are due to the target-mass corrections to the scaling structure functions $g_{1,2}(x, Q^2; M = 0)$.

To study the Bloom-Gilman quark hadron duality explicitly, we respectively plot the ratios of

$$\begin{aligned} R^{(n)}_{1,2}(Q^2)_{\rm TMCs} &= \bar{M}^n_{1,2}(Q^2)\big/\bar{A}^n_{1,2}(Q^2) - 1, \\ R^{(n)}_{1,2}(Q^2) &= \bar{M}^n_{1,2}(Q^2)\big/\bar{A}^n_{1,2}(Q^2; M = 0) - 1 \end{aligned} \tag{25}$$

in Fig. 5, where the effects of TMCs can be easily seen from the differences between the thin and thick curves. It is explicitly shown that the results with TMCs are almost independent of n and of Q^2 when Q^2 is large. Moreover, the values of the ratios with TMCs are closer to zero (or to x axis) than those without TMCs. This phenomenon occurs because TMCs enlarge the structure functions in the large-x region, particularly in the region of $x \to 1$. As a result, the target-mass corrections increase the denominators in the ratios of Eq. (25) and decrease the amplitudes of the ratios. Thus, we conclude that the target-mass corrections are important to get the Bloom-Gilman quark-hadron dualities of the proton spin structure functions although the Q^2 dependences of the two sets of the ratios (with or without TMCs) look similar. If no target-mass corrections are considered, we have $\xi \to x$, $g_{1,2}^{\rm TMCs}(\xi, Q^2) \to g_{1,2}(x, Q^2)$, and thus $\bar{A}^n_{1,2}(Q^2)$ turns to be $\bar{A}^n_{1,2}(Q^2; M = 0)$. However, when TMCs are taken into account, the corrections can be explicitly seen from the differences between the two truncated moments of Eqs. (23) and (24).

IV. THE TARGET-MASS CORRECTIONS TO THE LOCAL DUALITIES OF $g_{1,2}^p$ IN THE ELASTIC REGION

One can also check the target-mass corrections to the local dualities of the proton spin structure functions contributed by the nucleon elastic peak. Here, we calculate the other three truncated moments in the elastic region:

$$M^{(n)}_{1,2}(Q^2; {\rm el}) = \int_{\xi_\pi}^{1} d\xi\, \xi^{n-1} g^{\rm res}_{1,2}(\xi, Q^2), \tag{26}$$

$$\bar{A}^{(n)}_{1,2}(Q^2; {\rm el}) = \int_{\xi_\pi}^{1} d\xi\, \xi^{n-1} g^{\rm TMCs}_{1,2}(\xi, Q^2), \tag{27}$$

and

$$\bar{A}^{(n)}_{1,2}(Q^2; {\rm el}; M = 0) = \int_{\xi_\pi}^{1} d\xi\, \xi^{n-1} g_{1,2}(\xi, Q^2; M = 0). \tag{28}$$

Equation (26) can be rewritten in terms of the nucleon electric and magnetic form factors, because in the elastic region

$$g_1^{\rm res} = \frac{M\tau}{1+\tau} G_M(Q^2)[G_E(Q^2) + \tau G_M(Q^2)]\delta\left(\nu - \frac{Q^2}{2M}\right) \tag{29}$$

and

$$g_2^{\rm res} = \frac{M\tau^2}{1+\tau} G_M(Q^2)[G_E(Q^2) - G_M(Q^2)]\delta\left(\nu - \frac{Q^2}{2M}\right). \tag{30}$$

Thus, we get

$$\bar{M}^{(n)}_1(Q^2; {\rm el}) = \frac{G_M(Q^2)(G_E(Q^2) + \tau G_M(Q^2))}{2(1+\tau)} \frac{\xi_{\rm el}^{n+1}}{2 - \xi_{\rm el}}, \tag{31}$$

$$\bar{M}^{(n)}_2(Q^2; {\rm el}) = \frac{\tau G_M(Q^2)(G_E(Q^2) - G_M(Q^2))}{2(1+\tau)} \frac{\xi_{\rm el}^{n+1}}{2 - \xi_{\rm el}}, \tag{32}$$

where $\tau = Q^2/4M^2$ and $\xi_{\rm el} = 2/(1 + \sqrt{1 + 1/\tau})$. It should be stressed that the truncated moments of Eqs. (26)–(28) are contributed only by the nucleon elastic peak. In our calculation we replace the argument of the scaling structure functions in Eq. (28) directly by the Nachtmann variable ξ. Moreover, we simply employ the parametrizations of the nucleon form

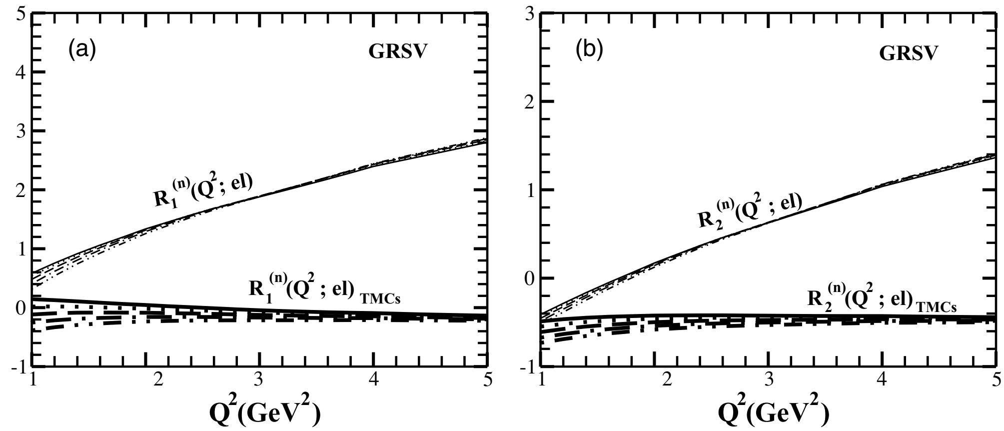

FIG. 6. Comparisons of $R^n_{1,2}(Q^2;\mathrm{el})_{\mathrm{TMCs}}$ (thick curves) with TMCs to $R^n_{1,2}(Q^2;\mathrm{el})$ (thin curves) without TMCs, for (a) g_1^p and (b) g_2^p. Notation as in Fig. 5

factors from Ref. [33]. To check the local dualities, we calculate the ratios of

$$R^{(n)}_{1,2}(Q^2;\mathrm{el})_{\mathrm{TMCs}} = \bar{M}^{(n)}_{1,2}(Q^2)\big/\bar{A}^{(n)}_{1,2}(Q^2;\mathrm{el}) - 1,$$
$$R^{(n)}_{1,2}(Q^2;\mathrm{el}) = \bar{M}^{(n)}_{1,2}(Q^2)\big/\bar{A}^{(n)}_{1,2}(Q^2;\mathrm{el};M=0) - 1. \quad (33)$$

In Fig. 6 these ratios are displayed by the thick and thin curves, respectively. We clearly see a remarkable role played by TMCs. If no TMCs are considered, the local quark-hadron dualities of the proton spin structure functions are obviously broken, as shown by the thin curves in the two figures, because the ratios increase continuously. However, when the target-mass corrections are taken into account, the ratios become almost independent of Q^2. This means that the quark-hadron dualities in the local elastic region may be preserved owing to the target-mass corrections. This phenomenon can also be easily understood according to our discussions in Sec. III. We know that the target-mass corrections remarkably enlarge the structure functions in the large-x region, particularly in the region of $x \to 1$. Thus, the denominators in the ratios of Eq. (33) increase and the amplitudes of the ratios decrease accordingly.

Note that the model-independent local duality contributed by the nucleon elastic peak was employed to obtain the polarized asymmetries of the nucleon [6]. However, it is argued that the estimated nucleon magnetic form factors extracted from the elastic local duality is well below the experimental points by a factor of 2–3 [12]. There are several discussions on the local duality of F_2 contributed by the elastic peak [34]. Here, our results show that the local dualities of the proton spin structure functions are possible if the target-mass corrections are taken into account.

V. CONCLUDING REMARKS

We have studied the target-mass corrections and their effects on the proton spin structures functions and on the well-known Bloom-Gilman quark-hadron dualities. The target-mass corrections to the CN moments and the Nachtmann moments are explicitly given. Our results show that TMCs play a remarkable effect on the proton spin structure functions, particularly in the low-Q^2 and large-x (or large-ξ) regions. The ξ dependences of $g^{\mathrm{TMCs}}_{1,2}(\xi, Q^2)$ indicate that when $Q^2 \geqslant 3$ GeV^2, we can approximately get a dual relation of $g^{\mathrm{TMCs}}_{1,2}(\xi, Q^2) \sim g_{1,2}(\xi, Q^2_{\mathrm{high}})$. The divergences from this relation are about 20%. Consequentially,

$$\bar{A}^n_{1,2}(Q^2) \sim \int_{\xi*}^{\xi_\pi} d\xi\, \xi^{n-1} g_{1,2}\big(\xi, Q^2_{\mathrm{high}}; M=0\big). \quad (34)$$

This relation means that TMCs may compensate the role played by the pQCD evolution, at least partially. We also find that the ratios $R^{(n)}_{1,2}(Q^2)_{\mathrm{TMCs}}$ are smaller and much closer to zero (or to x axis) than $R^{(n)}_{1,2}(Q^2)$ without TMCs. This suppression of the ratios is favored by the quark-hadron dualities. In fact, the Q^2 dependences of the two sets of ratios are similar in the large-Q^2 region. Such a role of target-mass corrections is clearly displayed by this suppression. Moreover, the local dualities in the elastic region are seen if the target-mass corrections are taken into account. The appearances of the dualities, both in the inelastic resonance production region and in the elastic one, result from the fact that TMCs increase the values of $g_{1,2}$ evidently in the large-x region, and therefore they remarkably enlarge the denominators of the ratios in Eqs. (25) and (33) and suppress the ratios. These conclusions remain the same for the other pQCD predictions of the scaling structure function $g_1(x, Q^2; M=0)$, such as those of AAC [29]. The remarkable role of TMCs in the ratios in the nucleon resonance production region and in the nucleon elastic region tells us that TMCs should be considered in the study of the Bloom-Gilman quark-hadron dualities. The occurrences of the dualities are expected to be $\geqslant$2 GeV^2. Here, we stress that we only examine the truncated moments, which are limited to the nucleon resonance production region or to the elastic scattering one. The dualities of those truncated moments cannot have any operator production expansion-based justification.

Our present results depend on the treatment of the target-mass corrections [19], on the parametrizations of g_1 and g_2 in the resonance region [31], on the pQCD predictions for $g_1(x, Q^2; M=0)$, and on the twist-2 WW relation for g_2.

It should be pointed out that, although the ratios in Fig. 5 indicate that they are almost Q^2 independent in the large-Q^2 region, they are not exactly zero as the Bloom-Gilman quark-hadron duality would indicate. The divergence is about 20%. Therefore, our calculation only indicates that the averages of the proton spin structure functions in the truncated resonance region are similar to those of the scaling functions in the DIS region. In fact, this phenomenon also appears in the study of the quark-hadron dualities of F_2^p [2,12]. One possible reason is the lack of higher twist terms, such as the twist-3 term $\tilde{d}_n$ and the twist-4 term $\tilde{f}_n$. Another is the difficulty in accurately modeling the large-ξ behaviors of the scaling spin structure functions. With increasing Q^2, the moments of Eqs. (22)–(24) and (26)–(28) are dominated by a smaller and smaller region near $\xi \sim 1$. For example, the upper (or lower) limits of ξ_π (or ξ^*) of the integrals in Eqs. (22)–(24) are about 0.563 (0.154), 0.832 (0.464), and 0.903 (0.628) for $Q^2 = 1$, 5, and 10 GeV^2, respectively. The corresponding values of x_π (or x^*) are about 0.780 (0.157), 0.947 (0.482), and 0.973 (0.651), respectively. Those kinematical regions of x are already beyond the one of the world data used in the pQCD predictions of the scaling spin structure functions. (The world data, covering the kinematical region of $0.005 \leqslant x \leqslant 0.75$ and $1 \leqslant Q^2 \leqslant 58$ GeV^2, are often employed.) To estimate the uncertainty in the large-ξ region, one may replace the upper limit (ξ_π) in the integral of Eq. (23) by $\xi'_\pi = \xi(x_{\max} = 0.75, Q^2)$. Then, those upper limits of Eqs. (22)–(24) become 0.55, 0.685, and 0.714 for the three cases of Q^2. As a result, one finds that the contributions from the large-ξ region ($x \geqslant 0.75$) to the integrals are around 1%, 8%, and 30% for the three Q^2 cases, respectively. Thus, one concludes that the scaling structure functions in the large-x region have a more important effect on the truncated integrals with a large Q^2 value than that with a small Q^2. Unfortunately, the available data of spin structure functions with a large x in the DIS region are very limited. The problem becomes even more evident when we discuss the local dualities in the elastic scattering region. Thus, new data in the large-x (large-ξ) region with a high precision are urgently required to test the duality quantitatively. If the duality is quantitatively confirmed, it would allow for a precise verification of our knowledge of the large-x parton distribution functions.

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