Structure function method applied to polarized and unpolarized electron-proton scattering: A solution of the $G_E(p)/G_M(p)$ discrepancy

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The cross section for polarized and unpolarized electron-proton scattering is calculated by taking into account radiative corrections in leading and next-to-leading logarithmic approximation. The expression of the cross section is formally similar to the cross section of the Drell-Yan process, where the structure functions of the electron play the role of Drell-Yan probability distributions. The interference of the Born amplitude with the two-photon exchange amplitude (box-type diagrams) is expressed as a contribution to the K factor. It is calculated under the assumption that proton form factors decrease rapidly with the momentum transfer squared and that the momentum is equally shared between the two photons. The calculation of the box amplitude is done when the intermediate state is the proton or the Δ resonance. The results of numerical estimations show that the present calculation of radiative corrections can bring into agreement the conflicting experimental results on proton electromagnetic form factors and that the two-photon contribution is very small.

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I. INTRODUCTION

Radiative corrections (RC) to elastic and inelastic electron-proton (*ep*) scattering cross sections can be classified in two types, according to the reaction mechanism assumed: one where a virtual photon is exchanged between electron and proton and a second one taking into account the two-virtual-photon exchange amplitude, arising from box-type Feynman diagrams in the lowest order of perturbation theory (PT). Both kinds of contributions to RC were considered in the literature, in detail, at the lowest order of PT for polarized and unpolarized cases.

The most elaborated consideration at the lowest order of PT was done in Ref. [1], where the approaches of previous papers (cf. the reference list in [1]) were considerably improved. The role of higher orders of PT was first considered for the unpolarized case in the limit of hard-photon emission in Ref. [2] and later for polarized case in Refs. [3] and [4].

The elastic *ep* cross section decreases very rapidly with the momentum transfer squared, $Q^2 = -q^2$, proportionally to Q^{-4}. The size of RC essentially depends on how the experiment was performed. For example, in experiments where only the angle of the scattered electron is measured, the initial electron emission can induce an enhancement of RC owing to the decrease of Q^2.

Initial and final lepton emission can be taken into account by writing the cross section in its Drell-Yan process form, where the structure functions (SFs) of the electron play the role of probability distributions [2]. The set of SFs obey the renormalization group equations (Lipatov's equations). Their solutions are well known [5]. The formalism of SFs allows one to obtain the cross section in the so-called leading logarithmic approximation (LLA), that is, taking correctly into account the terms of the order $[(\alpha/\pi)\ln(Q^2/m_e^2)]^n$. This approximation corresponds to collinear kinematics, where the photon is emitted in a direction close to the direction of the electron. By knowing the value of RC in the lowest order of PT, the nonleading contribution $(\alpha/\pi)[(\alpha/\pi)\ln(Q^2/m_e^2)]^n$ can be calculated.

A different source of cross-section enhancement is related to the so-called Weizsacker-Williams kinematics, where photons are emitted in noncollinear kinematics and provide almost zero momentum, $Q^2 < m_e^2$. This is not discussed in the present work.

A possible enhancement of the elastic cross section can be associated with the box-type Feynman diagram, when the momentum squared is equally shared between the two photons. The relative contribution of two-photon exchange, from simple counting in α, would be of the order of the fine structure constant, $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$: Any contribution of the two-photon exchange (through its interference with the one-photon mechanism) would not exceed 1%. But long ago it was observed [6] that the simple rule of α counting for the estimation of the relative role of the two-photon contribution to the amplitude of elastic electron hadron scattering does not hold at large momentum transfer. Using a Glauber approach for the calculation of multiple scattering contributions [7], it appeared that the relative role of two-photon exchange can increase significantly in the region of high momentum transfer, owing to the rapid decrease of proton form factors (FFs) in case of proton intermediate state. The relevant quantity is the square of the FFs, calculated at $Q^2/4$, and it can at least partially compensate the factor of α. A similar effect takes place when the $\Delta(33)$ resonance is present in the intermediate state of the box diagram, because the transition in the vertex $\gamma^* p\Delta$ also shows a rapid decrease with Q^2 [8].

By taking a simple model for nucleon FFs, based on the dipole parametrization

$$G_E(Q^2) = \frac{G_M(Q^2)}{\mu} = G_D(Q^2) = \frac{M_0^4}{\left(Q^2 + M_0^2\right)^2}, \quad M_0^2 = 0.71\,\mathrm{GeV}^2, \quad \mu = 2.79 \tag{1}$$

when both exchanged photons have momenta close to $q/2$, an enhancement factor appears in the loop calculation: the ratio of FFs in Born and box-type amplitudes. This specific kinematics differs from the "one soft photon" approach used in the past [1], when the box diagram is considered.

Recently, there has been a revival of interest in the two-photon contribution as a possible explanation of the discrepancy among experimental data on elastic *ed* scattering [9]. Electromagnetic proton FFs show a different behavior as a function of Q^2, when measured with two different methods: the polarization transfer method [10], which allows a precise measurement of the ratio of the electric to magnetic proton FFs [11], and the Rosenbluth separation from the unpolarized elastic *ep* cross section [12].

In Ref. [13] it was noted that the reason for the discrepancy lies in the slope of the reduced cross section as a function of ϵ, the virtual photon polarization. At the kinematics of the present experiments, RC on the cross section can reach up to 40% and affect very strongly this slope, changing even its sign, when $Q^2 \geqslant 2$ GeV^2.

In Ref. [14] it was claimed that the presence of the two-photon contribution can bring into agreement the data on the proton electromagnetic FFs from polarization transfer and the Rosenbluth methods. However, the kinematical properties related to fast decreasing FFs were not investigated in detail, nor were the possible presence of inelastic contributions in the intermediate state. A possible test of the model dependence of the calculation with an exactly solvable QED result is also absent.

On the other hand, Ref. [3] is very detailed. The SF method was applied to transferred polarization experiments. The size of this effect was an order of magnitude too small to bring the polarization data into agreement with the unpolarized ones. Therefore the conclusion of that paper was that one could not solve the discrepancy among the existing data.

The SF method was also applied to polarization observables in Ref. [4], where it was shown that the corrections can become very large, if one takes into account the initial-state photon emission. However, the corresponding kinematical region is usually rejected in the experimental analysis, by appropriate selection on the scattered electron energy.

The motivation of the present paper is to stress the need for present experiments to go beyond the lowest order of PT, using LLA and beyond. Radiation corrections traditionally applied are proportional to $\ln(\Delta E)/E \ln(-q^2/m^2)$, where E is the laboratory beam energy, q^2 is the momentum transfer squared, and ΔE is the maximum energy of the undetected photon. In recent experiments E is large and the experimental resolution is very good (allowing ΔE to be reduced). Therefore this RC term becomes sizable and one cannot safely neglect higher order corrections. A complete calculation of RC should take into account consistently all different terms that contribute at all orders (including the two-photon exchange contribution) and their interference. We derive an expression of the radiative-corrected cross section for *ep* elastic scattering, in both polarized and nonpolarized cases, which is easy to handle for experimentalists and which has a sufficient accuracy.

We will show first that the $G_E(p)/G_M(p)$ problem can be solved by taking into account initial-state emission, in the SF approach, and, second, that the two-photon exchange mechanism is irrelevant for the solution of this problem.

Our paper is organized as follows. In Sec. II we give the Drell-Yan formulas for cross sections in polarized and unpolarized cases. Section III is devoted to the calculation of the contribution of two-photon exchange, for the unpolarized cross section, and of the degree of transversal and longitudinal polarization of the recoil proton. Numerical results are presented and discussed in Sec. IV. Section V summarizes the main points of this work. Two appendices contain details of the calculation.

II. DRELL-YAN EXPRESSION OF THE *ep* CROSS SECTIONS IN UNPOLARIZED AND POLARIZED CASES

It is known [2] that the process of emission of hard photons by initial and scattered electrons plays a crucial role, which results in the presence of the radiative tail in the distribution on the scattered electron energy. The SF approach extends the traditional one [15], taking precisely into account the contributions of higher orders of PT and the role of initial-state photon emission. The cross section is expressed in terms of SF of the initial electron and of the fragmentation function of the scattered electron energy fraction. The dependence of the differential cross section on the angle and the energy fraction of the scattered electron $y = 1/\rho$ [where the recoil factor $\rho = 1 + (E/M)(1 - \cos\theta)$] can be written as

$$\frac{d\sigma}{d\Omega dy} = \int_{z_0}^{1} \frac{dz\rho_z}{z} D(z) D\left(\frac{y\rho_z}{z}\right) \frac{\Phi_0(z)}{|1 - \Pi\left(Q_z^2\right)|^2} \left(1 + \frac{\alpha}{\pi} K\right). \tag{2}$$

The term $K = K_e + K_p + K_{\mathrm{box}}$ is the sum of three contributions. K_e is related to nonleading contributions arising from the pure electron block and can be written as [2,5]

$$K_e = -\frac{\pi^2}{6} - \frac{1}{2} - \frac{1}{2}\ln^2\rho + Li_2(\cos^2\theta/2), \quad Li_2(z) = -\int_0^z \frac{dx}{x}\ln(1-x). \tag{3}$$

A second term, K_p, concerns proton emission. The emission of virtual and soft photons by the proton is not associated with a large logarithm L; therefore the whole proton contribution can be included as a K_p factor:

$$K_p = \frac{1}{\beta}\left\{-\frac{1}{2}\ln^2 x - \ln x \ln[4(1+\tau)] + \ln x - (\ln x - \beta)\ln\left[\frac{M^2}{4E^2(1-c)^2}\right] + \beta - Li_2\left(1-\frac{1}{x^2}\right) + 2Li_2\left(-\frac{1}{x}\right) + \frac{\pi^2}{6}\right\}, \quad (4)$$

where $x = (\sqrt{1+\tau}+\sqrt{\tau})^2$, $\beta = \sqrt{1-M^2/E'^2}$ is the velocity and $E' = E(1-1/\rho) + M$ is the energy of the scattered proton. The contribution of K_p to the K factor is of the order of $-2/1000$ for $c = 0.99$, $E = 21.5$ GeV, and $Q^2 = 31.3$ GeV^2 [1].

Lastly, $K_{\rm box}$ represents the interference of electron and proton emission. More precisely, both the interference between the two-virtual-photon exchange amplitude and the Born amplitude and the relevant part of the soft-photon emission (i.e., the interference between the electron and proton soft-photon emission) may be included in the term $K_{\rm box}$. This effect is not enhanced by the large logarithm (characteristic of SF) and can be considered among the nonleading contributions. It is an ϵ-independent quantity of the order of unity, which includes all the nonleading terms, such as two-photon exchange and soft-photon emission.

The nonsinglet SF is

$$D(z,\beta) = \frac{\beta}{2}\left[\left(1+\frac{3}{8}\beta\right)(1-z)^{\frac{\beta}{2}-1} - \frac{1}{2}(1+z)\right](1+O(\beta)), \quad (5)$$

$$\beta = \frac{2\alpha}{\pi}(L-1), \quad Q^2 = \frac{2E^2(1-\cos\theta)}{\rho}, \quad L = \ln\frac{Q^2}{m_e^2}, \quad (6)$$

where m_e is the electron mass. The method for using SF in the numerical calculation is described in Appendix A. The lower limit of integration, z_0, is related to the "inelasticity" cut c, necessary to select the elastic data:

$$z_0 = \frac{c}{\rho - c(\rho-1)}. \quad (7)$$

The Born cross section for the scattered electron, Φ_0, is

$$\Phi_0(Q^2,\epsilon) = \frac{\sigma_M}{\epsilon\rho(1+\tau)}\sigma_{\rm red}(Q^2,\epsilon), \quad \sigma_{\rm red}(Q^2,\epsilon) = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2), \quad (8)$$

where $\sigma_M = \alpha^2\cos^2(\theta/2)/[4E^2\sin^4(\theta/2)]$ is the Mott's cross section and

$$\tau = \frac{Q^2}{4M^2}, \quad \frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2(\theta/2). \quad (9)$$

The vacuum polarization for a virtual photon with momentum q, $q^2 = -Q^2 < 0$, is included as a factor $1/[1-\Pi(Q^2)]$. The main contribution to this term arises from the polarization of the electron-positron vacuum:

$$\Pi(Q^2) = \frac{\alpha}{3\pi}\left[L - \frac{5}{3}\right]. \quad (10)$$

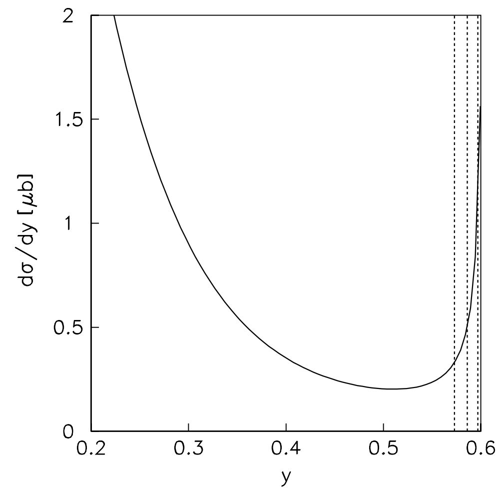

FIG. 1. The y dependence of the elastic differential cross section at $\theta = 32.4^\circ$ and $Q^2 = 3$ GeV^2.

The z-dependent kinematically corrected quantities (corrected for the shift in momentum owing to the photon emission) are obtained from the corresponding ones by replacing the initial electron energy E by zE.

The y dependence, at fixed momentum transfer and electron scattering angle, shows a steep rise, at small y owing to initial-state emission, and a rise in the vicinity of the elastic value, $y = 1/\rho$. As an example, such a dependence is shown in Fig. 1, for $\theta = 32.4^\circ$ and $Q^2 = 3$ GeV^2. The dashed lines show the kinematical cuts corresponding to $c = 0.95, 0.97$, and 0.99, from left to right.

In an experiment, the selection of elastic events requires a cut in the energy spectrum of the scattered electron, and one integrates over the events where the energy of the final electron, E_1', exceeds a threshold value $E_1' > Ey = Ec/\rho$, $\rho = 1 + (E/M)(1-\cos\theta)$, $c < 1$ (where E is the initial electron energy). The properties of the SF method allow the radiative corrections to be written in the form of initial- and final-state emission, although gauge invariance is conserved. This form obeys the Lee-Nauenberg-Kinoshita theorem, about the cancellation of mass singularities, when integrating on the final energy fraction. This results in omitting the final (fragmentation) SF, that is, in replacing the term $(\rho_z/z)D(y\rho_z/z)$ (associated with the final electron emission) by unity.

In Ref. [4] the expressions of the transversal and longitudinal components of the recoil proton polarization were derived in the framework of the Drell-Yan approach.

The relevant observables, $\Phi^{\rm SF}_{0,L,T}$, that is, the unpolarized and polarized (longitudinally and transversally) cross sections calculated in frame of the SF method, can be written as

$$\Phi^{\rm SF}_{0,L,T} = \int_{z_0}^1 dz D(z)\frac{\Phi_{0,L,T}(z)}{\left|1-\Pi\left(Q_z^2\right)\right|^2}\left[1 + \frac{\alpha}{\pi}K_{0,L,T}\right], \quad (11)$$

where the factors $K_{0,T,L}$ contain the contribution of the two-photon exchange diagrams, and they are estimated in the dipole approximation for FFs in the next section [5].

It is convenient to write the polarized and unpolarized cross section, in the framework of the SF method, in the form of a deviation from the expected Born expressions (where we omit RC of higher order):

$$\Phi^{\rm SF}_{0,L,T} = \Phi^{\rm Born}_{0,L,T}\left[1 + \Delta^{\rm SF}_{0,L,T} + \frac{\alpha}{\pi}K_{0,L,T}\right], \quad (12)$$

with

$$\Delta^{\rm SF}_{0,L,T} = \frac{\alpha}{\pi}\left\{\frac{2}{3}\left(L-\frac{5}{3}\right) - \frac{1}{2}(L-1)\left[2\ln\left(\frac{1}{1-z_0}\right) - z_0 - \frac{z_0^2}{2}\right] + \frac{1}{2}\frac{\rho(1+\tau)}{\Phi^{\rm Born}_{0,L,T}}(L-1)\int_{z_0}^{1}\frac{(1+z^2)dz}{1-z} \times \left[\frac{\Phi^{\rm Born}_{0,L,T}(z)}{\left|1-\Pi\left(Q_z^2\right)\right|^2} - \frac{\Phi^{\rm Born}_{0,L,T}(1)}{|1-\Pi(Q^2)|^2}\right]\right\}, \quad (13)$$

where $\Phi^{\rm Born}_{0,L,T}(1) = \Phi^{\rm Born}_{0,L,T}$ and $\Phi^{\rm Born}_{0} = \Phi_0(Q^2,\epsilon)$ from Eq. (8), and

$$\Phi^{\rm Born}_{T}(Q^2,\epsilon) = -2\lambda\left(\frac{1}{\rho}\right)^2\frac{\alpha^2}{Q^2}\sqrt{\frac{\tau}{\tan^2(\theta/2)(1+\tau)}} \times G_E(Q^2)G_M(Q^2), \quad (14)$$

$$\Phi^{\rm Born}_{L}(Q^2,\epsilon) = \lambda\frac{\alpha^2}{2M^2}\left(\frac{1}{\rho}\right)^2\sqrt{1+\frac{1}{\tan^2(\theta/2)(1+\tau)}} \times G_M^2(Q^2), \quad (15)$$

where $\lambda = \pm 1$ is the chirality of the initial electron.

III. CALCULATION OF THE K-FACTOR CONTRIBUTIONS FROM THE TWO-PHOTON EXCHANGE

A. Proton intermediate state

The box-type Feynman diagram is illustrated in Fig. 2, where the momenta of the particles are shown in brackets. Each of the photon carries approximatively half of the transferred momentum q. This assumption is justified on the bases of arguments developed in Ref. [6] and recalled in the Introduction. Let us stress that such an approximation leads to an overestimation of the two-photon contribution.

We parametrize the loop momentum of the box-type Feynman amplitude in such a way that the denominators of the Green function are $(\pm\kappa + q/2)^2$ for the photon, whereas for the electron (e) and the for the proton (p) they have a form $(e) = (\pm\kappa + \mathcal{P})^2 - m_e^2$, $(p) = (\kappa + \mathcal{Q})^2 - M^2$, with $\mathcal{P} = \frac{1}{2}(p_1 + p_1')$, $\mathcal{Q} = \frac{1}{2}(p + p')$. The $-$ sign for the electron corresponds to the Feynman diagram for the two-photon box

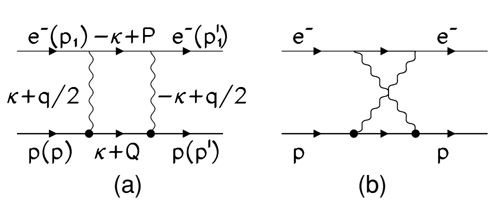

FIG. 2. Feynman diagrams for two-photon exchange in elastic ep scattering: (a) box diagram and (b) crossed box diagram.

[Fig. 2(a)] and the $+$ sign corresponds to the crossed box diagram [Fig. 2(b)].

The assumption of a rapid decreasing of FFs implies that we can neglect the dependence on the loop momentum κ in the denominators of the photon Green function as well as in the arguments of the FFs. This results in ultraviolet divergences of the loop momentum integrals. Therefore they should be understood as convergent integrals with the cutoff restriction $|\kappa^2| < M^2\tau$:

$$\int\frac{d^4\kappa}{i\pi^2}\frac{N_\pm(\mathcal{P},\mathcal{Q})}{\left((\pm\kappa+\mathcal{P})^2 - m_e^2\right)((\kappa+\mathcal{Q})^2 - M^2)} \times \theta(M^2\tau - |\kappa^2|) = I_\pm \cdot N_\pm(\mathcal{P},\mathcal{Q}), \quad (16)$$

where $\mathcal{P} = \frac{1}{2}(p_1 + p_1')$, $\mathcal{Q} = \frac{1}{2}(p + p')$. The explicit form of $I_\pm$ is given in Appendix B. $N_\pm(\mathcal{P},\mathcal{Q})$ is the Feynman diagram numerator defined in Eq. (20). The virtual photons Green function is written as

$$\frac{1}{\left|\frac{q}{2} \pm \kappa\right|^2} < \frac{1}{Q^2} = \frac{1}{M^2(1+\tau)}. \quad (17)$$

Then the expressions for K factors can be written as

$$K_i = -2\mathcal{N}(z)\frac{U^i(\mathcal{P},\mathcal{Q})}{\Phi_i}, \qquad i = 0, L, T, \quad (18)$$

$$\mathcal{N}(z) = (z+1)^2/[(z/4)+1]^4, \quad z = Q^2/M_0^2, \quad (19)$$

where $\mathcal{N}(z)$ is due to the dipole dependence of the FFs, which is extracted as an enhancement factor (see Fig. 3). One can see that this factor has a maximum equal to $\simeq 2$ for $z \simeq 2$, which corresponds to $Q^2 \simeq 1.4$ GeV^2. This behavior is consistent with the results of a rigorous QED calculation [16]. This calculation, which applies to $e\mu$ scattering, gives an upper limit of the two-photon contribution to ep scattering, when the muon mass is replaced by the proton mass.

The Born terms, Φ_i, $i = 0, L, T$, have been singled out in the definition of the K factor [see Eqs. (13)–(15)].

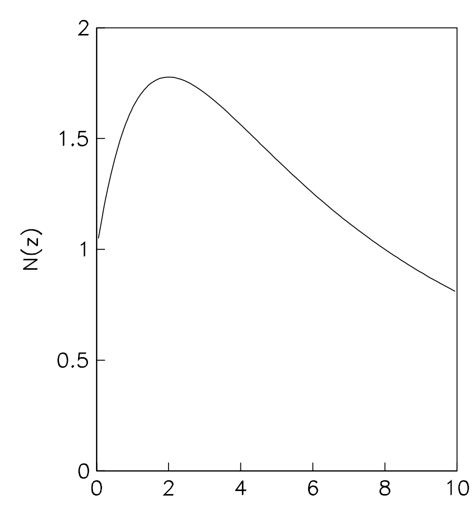

FIG. 3. z behavior of the enhancement factor $N(z)$.

In the unpolarized case the expression for $U_0(\mathcal{P},\mathcal{Q})$ is

$$\begin{aligned}U_0(\mathcal{P},\mathcal{Q}) &= \frac{\alpha^2 G_D^2(Q^2)}{16M^8\rho^2\tau(\tau+1)^2}\frac{1}{4}\,\mathrm{Tr}\left[(\hat{p}'+M)\Gamma_\lambda\left(\frac{q}{2}\right)\right.\\ &\quad\times(\hat{\mathcal{Q}}+M)\Gamma_\eta\left(\frac{q}{2}\right)(\hat{p}+M)\left.\bar{\Gamma}_\mu(q)\right]\\ &\quad\times\left\{I_+\cdot\frac{1}{4}\mathrm{Tr}[\hat{p}_1'\gamma_\lambda\hat{\mathcal{P}}\gamma_\eta\hat{p}_1\gamma_\mu]\right.\\ &\quad+\left.I_-\cdot\frac{1}{4}\mathrm{Tr}[\hat{p}_1'\gamma_\eta\hat{\mathcal{P}}\gamma_\lambda\hat{p}_1\gamma_\mu]\right\}\\ &= N_+I_+ + N_-I_-, \end{aligned} \tag{20}$$

where

$$\Gamma_\alpha(q) = \frac{F_1(Q^2)}{G_D(Q^2)}\gamma_\alpha - \frac{1}{4M}\frac{F_2(Q^2)}{G_D(Q^2)}[\gamma_\alpha,\hat{q}],$$
$$\bar{\Gamma}_\alpha(q) = \frac{F_1(Q^2)}{G_D(Q^2)}\gamma_\alpha + \frac{1}{4M}\frac{F_2(Q^2)}{G_D(Q^2)}[\gamma_\alpha,\hat{q}],$$

where $F_1(Q^2)$ and $F_2(Q^2)$ are the Pauli and Dirac FFs, related to the Sachs FFs by

$$F_1(Q^2) = \frac{G_E(Q^2)+\tau G_M(Q^2)}{1+\tau},$$
$$F_2(Q^2) = \frac{G_M(Q^2)-G_E(Q^2)}{1+\tau}.$$

The quantities $U_{T,L}(\mathcal{P},\mathcal{Q})$ for the polarized case can be obtained from Eq. (20) by the following replacements:

$$\gamma_\mu \to \gamma_\mu\gamma_5 \tag{21}$$

in the lepton traces and

$$(\hat{p}'+M) \to (\hat{p}'+M)\hat{a}_{T,L}\gamma_5 \tag{22}$$

in the proton traces. Here $a_{T,L}$ is the final proton polarization vector [i.e., $(a_{T,L}p')=0$] and corresponds to different orientations of the proton polarization. If the final proton is polarized along the x-axis, one finds

$$(a_T p) = 0, \quad (a_T p_1) = -\frac{E^2}{2M\rho}\frac{\sin\theta}{\sqrt{\tau(1+\tau)}}, \tag{23}$$

whereas in case of polarization along the z-axis,

$$\begin{aligned}(a_L p) &= 2M\sqrt{\tau(1+\tau)},\\ (a_L p_1) &= M\sqrt{\frac{\tau}{1+\tau}}\left(\frac{E}{M}-1-2\tau\right).\end{aligned} \tag{24}$$

B. The Δ resonance contribution

Let us write the structure of the vertex for the transition $\Delta(p)\to\gamma^*(q)+P(p')$, following the formalism of Refs. [17, 18] (and references therein):

$$\begin{aligned}M(\Delta\to\gamma^*P) &= eg_{\Delta N}\sqrt{3/2}\,\bar{u}(p',\eta)\left(\gamma_\mu-\frac{1}{M_\Delta}p'_\mu\right)\\ &\quad\times u_\nu(p,\lambda)\frac{F_{\mu\nu}(q)}{2\sqrt{Q^2}}G_D(Q^2),\end{aligned} \tag{25}$$

where $F_{\mu\nu}(q)=e_\mu(q)q_\nu-e_\nu(q)q_\mu$ is the Maxwell tensor, $e(q)$ is the polarization vector of virtual photon, η and λ are the chiral states of the nucleon and of the Δ resonance, and $\sqrt{3/2}g_{\Delta p}\approx 1.56\,\mu$ (with μ the anomalous magnetic moment of the proton). Moreover, the factor $G_D(Q^2)$ is explicitly extracted.

The Green function of the Δ resonance, neglecting its width, is

$$-i\frac{D_{\mu\nu}(p)}{p^2-M_\Delta^2+i0}, \tag{26}$$

with

$$\begin{aligned}D_{\mu\nu}(p) &= \sum_\lambda u_\mu(p,\lambda)\bar{u}_\nu(p,\lambda)\\ &= (\hat{p}+M_\Delta)\left[g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu\right.\\ &\quad\left.-\frac{1}{3p^2}(\hat{p}\gamma_\mu p_\nu+p_\mu\gamma_\nu\hat{p})\right],\\ D_{\mu\nu}(p)p_\mu &= D_{\mu\nu}(p)p_\nu = 0.\end{aligned} \tag{27}$$

The transition vertexes associated with FFs are of the same form as the dipole ones for the nucleons. The part of the virtual Compton scattering of the proton amplitude that enters in the box amplitude is

$$\bar{u}(p')[p']_\mu D_{\rho\sigma}(p_2)[p]_\nu u(p)F_{\mu\rho}(k_1)F^*_{\sigma\nu}(k_2),$$
$$k_{1,2}=\pm\kappa+\frac{q}{2},\quad p_2=\kappa+\mathcal{Q},\quad [p]_\mu=\gamma_\mu-\frac{1}{M_\Delta}p_\mu,$$

where $F_{\mu\nu}(k)=k_\mu e_\nu(k)-k_\nu e_\mu(k)$ is the Maxwell tensor and $e_\mu(k)$ is the photon polarization vector. Thus, in unpolarized case, the contribution of the Δ resonance to the K factor is

$$K_0^\Delta = -2\mathcal{N}(z)\frac{U_0^\Delta}{\Phi_0}, \tag{28}$$

with

$$\begin{aligned}U_0^\Delta &= -\frac{\alpha^2G_D^2(Q^2)(1.56\,\mu)^2}{64M^{10}\rho^2\tau^2(1+\tau)^2}\frac{1}{4}\mathrm{Tr}[(\hat{p}'+M)\\ &\quad\times[p']_\mu D_{\rho\sigma}(\mathcal{Q})[p]_\nu(\hat{p}+M)\bar{\Gamma}_\eta(q)]\\ &\quad\times\left\{I_+\cdot\frac{1}{4}\mathrm{Tr}[\hat{p}_1'P^{\mu\nu\rho\sigma}\hat{p}_1\gamma^\eta]\right.\\ &\quad\left.+I_-\cdot\frac{1}{4}\mathrm{Tr}[\hat{p}_1'R^{\mu\nu\rho\sigma}\hat{p}_1\gamma^\eta]\right\},\end{aligned} \tag{29}$$

where

$$P_{\mu\nu\rho\sigma} = \tfrac{1}{4}[\gamma_\rho q_\nu-\gamma_\nu q_\rho]\hat{\mathcal{P}}[\gamma_\sigma q_\mu-\gamma_\mu q_\sigma],$$
$$R_{\mu\nu\rho\sigma} = \tfrac{1}{4}[\gamma_\sigma q_\mu-\gamma_\mu q_\sigma]\hat{\mathcal{P}}[\gamma_\rho q_\nu-\gamma_\nu q_\rho].$$

The contributions in the polarized cases can be obtained from Eq. (29) via the same replacement rules [Eqs. (21) and (22)] and the corresponding denominators, $\Phi_{L,T}$.

IV. RESULTS AND DISCUSSION

The numerical results strongly depend on the experimental conditions, in particular on the inelasticity cut of the scattered

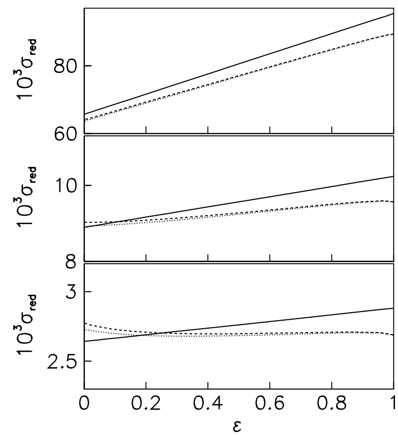

FIG. 4. The ϵ dependence of the elastic differential cross section, for $Q^2 = 1$, 3, and 5 GeV^2, from top to bottom: Born cross section (solid line), Drell-Yan cross section (dashed line), full calculation (dash-dotted line) (including Born, SF, and K-factor contributions).

electron energy spectrum. The results shown here correspond to $c = 0.97$. This value has been chosen because it corresponds to the energy resolution of modern experiments. The unpolarized cross section has been calculated by assuming the dependence of FFs on Q^2 given by Eq. (2). In Fig. 4 the results are shown as a function of ϵ, for $Q^2 = 1$, 3, and 5 GeV^2, from top to bottom. The calculation based on the structure function method is shown as dashed lines. The full calculation, including the two-photon exchange contribution, is shown as dash-dotted lines. For comparison the results corresponding to the Born reduced cross section are shown as solid lines.

One can see that the main effect of the present calculation is to modify and lower the slope of the reduced cross section. This effect gets larger with Q^2. Nonlinear effects are small and induced by the y integration. Including the two-photon exchange has little effect on the results in the kinematical range presented here.

The Q^2 dependence of the unpolarized cross section is shown in Fig. 5, for electron scattering angles equal to $\theta = 85^\circ, 60^\circ$, and 20°, from top to bottom. The $G_D^2(Q^2)$ dependence has been removed, to visually enhance the differences among the calculations. In spite of this, the corrections have very little effect on the Q^2 dependence of the reduced cross section.

The results for the polarized case are shown in Fig. 6 for the longitudinal (dashed lines) and the transversal (solid lines) components of the cross section. The ratio of the polarized cross section, corrected with the SF method [Eqs. (12) and (13)] to the Born polarized cross section [Eqs. (15) and (14)], is reported as a function of ϵ.

The relative effect of the corrections is of the order of the corrections to the unpolarized cross section and is very similar for the two polarized components. This means that the radiative corrections have very little effect on the ratio of the polarizations.

Again the effect of the two-photon contribution is negligible in both cases. In the specific kinematics considered here, the box-type contribution K_{box} does not depend on the soft-photon emission parameter, $\Delta E/E$. This quantity is of the order of

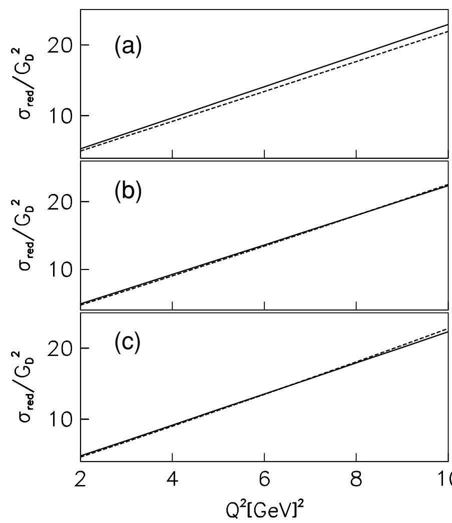

FIG. 5. The Q^2 dependence of the elastic differential cross section at $\theta = 85^\circ, 60^\circ$, and 20°. Notation as in Fig. 4.

the ΔE-independent contribution to the charge asymmetry $\Xi(E/M, \cos\theta)$ [corrected by the factor $N(Q^2/M_0^2)$] calculated for $e\mu$ elastic scattering in the frame of pure QED (see Fig. 2 of Ref. [16]). In the dipole approximation, the FFs reduce to $F_1(Q^2) \to G_D(Q^2)$ and $F_2(Q^2) \to 0$ when Q^2 is large.

It is particularly interesting to look at the ratio of the transverse to the longitudinal components of the proton polarization experimentally measured, which is directly related to the form factor ratio (Fig. 7, dashed line). The results are presented after normalization to the Born ratio, to compensate the kinematical factors. The correction to be applied to the experimental data, as predicted by the present calculation, is very small, within 1% at different θ values, for Q^2 up to 10 GeV^2. However, the

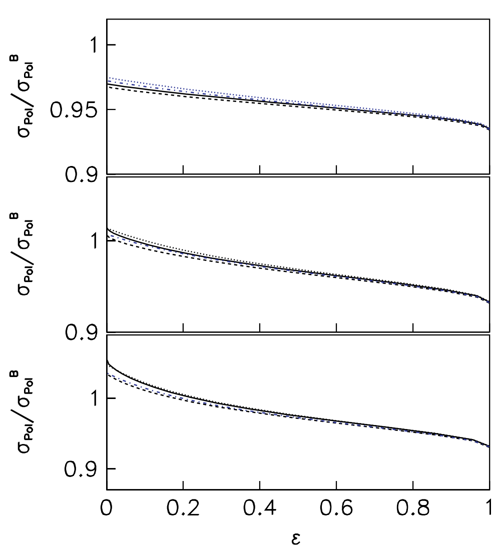

FIG. 6. (Color online) The ϵ dependence of the ratio of the polarized cross section, corrected by the SF method, to the corresponding component of the Born cross section, at $Q^2 = 1, 3$, and 5 GeV^2, from top to bottom. The calculation including (not including) the K factor is shown as the solid (dotted) line for the transversal component and as the dashed (dash-dotted) line for the longitudinal component.

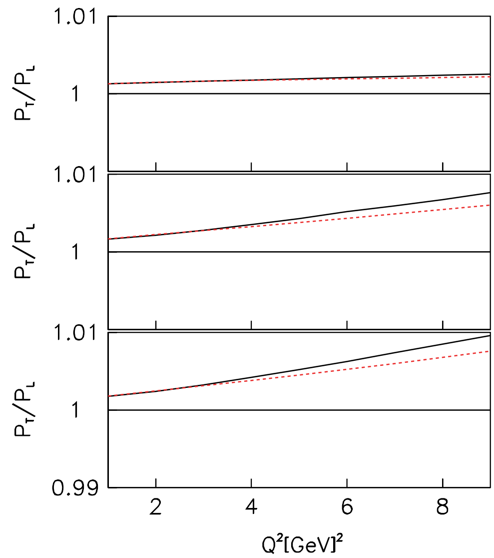

FIG. 7. (Color online) Q^2 dependence of the ratio of the transversal to longitudinal components of the proton polarization calculated in the SF method (dashed line), normalized to the same ratio in the Born approximation at $\theta = 85^\circ$, 60°, and 20°.

two-photon contribution depends on Q^2 and becomes larger as Q^2 increases.

One can tentatively extract the FF ratio, after correcting the measured unpolarized cross section by the ratio of the Born to the SF results. It is evidently an average correction, which is not applied event by event, but it takes into account the main feature of the SF calculation, that is, the lowering of the slope of the unpolarized cross section as a function of ϵ. We neglect the correction from the two-photon exchange, as it is negligible when compared to the high-order corrections. The Q^2 dependence of the ratio $R = \mu G_E/G_M$ is plotted in Fig. 8,

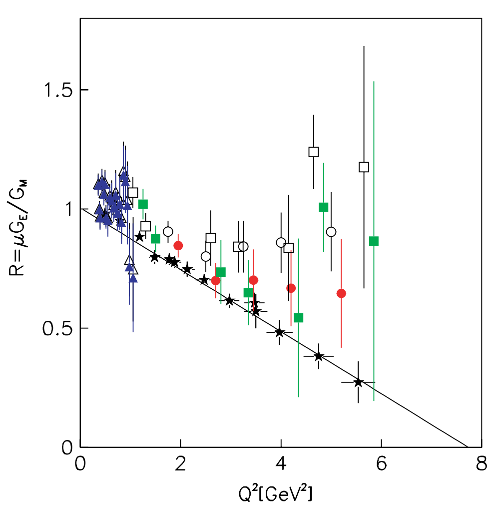

FIG. 8. (Color online) The Q^2 dependence of the FF ratio. Data from the Rosenbluth method before (open symbols) and after (solid symbols) correction ($c = 0.97$), from Ref. [19] (triangles), from Ref. [20] (squares), and from Ref. [21] (circles). Data from the polarization method [11] are also shown (stars). The line is a fit to the polarization data.

for the sets of data for which detailed information on RC has been published [20,21] (squares and circles, respectively). Open symbols refer to the published data; the corresponding solid symbols represent the corrected data. For comparison, a set of data at low Q^2 is also shown (triangles) [19]. Here RC are small, and high-order corrections have little effect on the results. Data from the polarization method are shown as stars. Although the corrections would have the effect of getting a larger ratio, the difference between points before and after correction is at most 1% and cannot be seen on this plot. The line corresponds to the following fit to polarization data (for $Q^2 \geqslant 1$ GeV^2) [11]:

$$R(Q^2) = 1 - (0.130 \pm 0.005)\{Q^2\,[\mathrm{GeV}^2] - (0.04 \pm 0.09)\}. \quad (30)$$

The present results suggest that an appropriate treatment of radiative corrections constitutes the solution of the discrepancy between FFs extracted by the Rosenbluth method or by the recoil polarization method.

V. CONCLUSION

We have considered radiative corrections in the case of quasielastic kinematics, when the scattered electron has energy close to the elastic value. We considered two types of corrections: real photon emission related to the electron vertex, which we calculated in the framework of the structure function approach, in leading and next to leading orders, with the latter expressed in terms of the K factor. We do not include the contribution to the K factor from proton emission, as it is known to be small. The enhancement of RC has been explicitly calculated in QED, owing to emission from the initial lepton.

The loop integral of the box diagram was calculated under the assumptions that the momentum transfer squared is equally shared between the two photons and that the proton FFs decrease rapidly with the momentum transfer squared.

The contribution to the K factor from the interference of electron and proton emission (two-photon exchange) was found to be small, not exceeding 1% for the unpolarized cross section. Its contribution is different for the polarized cross section and has a very small effect on the ratio of the longitudinal to transversal components. The K factor induces a large deviation for small ϵ values. Such behavior is not physical but is due to the approximation used for calculating the box diagram (see Appendix B). The present work is focused on the effect of higher order corrections to the cross section, which are responsible for the largest deviation from the ϵ dependence of the Born cross section. The nonleading contributions are calculated under the assumption that the momentum transfer is almost equally shared between the two exchanged photon. This assumption holds in the limit of a specific kinematics, which emphasizes the role of the two-photon contribution, at large Q^2.

The present results are consistent with a rigorous QED calculation of the two-photon contribution from Ref. [16], where the process $e^+e^- \to \mu^+ + \mu^-$ and the crossed process were calculated and shown to be of the order of 1%. The QED case can be considered as an upper limit for two-photon effects in electron proton scattering for at least two reasons: 1. The proton electromagnetic FFs F_1 and F_2 are smaller than unity, and 2. the contributions of the elastic and inelastic channels, in the intermediate state, should compensate each other. This compensation has already been pointed out in the literature [17]. We underline that the Δ resonance contribution should be considered as a model for all possible intermediate states, including πN and πNN. Using the analytical properties of the Compton scattering amplitude [22], one can expect a large cancellation of the elastic intermediate state (nucleon) and inelastic ones up to a level of 10%.

The quantitative results depend on the underlying approximations; however, these approximations lead to an overestimation of the box contribution, except at small angles. Our conclusions on the relevance of the two-photon exchange contradict a number of works presented in the literature [14]; however, they are consistent with a recent work [23] and with a pure QED calculation [16]. Further work is necessary. A complete evaluation of the box diagram for *ep* elastic scattering, in the framework of an analytical model, is in preparation [24].

The main effect of the present calculation of RC is visible on the unpolarized cross section: It changes noticeably the slope of the ϵ dependence of the reduced cross section, in comparison with the Born approximation. This slope is directly related to the electric FF; therefore applying RC as suggested here to the unpolarized cross section would resolve the discrepancy between FFs extracted from the Rosenbluth method and from the recoil polarization method.

In Ref. [4] it was shown that the corrections on the polarization observables can be very large, if the cut parameter is small (see Fig. 1). This is due to the initial-state photon emission, which is normally excluded in the experimental analysis. In this paper we considered the region near the elastic peak where the contribution to the polarized cross section ratio becomes small (of order 1%; see Fig. 7).

An average SF correction was applied to the data, and this correction significantly improves the consistency of the different sets. However, as pointed out in Ref. [25], this procedure is still applied as a global factor depending on the relevant variables, ϵ and Q^2, and does not solve the problem of the strong correlation between the parameters of the Rosenbluth fit. An event-by-event analysis, based on the SF method, should be done at the data-processing level. This is beyond the purpose of this paper.

In conclusion, the SF method can be successfully applied to calculate RC to elastic *ep* scattering. In particular, it takes precisely into account collinear photon emission. The two-photon contribution is negligible in the considered kinematical range. The correction to the ratio of longitudinal to transverse proton polarization is small. But the correction on the unpolarized cross section has the effect and the size required to resolve the discrepancy among proton FFs.

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APPENDIX A: METHOD FOR THE INTEGRATION OF THE *D* FUNCTION

Let us consider the integral

$$\mathcal{I} = \int_{x_0}^{1} D(x)\phi(x)dx. \tag{A1}$$

The partition function $D(x)$ has a $\delta(x-1)$-type behavior for $x = 1$ and has the following properties:

$$\int_0^1 D(x)dx = 1; \quad D(x)|_{x\neq 1} = \frac{\beta}{4}\frac{1+x^2}{1-x} + \mathcal{O}(\beta^2). \tag{A2}$$

Therefore Eq. (A2) becomes

$$\begin{aligned}\mathcal{I} &= \int_{x_0}^{1-\epsilon} dx\, D(x)\phi(x) + \int_{1-\epsilon}^{1} dx\, D(x)\phi(1)\\ &= \frac{\beta}{4}\int_{x_0}^{1-\epsilon} dx \frac{1+x^2}{1-x}\phi(x)\\ &\quad + \left(1 - \int_0^{1-\epsilon} dx \frac{\beta}{4}\frac{1+x^2}{1-x}\right)\phi(1) + \mathcal{O}(\beta^2).\end{aligned} \tag{A3}$$

After elementary integration, Eq. (A3) becomes

$$\begin{aligned}\mathcal{I} &= \frac{\beta}{4}\int_{x_0}^{1-\epsilon} dx \frac{1+x^2}{1-x}[\phi(x) - \phi(1) + \phi(1)]\\ &\quad + \phi(1)\left[1 - \frac{\beta}{4}\int_0^{1-\epsilon} dx \frac{1+x^2}{1-x}\right]\end{aligned} \tag{A4}$$

$$\begin{aligned}&= \phi(1)\left[1 - \frac{\beta}{4}\left(2\ln\frac{1}{1-x_0} - x_0 - \frac{x_0^2}{2}\right)\right]\\ &\quad + \frac{\beta}{4}\int_{x_0}^{1} dx \frac{1+x^2}{1-x}[\phi(x) - \phi(1)] + \mathcal{O}(\beta^2),\end{aligned} \tag{A5}$$

which removes the singularity.

APPENDIX B: CALCULATION OF $I_\pm$

In this appendix we perform the following integration:

$$\begin{aligned} I_\pm &= \mathrm{Re}\int \frac{d^4\kappa}{i\pi^2}\frac{\theta(M^2\tau - |\kappa^2|)}{(\mathcal{P}_\pm)(\mathcal{Q})},\\ (\mathcal{P}_\pm) &= (\pm\kappa + \mathcal{P})^2 - m_e^2,\\ (\mathcal{Q}) &= (\kappa + Q)^2 - M^2,\end{aligned} \tag{B1}$$

where $\mathcal{P} = \frac{1}{2}(p_1 + p_1')$, $\mathcal{Q} = \frac{1}{2}(p + p')$. Firstly we performed a Wick rotation ($\kappa_0 \to i\kappa_0$) and apply the cut-off provided by θ-function through the parametrization:

$$\begin{aligned}\mathrm{Re}\int \frac{d^4\kappa}{i\pi^2}\frac{\theta(M^2\tau - |\kappa^2|)}{(\mathcal{P}_\pm)(\mathcal{Q})} &= \frac{2}{\pi}\int_{-M\sqrt{\tau}}^{M\sqrt{\tau}} d\kappa_0 \int_0^{M\sqrt{\tau - k_0^2/M^2}} dk\, k^2\\ &\quad\times \int_{-1}^{1} d(\cos\theta_\kappa)\,\mathrm{Re}\frac{1}{(\mathcal{P}_\pm)(\mathcal{Q})},\end{aligned} \tag{B2}$$

where $k = |\vec{\kappa}|$. We also performed the integration over the azimuthal angle ϕ_κ. Now let us consider the integral in the Breit system where $q_0 = 0$ and $\vec{p}_1 = -\vec{p}_1'$. Thus $\vec{\mathcal{P}} = 0$, $p_0 = p_0' = E'$, $|\vec{p}_1| = M\sqrt{\tau}$, $\vec{\mathcal{Q}}^2 = M^2 \cot^2(\theta_e/2)$, and $E' = M\sqrt{\tau + 1/\sin^2(\theta_e/2)}$, where θ_e is the electron scattering angle in the laboratory frame.

Before integrating over angle θ_κ let us write the explicit expression for real part of the integrand:

$$\mathrm{Re}\frac{1}{(\mathcal{P}_\pm)(\mathcal{Q})} = \frac{a(a + b\cos\theta_\kappa) \mp \delta_1\delta_2}{\left(a^2 + \delta_1^2\right)\left((a + b\cos\theta_\kappa)^2 + \delta_2^2\right)},$$

where $a = -\kappa_0^2 - k^2 + M^2\tau$, $b = -2k|\vec{\mathcal{Q}}|$, $\delta_1 = 2\kappa_0 M\sqrt{\tau}$, and $\delta_2 = 2\kappa_0 E'$. The integration over θ_κ is straightforward and results in

$$I_\pm = -\frac{1}{\pi|\vec{\mathcal{Q}}|}\int_{-M\sqrt{\tau}}^{M\sqrt{\tau}} d\kappa_0 \int_0^{M\sqrt{\tau - k_0^2/M^2}} dk\, k \frac{1}{a^2 + \delta_1^2} \times \left\{\frac{a}{2}\ln\left(\frac{(a+b)^2 + \delta_2^2}{(a-b)^2 + \delta_2^2}\right) \mp \delta_1 \arctan\left(\frac{2b\delta_2}{a^2 - b^2 + \delta_2^2}\right)\right\}. \quad \text{(B3)}$$

The limit of theses integrals for small values of $|\vec{\mathcal{Q}}|$ is

$$I_\pm|_{|\vec{\mathcal{Q}}|\to 0} = \frac{4}{\pi}\int_{-M\sqrt{\tau}}^{M\sqrt{\tau}} d\kappa_0 \int_0^{M\sqrt{\tau - k_0^2/M^2}} \frac{dk\, k^2(a^2 \mp \delta_1\delta_2)}{\left(a^2 + \delta_2^2\right)\left(a^2 + \delta_1^2\right)}.$$

One can see that, in the limiting case $\epsilon \to 0$ and $\theta \to \pi$, $I_\pm$ is finite, whereas for $\epsilon \to 1$ and $\theta \to 0$, $I_\pm \to 0$. This is responsible for the behavior of the two-photon exchange contribution as a function of ϵ.

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