Asymmetry dependence of the caloric curve for mononuclei

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The asymmetry dependence of the caloric curve, for mononuclear configurations, is studied as a function of neutron-to-proton asymmetry with a model that allows for independent variation of the neutron and proton surface diffusenesses. The evolution of the effective mass with density and excitation is included in a schematic fashion and the entropies are extracted in a local density approximation. The plateau in the caloric curve displays only a slight sensitivity to the asymmetry.

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This report presents the results of extending the model reported in Refs. [1,2], which calculates the caloric curve (temperature T versus excitation energy per particle ϵ) to explicitly treat the neutron/proton asymmetry degree of freedom. In making this extension, we have taken the surface diffusenesses for neutrons and protons (b_n and b_p) as the shape degrees of freedom. The self-similar expansion (c) degree of freedom is equilibrated at low excitation energy but not allowed to vary with excitation energy.

The objective of our model is to calculate the entropy of a mononucleus as a function of excitation energy in a manner that takes into account, in a plausible fashion, the effects of both expansion and the inevitable loss of collectivity. The present work then answers the question of whether the expected plateau in $T(\epsilon)$, for finite isolated Fermi systems, has a significant asymmetry dependence. We present results for $A = 208$ with overall asymmetries $I \equiv (N - Z)/(N + Z)$ of 0.1, 0.21154, 0.3, and 0.4. (Asymmetry is sign sensitive with increasingly positive values, the only sign studied in this work, corresponding to increasing neutron excess.) We refer to the radially dependent asymmetry as $\delta(r) \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$. As a complete description of the model is presented in Ref. [2], we only discuss the model in outline form and mention the modifications to treat the asymmetry degree of freedom.

Of particular interest is the comparison of the asymmetry dependence of the plateau temperatures we find for our metastable (isolated) nuclei to the limiting temperature in true equilibrium calculations [3]. (The limiting temperature is a term coined when there is no solution to the paired Gibbs's equations requiring each component to have the same chemical potential in the coexisting liquid and vapor phases.)

The temperature, an auxiliary thermodynamic parameter for an isolated system, is taken to be the inverse of the rate of change of the maximum entropy $s_M \equiv \max[s(b_n, b_p)]$, with excitation energy ε

$$T = 1 \bigg/ \left[\frac{\partial s_M}{\partial \varepsilon}\right]. \tag{1}$$

It is the change of these statistical temperatures with excitation energy that defines our mononuclear caloric curve $T(\varepsilon)$.

In this work, the density profiles $\rho_\tau(r)$ of the two isospin partners ($\tau = n, p$) are of the same functional form but are individually scaled from a spherically symmetric native radial profile of the "standard" type

$$\rho_\tau^n(r, b_\tau) = \frac{\rho_\tau^o}{2}\left\{1 - \text{erf}\left[\frac{\mathrm{r} - \mathrm{C}_\tau(\mathrm{b}_\tau)}{\sqrt{2}\mathrm{b}_\tau}\right]\right\}. \tag{2}$$

The central radii C_τ, and thus the central densities ρ_τ^o, are defined in terms of effective sharp radii $R_\tau = r_\tau A^{1/3}$ and the surface widths b_τ by extending the expansion derived by Süssmann [4] to ensure volume conservation (of 1 part in 10^4) out to $b = 3$ fm. The shapes are limited to the spherically symmetric family $\rho(r, b_\tau, c_\tau) = c_\tau^3 \rho^n(c_\tau r, b_\tau)$. The effective sharp charge radius (or equivalently the central density or the self-similar expansion parameter c_τ) was adjusted to best reproduce the charge density of ^{208}Pb. The effective sharp radius for neutrons was determined for each asymmetry by maximizing the binding energy with nominal surface widths of $b_{n,p} = 1$ fm. Having determined the proton and neutron sharp radii (for $\varepsilon = 0$) only the two surface widths were allowed to vary with excitation energy. (Our previous isoscalar treatment, using (b and c) as independent variables, justifies this logic as we found that the surface expansion is the primary expansion degree of freedom.) The neutron skin thicknesses for $I = 0.1$, 0.21154, 0.3, and 0.4 were found to be $t(\text{fm}) =$ 0.10, 0.23, 0.36, and 0.50, respectively.

For any given excitation energy, the expansion parameters are found by maximizing the entropy in the (b_n, b_p) space. The entropy depends on the thermal energy u, which is less than the total energy by the energy needed for expansion ε_E, i.e., $u = \varepsilon - \varepsilon_E$. (The energies and entropies are expressed on a per nucleon basis.) The collective energy involved in expansion is taken from a simple energy-density formalism used in Ref. [2], which makes use of the equation of state (EOS) offered by the Thomas-Fermi model of Myers and Swiatecki [5]. In this model the binding energy (ε_{ED}) of a drop is the sum of three terms, corresponding to (a) the binding energy of the drop neglecting gradient corrections, (b) the gradient correction term, and (c) the Coulomb integral

$$\varepsilon_{\text{ED}} = \varepsilon_{\text{md}} + \varepsilon_{\text{gr}} + \varepsilon_{\text{coul}}, \tag{3}$$

where

$$\varepsilon_{\text{md}} = \int_0^\infty e_b(\rho, \delta)\rho(r)d\mathbf{r}, \tag{4}$$

$$\varepsilon_{\rm gr} = \frac{\hbar^2 b_{gt}}{8m}\int_0^\infty |\nabla\rho(b,c,r)|^2 d\mathbf{r}, \quad (5)$$

$$\varepsilon_{\rm coul} = -\int_0^\infty \left(\frac{4}{3}\pi r^3\rho_p\right)\frac{1}{r}(4\pi r^2\rho_p dr). \quad (6)$$

The strength of the gradient term ($b_{\rm gt}$) was adjusted to reproduce macroscopic mass trends [2]. The analytic Coulomb integral plus the small exchange correction were taken from Ref. [4]. In this model the asymmetry energy is not taken as a function of the extracted temperature variable and thus the change in the contribution of the asymmetry energy (to the total binding energy) with excitation energy is only a result in the change in the density profile.

The top panels of Fig. 1 display $\varepsilon_E(b_n, b_p)$ for $I = 0.1$ and 0.4. The absolute minima are near $(b_n, b_p) = (1.0, 1.0)$, with the equilibrium neutron width (at $\varepsilon = 0$ MeV) increasing with asymmetry. The (binding) energy cost of increasing the neutron (proton) surface width decreases (increases) with increasing asymmetry.

The dominant term in the expression for the entropy of a quantum drop of degenerate Fermi liquid can be written

$$S = 2\sqrt{aA(\varepsilon-\varepsilon_E)}, \quad (7)$$

where a is the level-density parameter and the total and expansion energies per nucleon are ε and $\varepsilon_{E,}$ respectively.

In the local density approximation, the level-density parameter depends on the nuclear profile, the local Fermi momentum $k_{F\tau}$, and the effective mass m_τ (for each isospin partner τ) [6,7]

$$a = \frac{\pi^2}{4}\sum_\tau\int\frac{\rho_\tau(r)}{\left[\hbar^2 k_{F\tau}^2(r)/2m_\tau^*\right]}d\mathbf{r}. \quad (8)$$

We adopt the factorization of the effective mass into a momentum m_k and frequency-dependent m_ω terms as suggested by Mahaux [8]. The isospin splitting in m_k is taken from the theoretical work of Rizzo *et al.* [9] (and Ref. [10], case 1), whereas the phenomenological m_ω dependence is that employed by De *et al.* [11]

$$\left(\frac{m^*}{m}\right)_\tau = [m_k]_\tau[m_\omega] = [m_k(\bar\rho,\delta)]_\tau[1-\beta(T)\bar\rho'(r)], \quad (9)$$

with the reduced nucleon density $\bar\rho = \rho/\rho_o$ ($\rho_o = 0.16$ fm^{-3}) and $\beta(T) = 0.4A^{1/3}\exp[-({\rm TA}^{1/3}/21)^2]$. The isospin splitting in the "k mass," drives this factor for neutrons (protons) up (down) from the nominal value of 0.7 with increasing asymmetry (neutron richness). This trend is expected from the larger strength of the n-p relative to the n-n and p-p interactions [12]. For both neutrons and protons, the k-mass factor increases to one with decreasing density.

In the present work we have left the "ω-mass" factor asymmetry independent. Although a low-lying nuclear structure will provide a surface peaked effective mass component [13], which could be interpreted as $m_\omega(\delta)$, the influence on this term on the level-density parameter a is largely washed out by $\varepsilon = 1$ MeV/u.

Entropy maps $S/A = s(b_n, b_p)$ for $I = 0.1$ and 0.4 are shown the bottom panels of Fig. 1 for $\varepsilon = 1, 2, 3, 4,$ and 5 MeV. The maximum entropies are $s = 0.65, 0.90, 1.12,$ 1.34, and 1.55 (0.69, 0.97, 1.21, 1.46, 1.7) for $\varepsilon = 1, 2, 3, 4,$ and 5 MeV for $I = 0.1$ (0.4), respectively. The maximum in the entropy rapidly shifts to wider surface widths with higher energy and the equilibrium profile also has a substantially more diffuse neutron distribution with increasing neutron excess.

Caloric curves for all asymmetries show the same general character, typified by a well-defined approximate plateau; see Fig. 2. Although this approximate plateau is not as flat as that found in our previous work employing the isospin-independent (b, c) degrees of freedom, the plateau found in this work is reasonably well established by $\varepsilon = 2$ MeV/u. There is a only a very slight decrease in the plateau temperature with increasing asymmetry.

As found in our previous work, the general shape of the caloric curve is controlled by two factors. At low excitation energy the surface peak of the effective mass disappears,

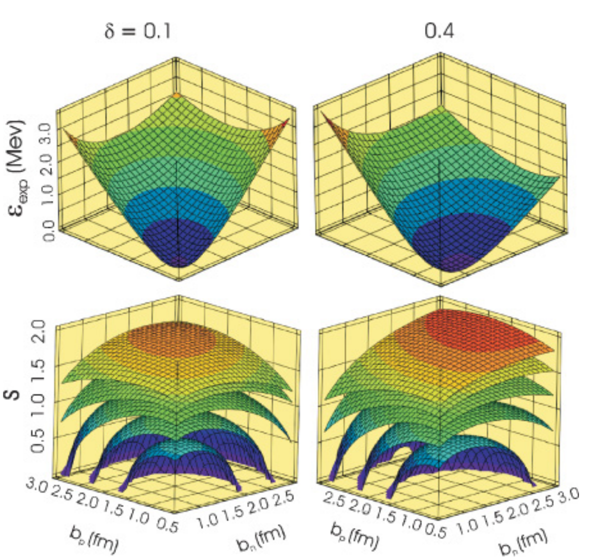

FIG. 1. (Color online) (Top) Expansion energy, $\varepsilon(b_n, b_p)$ for $\delta =$ 0.21154 and 0.4. (Bottom) $s(b_n, b_p)$ maps for $\varepsilon = 1, 2, 3, 4,$ and 5 MeV.

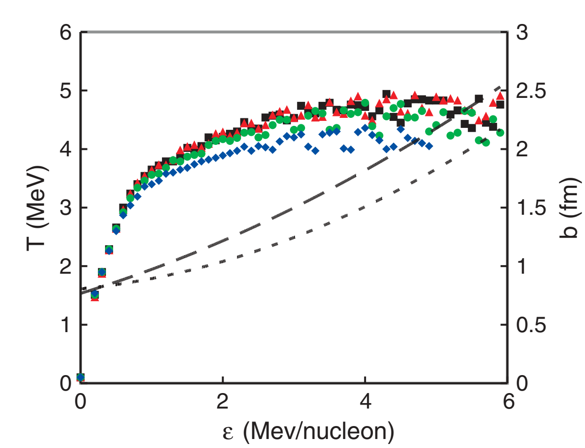

FIG. 2. (Color online) (Top) Caloric curves for $\delta = 0.1$ (triangles), 0.21154 (squares), 0.3 (circles), and 0.4 (diamonds). The equilibrium neutron (dashed) and proton (dotted) surface widths are also shown for $\delta = 0.21154$.

causing a general slowing down of the rate of increase of the density of states. With increasing excitation, expansion, in the present case modeled by an increase surface width, is the principal effect causing a leveling of the Caloric curve. This point was first made by Toke *et al*. [14]. The evolution of the k mass, in this case with a splitting, induces some positive feedback into the expansion process, thus causing additional leveling of the caloric curve. The contribution of this work is showing that the asymmetry has only a minor effect on the caloric curve. In retrospect this is not a surprising result as the level-density parameter is proportional to the effective masses and the asymmetry dependence of the k masses for neutrons and protons are roughly linear with opposite slopes.

The weak sensitivity of the plateau temperature found in this work is in stark contrast to that found for the "limiting temperature" for two-phase systems [3]. The two-phase limiting temperature T_L increases with increasing asymmetry, from about 6 to 8.5 MeV as δ *increases* from 0.211 to 0.4 (for $A = 208$, see Fig. 1 of Ref. [3]). The physical difference between the present work and that of Ref. [3] is the nuclear boundary condition. In the true equilibrium case, a solution to the Gibbs criteria is found (or not) when the nuclear "bulk" and Coulomb pressures of the drop are balanced by the surface tension and the (external) vapor pressure. Our calculation has no external vapor, $p_{\mathrm{surface}} = 0$. The high values T_L for extremely neutron-rich nuclei (even those well beyond the drip line) is a consequence of the vapor.

The strong dependence of the plateau tempertaure on pressure was studied by Kolomietz *et al*. [15] (also see Ref. [16]). There is very little difference between the caloric curves for a mononucleus (with equilibrated surface and no external pressure) and those found for the two-phase solution (with each phase uniform) at low pressure [16]. The unique boundary condition we employ (which does not allow for true equilibrium but is experimentally plausible) removes the asymmetry dependence. This result is also consistent with the weak asymmetry dependence found with the equilibrium two-phase calculation at low pressure [15].

One can thus conclude that without a vapor phase the equilibrium-limiting temperatures (which one should expect to be relevant for boxed matter) cannot be reached and that the relevant (lower) limit is that of the metastable objects modeled here. That is, with increasing excitation energy, the temperature is limited by the surface expanding without limit (ultimately leading to fragment production on the surface), a process that occurs at lower temperatures and pressures than one would find with coexisting phases. This suggests the interesting perspective that support for the argument that mixed phases are generated in reactions could be found from an experimental finding of a marked asymmetry dependence of the limiting temperature.

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