Tensor electric polarizability of the deuteron in storage-ring experiments

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(Received 10 May 2006; revised manuscript received 30 November 2006; published 12 January 2007)

The tensor electric polarizability of the deuteron gives important information about spin-dependent nuclear forces. If a resonant horizontal electric field acts on a deuteron beam circulating into a storage ring, the tensor electric polarizability stimulates the buildup of the vertical polarization of the deuteron (the Baryshevsky effect). General formulas describing this effect have been derived. Calculated formulas agree with the earlier obtained results. The problem of the influence of tensor electric polarizability on spin dynamics in such a deuteron electric-dipole-moment experiment in storage rings has been investigated. Doubling the resonant frequency used in this experiment dramatically amplifies the Baryshevsky effect and provides the opportunity to make high-precision measurements of the deuteron's tensor electric polarizability.

DOI: 10.1103/PhysRevC.75.014003 PACS number(s): 21.45.+v, 11.10.Ef, 21.10.Ky, 24.70.+s

I. INTRODUCTION

Electric and magnetic polarizabilities are important properties of deuteron and other nuclei. Tensor electric and magnetic polarizabilities are defined by spin interactions of nucleons. In particular, measurement of the tensor electric polarizability of the deuteron gives important information about the interaction between spins of nucleons and provides a good possibility for examining the theory of spin-dependent nuclear forces.

A method for determining this important electromagnetic property of deuterons has been proposed by V. Baryshevsky and co-workers [1–3]. If an electric field acts on a deuteron beam circulating into a storage ring, the presence of the tensor electric polarizability leads to the appearance of an interaction quadratic in the spin. When the electric field in the particle rest frame oscillates at the resonant frequency, an effect similar to nuclear magnetic resonance (NMR) takes place. This effect stimulates the buildup of the vertical polarization (BVP) of the deuteron beam [1–3].

In the present work, we derive general formulas describing the BVP caused by the tensor electric polarizability of the deuteron in storage rings (the Baryshevsky effect). Another effect defined by the tensor magnetic polarizability of the deuteron is the spin rotation in the horizontal plane at two frequencies instead of the expected rotation at the $g-2$ frequency [1–3]. In those works, the approach based on equations defining the dynamics of the polarization vector and polarization tensor has been used. To check the results obtained and develop a more general theory, we followed a quite different method of spin amplitudes (see Refs. [4,5]). In the present work, this method is partially changed. We use the matrix Hamiltonian for determining an evolution of the spin wave function.

The Baryshevsky effect should be taken into account when searching for the electric dipole moment (EDM) of the deuteron [1–3]. The existence of the deuteron EDM also leads to the BVP. Researchers plan to measure the BVP in a deuteron EDM experiment in storage rings [6–8]. Since the Baryshevsky effect can imitate the existence of an EDM, the spin dynamics caused by this effect should be investigated in detail.

We use the relativistic system of units $\hbar = c = 1$.

In the next section, we review the main aspects of the Hamiltonian approach used in the method of spin amplitudes. In Sec. III, we briefly discuss the form of the Hamiltonian operator in a cylindrical coordinate system. Section IV is devoted to the calculation of corrections to the Hamiltonian operator for the tensor polarizabilities of the deuteron. A solution for the matrix Hamiltonian equation is given in Sec. V. In Sec. VI, we calculate the spin dynamics expressed by the evolution of the vertical component of the polarization vector. A detailed analysis of new experimental possibilities for measuring the tensor electric polarizability of the deuteron is performed in Sec. VII. Section VIII is dedicated to the differentiation of the effects of the EDM and the tensor electric polarizability in a deuteron EDM experiment. Finally, in Sec. IX we discuss previously obtained formulas and summarize the main results of the work.

II. HAMILTONIAN APPROACH IN THE METHOD OF SPIN AMPLITUDES

The method of spin amplitudes uses quantum mechanics formalism to more easily describe spin dynamics (see Refs. [4, 5]). For spin-1/2 particles, it is mathematically advantageous to use this formalism, because transporting the two-component spin wave function (spinor) Ψ is simpler than transporting the three-dimensional polarization vector $\boldsymbol{P}$. The relationship between them is given by the expectation value of the Pauli spin vector $\boldsymbol{\sigma}$

$$\boldsymbol{P} = \Psi^\dagger \boldsymbol{\sigma} \Psi, \quad \Psi = \begin{pmatrix} C_{+1/2}(t) \\ C_{-1/2}(t) \end{pmatrix}, \tag{1}$$

where $C_{+1/2}(t)$ and $C_{-1/2}(t)$ are the time-dependent amplitudes. Together with the identity matrix, the Pauli matrices generate an irreducible representation of the SU(2) group.

Algebraically, the SU(2) group is a double cover of the three-dimensional rotation group SO(3). Therefore, the formalism based on the Pauli matrices is applicable to

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particles/nuclei with arbitrary spin if an effect of spin rotation is analyzed. The spin rotation can also be exhaustively described with the polarization vector $\boldsymbol{P}$, which is defined by

$$P_i = \frac{\langle S_i \rangle}{S}, \quad i, j = x, y, z, \tag{2}$$

where S_i are corresponding spin matrices and S is the spin quantum number. The polarization vector being an average spin is a strictly classical quantity (see Ref. [9]) whose evolution can be investigated in the framework of classical spin physics.

Particles with spin $S \geqslant 1$ also possess a tensor polarization. Main characteristics of such a polarization are specified by the polarization tensor P_{ij}, which is given by [10]

$$P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2S(S+1)\delta_{ij}}{2S(2S-1)}, \quad i, j = x, y, z. \tag{3}$$

The polarization tensor satisfies the conditions $P_{ij} = P_{ji}$ and $P_{xx} + P_{yy} + P_{zz} = 1$ and therefore has five independent components. Additional tensors composed of products of three or more spin matrices are needed only for the exhaustive description of polarization of particles/nuclei with spin $S \geqslant 3/2$.

The spin matrices for spin-1 particles have the forms

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{4}$$

Possibly, the nontrivial spin dynamics predicted in Refs. [1–3] and conditioned by the tensor electric polarizability of deuteron is the first example of the importance of spin tensor interactions in the physics of polarized beams. Tensor interactions of the deuteron can also be described with the method of spin amplitudes. In this case, three-component spinors and 3×3 matrices should be used. The method of spin amplitudes is mathematically advantageous because transporting the three-component spinor is much simpler than transporting the three-dimensional polarization vector $\boldsymbol{P}$ and five independent components of the polarization tensor P_{ij} together.

We follow the traditional quantum mechanical approach perfectly expounded by Feynman [11] and use the matrix Hamiltonian equation and the matrix Hamiltonian H for determining an evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \quad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}, \quad H_{ij} = \langle i|\mathcal{H}|j\rangle, \tag{5}$$

where H is a 3×3 matrix, Ψ is the three-component spin wave function (spinor), $H_{ij} = H_{ji}^*$, and $i, j = 1, 0, -1$. In this equation, H_{ij} are matrix elements of the Hamiltonian operator $\mathcal{H}$.

A determination of spin dynamics can be divided into several stages, namely, (i) a solution of Hamiltonian equation (5) and a determination of eigenvalues and eigenvectors of the Hamiltonian matrix H, (ii) a derivation of spin wave function consisting of a solution of a set of three linear algebraic equations, and (iii) a calculation of time evolution of polarization vector and polarization tensor.

III. HAMILTONIAN OPERATOR IN A CYLINDRICAL COORDINATE SYSTEM

The spin dynamics can be analytically calculated when a storage ring is either circular or divided into circular sectors by empty spaces. In this case, the use of cylindrical coordinates can be very successful. The particle spin motion in storage rings is usually specified with respect to the particle trajectory. Main fields are commonly defined relative to the cylindrical coordinate axes. Therefore, the use of the cylindrical coordinates considerably simplifies the analysis of spin rotation in the horizontal plane ($g-2$ precession) and other spin effects. Equation of spin motion in storage rings in a cylindrical coordinate system has the form [12]

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{\omega}_a \times \boldsymbol{S},$$
$$\boldsymbol{\omega}_a = -\frac{e}{m}\left\{a\boldsymbol{B} - \frac{a\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) + \left(\frac{1}{\gamma^2-1} - a\right)(\boldsymbol{\beta}\times\boldsymbol{E}) + \frac{1}{\gamma}\left[\boldsymbol{B}_\parallel - \frac{1}{\beta^2}(\boldsymbol{\beta}\times\boldsymbol{E})_\parallel\right] + \frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right\}, \tag{6}$$

where $a = (g-2)/2$, $g = 2\mu m/(eS)$, $\eta = 2dm/(eS)$, and d is the EDM. The sign $\parallel$ means a horizontal projection for any vector. In this work, we do not consider effects caused by perturbations of particle trajectory investigated in Ref. [12]. The quantity $\boldsymbol{\omega}_a$ is also equal to [12]

$$\boldsymbol{\omega}_a = \boldsymbol{\Omega} - \dot{\phi}\boldsymbol{e}_z, \tag{7}$$

where $\boldsymbol{\Omega}$ is the Thomas-Bargmann-Michel-Telegdi (T-BMT) frequency [13] corrected for the EDM [12,14–16], and $\dot{\phi}\boldsymbol{e}_z$ is the instantaneous angular frequency of orbital revolution.

If we used the Hamiltonian of the particle with the EDM given in the laboratory frame [16], we would present the matrices S_ρ and S_ϕ in the form

$$S_\rho = S_x\cos\phi + S_y\sin\phi, \quad S_\phi = -S_x\sin\phi + S_y\cos\phi. \tag{8}$$

However, this representation of spin matrices S_ρ, S_ϕ leads to cumbersome calculations because the azimuth ϕ defined by a particle position is time dependent. Therefore, it is helpful to consider spin effects in the frame rotating at the instantaneous angular frequency of orbital revolution which is almost equal to the cyclotron frequency. In this frame, the motion of particles is relatively slow because it can be caused only by beam oscillations and other deflections of particles from the ideal trajectory. The equation of spin motion in the rotating frame coincides with that in the cylindrical coordinate system because the horizontal axis of this system rotates at the instantaneous angular frequency of orbital revolution.

The Hamiltonians of the particle in the rotating frame and in the laboratory one ($\mathcal{H}$ and $\mathcal{H}_{\rm lab}$, respectively) are related by [17]

$$\mathcal{H} = \mathcal{H}_{\text{lab}} - \boldsymbol{S} \cdot \boldsymbol{\omega}, \quad (9)$$

where $\boldsymbol{\omega}$ is the observer's proper frequency of rotation [18]. In the considered case, this frequency coincides with the instantaneous angular frequency of orbital revolution. The relation between the Hamiltonian in the laboratory frame and the T-BMT frequency corrected for the EDM is given by

$$\mathcal{H}_{\text{lab}} = \mathcal{H}_0 + \boldsymbol{S} \cdot \boldsymbol{\Omega},$$

where $\mathcal{H}_0$ is a sum of spin-independent operators. Therefore, the Hamiltonian in the rotating frame has the form

$$\mathcal{H} = \mathcal{H}_0 + \boldsymbol{S} \cdot \boldsymbol{\omega}_a, \quad (10)$$

where $\boldsymbol{\omega}_a$ is defined by Eq. (6). Evidently, Hamiltonian (10) is consistent with Eq. (6).

The particle in the rotating frame is localized and ideally is in rest. Therefore, we can direct the x and y axes in this frame along the radial and longitudinal axes, respectively. This procedure is commonly used (see Refs. [4,5,10]) and results in the direct substitution of spin matrices (4) for S_ρ and S_ϕ:

$$S_\rho = S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_\phi = S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (11)$$

The matrix S_z remains unchanged. Similar substitution can be performed for the matrices Π_{ij}. The use of definition (11) strongly simplifies calculations.

If an interaction causes a correction to the Hamiltonian operator (10) and does not appreciably influence the particle motion, this correction is the same in the laboratory frame and the rotating one.

In this work, we suppose $\boldsymbol{B}$ to be upward. For the deuteron, $a < 0$ and $(\omega_a)_z > 0$.

IV. CORRECTIONS TO THE HAMILTONIAN OPERATOR FOR TENSOR POLARIZABILITIES OF THE DEUTERON

Corrections to the Hamiltonian operator for the deuteron polarizabilities contain scalar and tensor parts. The scalar part is spin independent and can be disregarded. General formulas used in Refs. [1–3] are within first-order terms in the normalized velocity β. In the present work, we derive exact formulas for the configuration of main fields related to a resonant deuteron EDM experiment (see Refs. [6,7]). Because the Lorentz factor is planned to be $\gamma = 1.28$ [8] in that experiment, exact relativistic formulas are needed.

Within first-order terms in β, the interaction Hamiltonian depending on the electric and magnetic polarizabilities is given by

$$V = V_e + V_m = -\tfrac{1}{2}\alpha_{ik} E'_i E'_k - \tfrac{1}{2}\beta_{ik} B'_i B'_k, \quad \boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}, \quad \boldsymbol{B}' = \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}, \quad (12)$$

where α_{ik} and β_{ik} are the tensors of electric and magnetic polarizabilities, $\boldsymbol{E}'$ and $\boldsymbol{B}'$ are effective fields acting on a particle (fields in the particle rest frame, i.e., in the rotating frame). In this approximation, the spin-dependent part of the Hamiltonian defined by the tensor electric and magnetic polarizabilities is equal to [1–3]

$$V = -\alpha_T (\boldsymbol{S} \cdot \boldsymbol{E}')^2 - \beta_T (\boldsymbol{S} \cdot \boldsymbol{B}')^2, \quad (13)$$

where α_T and β_T are the tensor electric and magnetic polarizabilities, respectively.

The Baryshevsky effect takes place when one stimulates coherent longitudinal oscillations of particles at a resonance frequency. The angular frequency of forced longitudinal oscillations, ω, is equal to the difference between two radio frequencies (see Ref. [8]). It should be very close to the angular frequency of spin rotation ($g - 2$ frequency), ω_0, and close to the eigenfrequency of free synchrotron oscillations (synchrotron frequency) [8]. The resonant electric field in the particle rest frame possesses the oscillating longitudinal component E'_ϕ defined by the Lorentz transformation of a longitudinal electric field met by the oscillating particles and the radial one E'_ρ caused by the Lorentz transformation of vertical magnetic field. The latter component has a resonance part because of the modulation of the particle velocity. In the present work, we consider only the effects of resonant fields on the BVP in ideal conditions and disregard the systematic errors listed in Sec. VII. Thus, we take into consideration the constant vertical magnetic field and the oscillating longitudinal electric one.

The relativistic formulas for the fields in the particle rest frame, $\boldsymbol{E}'$ and $\boldsymbol{B}'$, have the form

$$\boldsymbol{E}' = \beta\gamma B_z \boldsymbol{e}_\rho + E_\phi \boldsymbol{e}_\phi, \quad \boldsymbol{B}' = \gamma B_z \boldsymbol{e}_z. \quad (14)$$

The fields and the electromagnetic moments in the laboratory frame are unprimed.

Induced electric and magnetic dipole moments in the particle rest frame caused by the tensor polarizabilities are equal to

$$\boldsymbol{d}' = \alpha_T \{\boldsymbol{S}, (\boldsymbol{S} \cdot \boldsymbol{E}')\}, \quad \boldsymbol{m}' = \beta_T \{\boldsymbol{S}, (\boldsymbol{S} \cdot \boldsymbol{B}')\}, \quad (15)$$

where $\{\ldots, \ldots\}$ means an anticommutator.

The correction to the Hamiltonian operator for the tensor polarizabilities of the deuteron is equal to

$$V = V_{\text{lab}} = -\tfrac{1}{2}(\boldsymbol{d} \cdot \boldsymbol{E} + \boldsymbol{m} \cdot \boldsymbol{B}). \quad (16)$$

This correction does not change the angular frequency of orbital revolution. According to Eq. (9), the correction is the same in the rotating frame and the laboratory one. Since the induced dipole moments are proportional to the effective fields in the particle rest frame, the factor 1/2 appears.

To obtain the dipole moments in the laboratory frame, $\boldsymbol{d}$ and $\boldsymbol{m}$, we can use the Hamiltonian operator for relativistic particles with electric and magnetic dipole moments. For spin-1/2 particles, it has been derived in Ref. [16]. The Hamiltonian operator for spin-1 particles is similar, because it should be consistent with the corresponding equation of spin motion (modified T-BMT equation) which is valid for any spin.

If we neglect the normal magnetic moment $\mu_0 = eS/m$, which is small for nuclei, the relations between the electromagnetic moments in two frames are given by

$$\boldsymbol{d} = \boldsymbol{d}' - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{d}'\cdot\boldsymbol{\beta}) - \boldsymbol{\beta}\times\boldsymbol{m}',$$
$$\boldsymbol{m} = \boldsymbol{m}' - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{m}'\cdot\boldsymbol{\beta}) + \boldsymbol{\beta}\times\boldsymbol{d}'. \quad (17)$$

When $\boldsymbol{d} = e\boldsymbol{l}$, the relation between $\boldsymbol{d}$ and $\boldsymbol{d}'$ arises from the Lorentz transformation of the length of electric dipole, $\boldsymbol{l}$, because the charge e is a relativistic invariant. Relations (17) remain valid for induced electromagnetic moments.

As a result, the correction to the Hamiltonian operator in the rotating frame takes the form

$$V = -\frac{1}{2\gamma}\left(\boldsymbol{d}'\cdot\boldsymbol{E}' + \boldsymbol{m}'\cdot\boldsymbol{B}'\right) = -\frac{\alpha_T}{\gamma}(\boldsymbol{S}\cdot\boldsymbol{E}')^2 - \frac{\beta_T}{\gamma}(\boldsymbol{S}\cdot\boldsymbol{B}')^2. \quad (18)$$

Equation (13) is an approximate version of Eq. (18).

The equation of oscillatory motion of the particle has the form

$$\frac{d\boldsymbol{p}}{dt} = e\boldsymbol{E}. \quad (19)$$

The quantity $\boldsymbol{E}$ in Eq. (19) is the electric field met by the particle. As a result of the coherent beam oscillations, this field oscillates in the particle rest frame. The angular frequency of velocity modulation ω significantly differs from that of the resonator (see Refs. [6,8]) but should be very close to the angular frequency of spin rotation ω_0. The latter quantity is almost equal to the vertical component of $\boldsymbol{\omega}_a$, because other components of this vector are relatively small, that is,

$$\omega_0 \equiv (\omega_a)_z = -\frac{ea}{m}B_z. \quad (20)$$

For the deuteron, $\omega_0 > 0$. The modulation of normalized velocity can be given by (see Refs. [6,8])

$$\boldsymbol{\beta} = \frac{\boldsymbol{p}}{\sqrt{m^2+p^2}} = \boldsymbol{\beta}_0 + \Delta\beta_0\cdot\cos(\omega t+\varphi)\boldsymbol{e}_\phi, \quad (21)$$

where

$$\boldsymbol{\beta}_0 = \frac{\boldsymbol{p}_0}{\sqrt{m^2+p_0^2}}, \quad \gamma_0 = \frac{\sqrt{m^2+p_0^2}}{m}.$$

Owing to this modulation, the radial electric field in the particle rest frame has the oscillatory part. The effect of the modulation on the BVP is described by the last term in Eq. (6) proportional to $\boldsymbol{\beta}\times\boldsymbol{B}$.

We can calculate the electric field acting on the particle to within first-order terms in $\Delta\beta_0$. The particle momentum is defined by the equation

$$\boldsymbol{p} = \frac{m\boldsymbol{\beta}}{\sqrt{1-\beta^2}} = \boldsymbol{p}_0 + \gamma_0^3 m\Delta\beta_0\cdot\cos(\omega t+\varphi)\boldsymbol{e}_\phi. \quad (22)$$

According to the result of differentiation of Eq. (22) on time, Eq. (19) takes the form

$$\boldsymbol{E} = -E_0\sin(\omega t+\varphi)\boldsymbol{e}_\phi, \quad (23)$$

where

$$E_0 = \frac{\gamma_0^3 m\omega}{e}\Delta\beta_0. \quad (24)$$

Equation (18) can be transformed to the form

$$V = -\frac{\alpha_T}{\gamma}(\beta\gamma B_z S_\rho + E_\phi S_\phi)^2 - \beta_T\gamma B_z^2 S_z^2. \quad (25)$$

An estimate of two terms in the formula for the effective electric field $\boldsymbol{E}'$ [see Eq. (14)] shows that the term proportional to the magnetic field B_z is much greater for the deuteron. To simplify the calculation, we neglect the effect of the longitudinal electric field and use the approximation

$$V = -\gamma B_z^2\left(\alpha_T\beta^2 S_\rho^2 + \beta_T S_z^2\right). \quad (26)$$

The quantities γ and $\beta^2\gamma$ can be expanded in series of $\Delta\beta_0$ such that

$$\gamma = \gamma_0 + \beta_0\gamma_0^3\cdot\Delta\beta_0\cos(\omega t+\varphi) + \tfrac{1}{4}\left(1+3\beta_0^2\gamma_0^2\right)\gamma_0^3(\Delta\beta_0)^2\{1+\cos[2(\omega t+\varphi)]\},$$
$$\beta^2\gamma = \beta_0^2\gamma_0 + \left(2+\beta_0^2\gamma_0^2\right)\beta_0\gamma_0\cdot\Delta\beta_0\cos(\omega t+\varphi) + \tfrac{1}{4}\left(2+5\beta_0^2\gamma_0^2+3\beta_0^4\gamma_0^4\right)\gamma_0(\Delta\beta_0)^2 \times\{1+\cos[2(\omega t+\varphi)]\}. \quad (27)$$

Equations (26) and (27) define the corrections to the Hamiltonian operator for the tensor polarizabilities of the deuteron.

V. SOLUTION OF MATRIX HAMILTONIAN EQUATION

Nonzero matrix elements of the spin operators contained by Eq. (26) are

$$\left(S_\rho^2\right)_{11} = \left(S_\rho^2\right)_{1,-1} = \left(S_\rho^2\right)_{-1,1} = \left(S_\rho^2\right)_{-1,-1} = \tfrac{1}{2},$$
$$\left(S_\rho^2\right)_{00} = 1, \quad \left(S_z^2\right)_{11} = \left(S_z^2\right)_{-1,-1} = 1. \quad (28)$$

Therefore, matrix Hamiltonian (5) takes the form

$$H = \begin{pmatrix} E_0+\omega_0+\mathcal{A}+\mathcal{B} & 0 & \mathcal{A} \\ 0 & E_0+2\mathcal{A} & 0 \\ \mathcal{A} & 0 & E_0-\omega_0+\mathcal{A}+\mathcal{B} \end{pmatrix}, \quad (29)$$

where

$$\mathcal{A} = a_0 + a_1\cos(\omega t+\varphi) + a_2\cos[2(\omega t+\varphi)],$$
$$\mathcal{B} = b_0 + b_1\cos(\omega t+\varphi) + b_2\cos[2(\omega t+\varphi)],$$
$$a_0 = -\tfrac{1}{2}\alpha_T B_z^2\gamma_0\left[\beta_0^2 + \tfrac{1}{4}\left(2+5\beta_0^2\gamma_0^2+3\beta_0^4\gamma_0^4\right)(\Delta\beta_0)^2\right],$$
$$a_1 = -\tfrac{1}{2}\alpha_T B_z^2\left(2+\beta_0^2\gamma_0^2\right)\beta_0\gamma_0\cdot\Delta\beta_0,$$
$$a_2 = -\tfrac{1}{8}\alpha_T B_z^2\left(2+5\beta_0^2\gamma_0^2+3\beta_0^4\gamma_0^4\right)\gamma_0(\Delta\beta_0)^2,$$
$$b_0 = -\beta_T B_z^2\gamma_0\left[1+\tfrac{1}{4}\left(1+3\beta_0^2\gamma_0^2\right)\gamma_0^2(\Delta\beta_0)^2\right],$$
$$b_1 = -\beta_T B_z^2\beta_0\gamma_0^3\cdot\Delta\beta_0,$$
$$b_2 = -\tfrac{1}{4}\beta_T B_z^2\left(1+3\beta_0^2\gamma_0^2\right)\gamma_0^3(\Delta\beta_0)^2, \quad (30)$$

and E_0 is the zero energy level.

In Hamiltonian (29), the EDM effect is not taken into account. We consider the spin dynamics near a resonance.

At the first stage, it is useful to pass on to new amplitudes (see Ref. [11]). This transformation brings real parts of diagonal elements of the matrix Hamiltonian to zero. However, it does not nullify the imaginary parts of diagonal elements for unstable particles (see Ref. [19]). Evidently, Hamiltonian (29) is real, and the new amplitudes are equal to

$$\begin{aligned}\gamma_1(t) &= \exp\left\{i\left[k_1 t + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}C_1(t),\\ \gamma_0(t) &= \exp\left\{i\left[k_0 t + \frac{2a_1}{\omega}f(t) + \frac{a_2}{\omega}g(t)\right]\right\}C_0(t),\\ \gamma_{-1}(t) &= \exp\left\{i\left[k_{-1} t + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}C_{-1}(t),\\ k_1 &= E_0+\omega_0+a_0+b_0, \quad k_0 = E_0+2a_0, \\ k_{-1} &= E_0-\omega_0+a_0+b_0,\\ f(t) &= \sin(\omega t+\varphi)-\sin(\varphi),\\ g(t) &= \sin[2(\omega t+\varphi)]-\sin(2\varphi).\end{aligned} \tag{31}$$

The dynamics of these amplitudes does not depend on the tensor magnetic polarizability and is given by

$$\begin{cases} i\dfrac{d\gamma_1}{dt} = \mathcal{A}\exp(2i\omega_0 t)\gamma_{-1},\\ i\dfrac{d\gamma_0}{dt} = 0,\\ i\dfrac{d\gamma_{-1}}{dt} = \mathcal{A}\exp(-2i\omega_0 t)\gamma_1.\end{cases} \tag{32}$$

Equations (31) and (32) result in

$$C_0(t) = \exp\left\{-i\left[k_0 t + \frac{2a_1}{\omega}f(t) + \frac{a_2}{\omega}g(t)\right]\right\}C_0(0). \tag{33}$$

Zero component of spin is not mixed with other components.

At the second stage, we can average over a much longer time than the oscillation period [11]. The relation

$$\cos(\zeta t+\eta) = \tfrac{1}{2}\{\exp[i(\zeta t+\eta)] + \exp[-i(\zeta t+\eta)]\}$$

can be used. There are two resonant frequencies, $\omega=\omega_0$ and $\omega = 2\omega_0$. The first of them corresponds to the resonance condition in the deuteron EDM experiment [6,7]. We will consider this case first.

When $\omega\approx\omega_0$, averaging over time results in

$$\begin{cases} i\dfrac{d\gamma_1}{dt} = \dfrac{a_2}{2}\gamma_{-1}\exp\{2i[(\omega_0-\omega)t-\varphi]\},\\ i\dfrac{d\gamma_{-1}}{dt} = \dfrac{a_2}{2}\gamma_1\exp\{-2i[(\omega_0-\omega)t-\varphi]\}.\end{cases} \tag{34}$$

At the third stage, we can use the following transformation:

$$\begin{aligned} D_1(t) &= \exp[-i(\omega_0-\omega)t]\gamma_1(t),\\ D_{-1}(t) &= \exp[i(\omega_0-\omega)t]\gamma_1(t).\end{aligned} \tag{35}$$

A transformed Eq. (34) can be written in the matrix form

$$i\frac{dD}{dt} = H'D, \quad H' = \begin{pmatrix}\omega_0-\omega & \dfrac{a_2}{2}\exp(-2i\varphi)\\ \dfrac{a_2}{2}\exp(2i\varphi) & -(\omega_0-\omega)\end{pmatrix}, \quad D = \begin{pmatrix}D_1(t)\\ D_{-1}(t)\end{pmatrix}. \tag{36}$$

Equation (36) can then be analytically solved as

$$\begin{aligned} D_1(t) &= \left[\frac{\omega'+\omega_0-\omega}{2\omega'}D_1(0) + \frac{\mathcal{E}}{2\omega'}D_{-1}(0)\right]\exp(-i\omega' t)\\ &\quad + \left[\frac{\omega'-(\omega_0-\omega)}{2\omega'}D_1(0) - \frac{\mathcal{E}}{2\omega'}D_{-1}(0)\right]\exp(i\omega' t),\\ D_{-1}(t) &= \left[\frac{\mathcal{E}^*}{2\omega'}D_1(0) + \frac{\omega'-(\omega_0-\omega)}{2\omega'}D_{-1}(0)\right]\exp(-i\omega' t)\\ &\quad + \left[-\frac{\mathcal{E}^*}{2\omega'}D_1(0) + \frac{\omega'+\omega_0-\omega}{2\omega'}D_{-1}(0)\right]\exp(i\omega' t),\end{aligned} \tag{37}$$

or

$$\begin{aligned} D_1(t) &= \left[\cos(\omega' t) - i\frac{\omega_0-\omega}{\omega'}\sin(\omega' t)\right]\\ &\quad\times D_1(0) - i\frac{\mathcal{E}}{\omega'}\sin(\omega' t)D_{-1}(0),\\ D_{-1}(t) &= -i\frac{\mathcal{E}^*}{\omega'}\sin(\omega' t)D_1(0)\\ &\quad + \left[\cos(\omega' t) + i\frac{\omega_0-\omega}{\omega'}\sin(\omega' t)\right]D_{-1}(0),\end{aligned} \tag{38}$$

where

$$\omega' = \sqrt{(\omega_0-\omega)^2 + \mathcal{E}\mathcal{E}^*}, \quad \mathcal{E} = \frac{a_2}{2}\exp(-2i\varphi). \tag{39}$$

The angular frequency of spin oscillation is equal to $2\omega'$. The initial spin amplitudes take the form

$$\begin{aligned} C_1(t) &= \exp\left\{-i\left[(E_0+\omega+a_0+b_0)t\right.\right.\\ &\quad\left.\left. + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_1(t),\\ C_{-1}(t) &= \exp\left\{-i\left[(E_0-\omega+a_0+b_0)t\right.\right.\\ &\quad\left.\left. + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_{-1}(t),\\ C_1(0) &= D_1(0), \quad C_{-1}(0) = D_{-1}(0).\end{aligned} \tag{40}$$

The resonance at the doubled frequency $\omega\approx 2\omega_0$ can be investigated in a similar way. The evolution of the spin amplitudes is given by

$$\begin{aligned} C_1(t) &= \exp\left\{-i\left[\left(E_0+\frac{\omega}{2}+a_0+b_0\right)t\right.\right.\\ &\quad\left.\left. + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_1(t),\\ C_0(t) &= \exp\left\{-i\left[(E_0+2a_0)t + \frac{2a_1}{\omega}f(t) + \frac{a_2}{\omega}g(t)\right]\right\}C_0(0),\\ C_{-1}(t) &= \exp\left\{-i\left[\left(E_0-\frac{\omega}{2}+a_0+b_0\right)t\right.\right.\\ &\quad\left.\left. + \frac{a_1+b_1}{\omega}f(t) + \frac{a_2+b_2}{2\omega}g(t)\right]\right\}D_{-1}(t),\\ C_1(0) &= D_1(0), \quad C_{-1}(0) = D_{-1}(0),\end{aligned} \tag{41}$$

where

$$D_1(t) = \left(\cos\frac{\omega'' t}{2} - i\frac{2\omega_0-\omega}{\omega''}\sin\frac{\omega'' t}{2}\right) \times D_1(0) - i\frac{2\mathcal{E}'}{\omega''}\sin\frac{\omega'' t}{2}D_{-1}(0),$$
$$D_{-1}(t) = -i\frac{2\mathcal{E}'^*}{\omega''}\sin\frac{\omega'' t}{2}D_1(0) + \left(\cos\frac{\omega'' t}{2} + i\frac{2\omega_0-\omega}{\omega''}\sin\frac{\omega'' t}{2}\right)D_{-1}(0), \quad (42)$$
$$\mathcal{E}' = \frac{a_1}{2}\exp(-i\varphi),$$

and the angular frequency of spin oscillation is equal to

$$\omega'' = \sqrt{(2\omega_0-\omega)^2 + 4\mathcal{E}'\mathcal{E}'^*}. \quad (43)$$

VI. SPIN DYNAMICS CAUSED BY TENSOR POLARIZABILITIES OF DEUTERON

For spin-1 particles, the three components of the polarization vector and related components of the polarization tensor are defined by

$$P_\rho = \frac{1}{\sqrt{2}}(C_1C_0^* + C_1^*C_0 + C_0C_{-1}^* + C_0^*C_{-1}),$$
$$P_\phi = \frac{i}{\sqrt{2}}(C_1C_0^* - C_1^*C_0 + C_0C_{-1}^* - C_0^*C_{-1}),$$
$$P_z = (C_1C_1^* - C_{-1}C_{-1}^*), \quad (44)$$
$$P_{\rho\rho} = \frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} + C_0C_0^*) - \frac{1}{2},$$
$$P_{\phi\phi} = -\frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} - C_0C_0^*) - \frac{1}{2},$$
$$P_{\rho\phi} = i\frac{3}{2}(C_1C_{-1}^* - C_1^*C_{-1}).$$

The horizontal components P_ρ and P_ϕ do not give the necessary information about the investigated effect because they undergo fast oscillations caused by the $g-2$ spin precession. The change of the vertical component P_z is a relatively slow process.

The quantity P_z does not depend on C_0. Since $C_1C_1^* = D_1D_1^*$ and $C_{-1}C_{-1}^* = D_{-1}D_{-1}^*$, the BVP is caused by the tensor electric polarizability and is not affected by the tensor magnetic one. However, this conclusion is not valid if the deuteron possesses an EDM. In this case, the tensor magnetic polarizability leads to splitting of the resonance frequency [1–3].

When $\omega \approx \omega_0$, the evolution of the vertical component of the polarization vector is expressed by

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}[1-\cos(2\omega' t)]\right]P_z(0) + \frac{2\mathcal{E}_0}{3\omega'}\left\{\frac{1}{2}[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{\omega_0-\omega}{\omega'}\cos(2\varphi) \times [1-\cos(2\omega' t)] - \sin(2\varphi)\sin(2\omega' t)\right] + P_{\rho\phi}(0)\left[\frac{\omega_0-\omega}{\omega'}\sin(2\varphi)[1-\cos(2\omega' t)] + \cos(2\varphi)\sin(2\omega' t)\right]\right\}, \quad \mathcal{E}_0 = \frac{a_2}{2}, \quad (45)$$

where the quantities a_2 and ω' are defined by Eqs. (30) and (39), respectively.

For a vector-polarized deuteron beam, the related components of polarization vector and polarization tensor have the form

$$P_z = \cos(\theta), \quad P_{\rho\rho} = \tfrac{1}{2}\left[3\sin^2(\theta)\cos^2(\psi) - 1\right],$$
$$P_{\phi\phi} = \tfrac{1}{2}\left[3\sin^2(\theta)\sin^2(\psi) - 1\right], \quad P_{\rho\phi} = \tfrac{3}{4}\sin^2(\theta)\sin(2\psi), \quad (46)$$

where θ and ψ are the spherical angles defining the spin direction in the rotating frame. The azimuth $\psi = 0$ characterizes the spin directed radially outward.

The projection of deuteron spin onto the preferential direction can be equal to zero. The beam possessing such a polarization is tensor polarized. The vector polarization of this beam is zero. The components of polarization vector and polarization tensor take the form

$$P_\rho = P_\phi = P_z = 0, \quad P_{\rho\rho} = -3\sin^2(\theta)\cos^2(\psi) + 1,$$
$$P_{\phi\phi} = -3\sin^2(\theta)\sin^2(\psi) + 1, \quad P_{\rho\phi} = -\tfrac{3}{2}\sin^2(\theta)\sin(2\psi), \quad (47)$$

where θ and ψ are the spherical angles stated above. When the polarization of the deuteron beam is the vector, Eqs. (45) and (46) result in

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}[1-\cos(2\omega' t)]\right]\cos(\theta) + \frac{\mathcal{E}_0}{2\omega'}\sin^2(\theta)\left\{\frac{\omega_0-\omega}{\omega'}\cos[2(\psi-\varphi)][1-\cos(2\omega' t)] + \sin[2(\psi-\varphi)]\sin(2\omega' t)\right\}. \quad (48)$$

If the resonance is perfect, the initial beam polarization is horizontal $[P_z(0) = 0]$, and $\omega' t \ll 0$, then Eq. (45) takes the form

$$P_z(t) = \frac{2}{3}a_2 t\left[P_{\rho\phi}(0)\cos(2\varphi) - \frac{P_{\rho\rho}(0) - P_{\phi\phi}(0)}{2}\sin(2\varphi)\right]. \quad (49)$$

In this case, the vertical spin component grows linearly with time, and

$$\frac{dP_z}{dt} = \frac{2}{3}a_2\left[P_{\rho\phi}(0)\cos(2\varphi) - \frac{P_{\rho\rho}(0) - P_{\phi\phi}(0)}{2}\sin(2\varphi)\right]. \quad (50)$$

For the resonance at the doubled frequency $\omega \approx 2\omega_0$, the evolution of the vertical component of the polarization vector is given by

$$P_z(t) = \left[1 - \frac{4\mathcal{E}_0'^2}{\omega''^2}[1-\cos(\omega'' t)]\right] P_z(0) + \frac{2\mathcal{E}_0'}{3\omega''}\left\{[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{2\omega_0-\omega}{\omega''}\cos(\varphi) \times [1-\cos(\omega'' t)] - \sin(\varphi)\sin(\omega'' t)\right] + 2P_{\rho\phi}(0)\left[\frac{2\omega_0-\omega}{\omega''}\sin(\varphi)[1-\cos(\omega'' t)] + \cos(\varphi)\sin(\omega'' t)\right]\right\}, \quad \mathcal{E}_0' = \frac{a_1}{2}, \tag{51}$$

where the quantities a_1 and ω'' are defined by Eqs. (30) and (43), respectively.

If the resonance is perfect, the initial beam polarization is horizontal, and $\omega'' t \ll 0$, then Eq. (51) takes the form

$$P_z(t) = \frac{2}{3}a_1 t\left[P_{\rho\phi}(0)\cos(\varphi) - \frac{P_{\rho\rho}(0) - P_{\phi\phi}(0)}{2}\sin(\varphi)\right]. \tag{52}$$

Equations (30), (49), and (52) show that resonance frequency doubling leads to a dramatic amplification of the Baryshevsky effect. When the frequency is doubled, the EDM effect becomes nonresonant. In this case, it does not influence the spin dynamics.

VII. MEASUREMENT OF TENSOR ELECTRIC POLARIZABILITY OF DEUTERON IN STORAGE-RING EXPERIMENTS

To discover the Baryshevsky effect, it is necessary to stimulate the BVP conditioned by the tensor electric polarizability of deuteron and to avoid a similar effect caused by the magnetic moment. It is known that the magnetic resonance takes place when the particle placed in a uniform vertical magnetic field is also affected by a horizontal magnetic field oscillating at a frequency close to the frequency of spin rotation (see, e.g., Ref. [20]). The magnetic resonance results in the BVP for a horizontally polarized beam.

Evidently, the magnetic resonance cannot take place when the electric field is longitudinal, because nothing but the oscillating electric field appears in the particle rest frame. Since the frequencies of betatron oscillations are chosen to be far from resonances, these oscillations cannot lead to the resonance effect. However, the resonance is caused by the tensor electric polarizability of the deuteron. The electric field in the particle rest frame possesses the oscillating longitudinal component E'_ϕ and the radial one E'_ρ caused by the Lorentz transformation of the vertical magnetic field. The latter component has a resonance part because of the modulation of the particle velocity.

To measure the effect, some resonators (rf cavities) should be used. The electric field in a resonator is generated along the central line, and the magnetic field is orthogonally directed [21]. The magnetic field along the central line is equal to zero. If the rf cavities are perfectly placed and longitudinally directed, the magnetic field cannot stimulate any resonance effect. In this case, the observed BVP corresponds to the definite value of the tensor electric polarizability.

However, both a displacement and an angular deviation of the center line of the rf cavities away from an average particle trajectory lead to a similar behavior of spin imitating the Baryshevsky effect. As a result, they create systematic errors in the measurement of the tensor electric polarizability. Most of these errors are not in resonance with the spin precession in the horizontal plane. Therefore, they create background and result in fast oscillations of the vertical component of the polarization vector (see Refs. [6–8,22]). Besides this effect, the systematic error can be caused by a radial magnetic field in the particle rest frame oscillating at the resonant frequency. In the deuteron EDM experiment, a similar error will be eliminated by alternately producing two sub-beams with different betatron tunes [6,8,22]. In the deuteron tensor-electric-polarizability one, the resonant radial magnetic field in the particle rest frame is much less important when a tensor polarized beam is used (see below). We calculate only the effects of resonant fields on the BVP in ideal conditions and disregard systematic errors. Thus, we take into consideration the constant vertical magnetic field and the oscillating longitudinal electric one.

The measurement of the tensor electric polarizability of the deuteron in a storage ring needs the field configuration similar to that proposed for the deuteron EDM experiment [6–8]. However, the resonance frequency should be doubled ($\omega \approx 2\omega_0$). Resonance frequency doubling cannot be implemented in the designed EDM ring. In this ring, the eigenfrequency of free synchrotron oscillations must be chosen close to the $g-2$ frequency, ω_a, and the resonance effect is created by the beatings between two rf frequencies [8]. Therefore, the measurement of the tensor electric polarizability of deuteron needs another ring or at least rf cavities different from that developed for the deuteron EDM experiment.

However, the Baryshevsky effect caused by the tensor electric polarizability of the deuteron should be taken into account when performing the deuteron EDM experiment [1–3]. This effect results in the similar BVP and can imitate the presence of the deuteron EDM of order of $d \sim 10^{-29}\, e$ cm. An attainment of such an accuracy is the goal of the storage-ring EDM experiment [6–8].

The EDM-dependent evolution of deuteron spin in this experiment has been calculated in detail in Ref. [23]. The dynamics of the vertical component of the polarization vector is given by

$$P_z^{(\mathrm{EDM})}(t) = \frac{\mathcal{E}_0''}{\Omega'}\left\{\frac{\omega_0-\omega}{\Omega'}\cos(\psi-\varphi)[1-\cos(\Omega' t)] + \sin(\psi-\varphi)\sin(\Omega' t)\right\}, \tag{53}$$

where

$$\Omega' = |\boldsymbol{\Omega}'| = \sqrt{(\omega_0-\omega)^2 + \mathcal{E}_0''^2}, \tag{54}$$

$$\mathcal{E}_0'' = -\frac{1}{2} d B_z \cdot \Delta\beta_0 \left(1 + \frac{a\gamma_0^2\omega}{\omega_0}\right), \tag{55}$$

and the azimuth ψ defines the direction of spin at zero time. The initial polarization is supposed to be horizontal.

When $\Omega' t \ll 1$,

$$P_z^{(\mathrm{EDM})} = \mathcal{E}_0'' t \sin(\psi - \varphi) = -\frac{1}{2} d B_z \Delta\beta_0 \left(1 + \frac{a\gamma_0^2 \omega}{\omega_0}\right) t \sin(\psi - \varphi). \quad (56)$$

We can evaluate the expected sensitivity in the measurement of the tensor electric polarizability of the deuteron with the comparison of Eqs. (51) and (53) and the use of Eq. (46) and the sensitivity of the deuteron EDM experiment estimated in Ref. [8]. For the deuteron, $a = a_d = -0.14299$. The sensitivity to the EDM of $d = 1 \times 10^{-29}$ e cm corresponds to the accuracy of $\delta\alpha_T = 1.2 \times 10^{-43}$ cm^3 when $\omega \approx 2\omega_0$ and Eqs. (51), (53)–(55) are used. This estimate is based on the values of $\gamma_0 = 1.28$, $\beta_0 = 0.625$, $\Delta v_0 = 3.5 \times 10^6$ m/s, and $B_z = 3$ T [8]. There are three independent theoretical predictions for the value of the tensor electric polarizability of deuteron, namely $\alpha_T = -6.2 \times 10^{-41}$ cm^3 [24], -6.8×10^{-41} cm^3 [25], and 3.2×10^{-41} cm^3 [26]. Two first values are very close to each other, but they do not agree with the last result.

In all probability, the best sensitivity in the measurement of α_T can be achieved with the use of a tensor polarized deuteron beam. The initial preferential direction of deuteron polarization should be horizontal. When the vector polarization of such a beam is zero, any spin rotation does not occur. In this case, there are no related systematic errors caused by the radial magnetic field and some other reasons. In the general case, such systematic errors are proportional to a residual vector polarization of the beam. This advantage leads to a sufficient increase in experimental accuracy. When $\omega \approx 2\omega_0$, the equation describing the evolution of beam polarization takes the form

$$P_z(t) = -\frac{2\mathcal{E}_0'}{\omega''} \sin^2(\theta) \left\{ \frac{2\omega_0 - \omega}{\omega''} \cos(2\psi - \varphi)[1 - \cos(\omega'' t)] + \sin(2\psi - \varphi) \sin(\omega'' t) \right\}. \quad (57)$$

In this case, the preliminary estimate of experimental accuracy is $\delta\alpha_T \sim 10^{-45}$–$10^{-44}$ cm^3.

When $\theta = \pi/2$, the natural choice of phase

$$\varphi = 2\psi \pm \frac{\pi}{2}$$

brings Eq. (57) to the form

$$P_z(t) = \pm \frac{2\mathcal{E}_0'}{\omega''} \sin(\omega'' t). \quad (58)$$

Other possibilities, $\varphi = 2\psi$ and $\varphi = 2\psi \pm \pi$, lead to the equation

$$P_z(t) = \pm \frac{4\mathcal{E}_0'(2\omega_0 - \omega)}{\omega''^2} \sin^2\left(\frac{\omega'' t}{2}\right). \quad (59)$$

The dependence of $P_z(t)$ on time becomes quadratic when $\omega'' t \ll 1$. Therefore, these possibilities are less useful. However, they can be used for checking the result.

The deuteron tensor-electric-polarizability experiment essentially differs from the deuteron EDM one by a nonnecessity of carefully checking the systematic errors caused by the horizontal magnetic field in the particle rest frame. The use of a tensor polarized beam makes it possible to avoid any spin rotations and to cancel all related systematic errors. The BVP of such a beam is defined only by the tensor electric polarizability of deuteron. It has a great advantage because the elimination of similar systematic errors is one of the main problems for the deuteron EDM experiment [8,22]. A residual vector polarization of the beam together with a resonant magnetic field in the particle rest frame can result in a false signal. However, a necessary correction into the BVP can be made with a longitudinally vector polarized beam. It is important that the BVP caused by any systematic error is overturned and that the BVP defined by the tensor electric polarizability remains the same when reversing the polarization of this beam [see Eq. (48)]. This property brings an easy differentiation between the Baryshevsky effect and false signals for vector-polarized beams. In all probability, the deuteron tensor-electric-polarizability experiment can be made at one of the existing rings.

VIII. DIFFERENTIATION OF EFFECTS OF EDM AND TENSOR ELECTRIC POLARIZABILITY IN THE DEUTERON EDM EXPERIMENT

Equations (45) and (53)–(55) describing the effects of the tensor electric polarizability and the EDM on the spin dynamics in the EDM experiment essentially differ. Therefore, these effects can be differentiated.

When the initial polarization of deuteron beam is horizontal, Eq. (48) takes the form

$$P_z^{(\mathrm{tensor})}(t) = \frac{\mathcal{E}_0}{2\omega'} \left\{ \frac{\omega_0 - \omega}{\omega'} \cos[2(\psi - \varphi)][1 - \cos(2\omega' t)] + \sin[2(\psi - \varphi)] \sin(2\omega' t) \right\}. \quad (60)$$

For the EDM experiment, the choice of phase

$$\varphi = \psi \pm \frac{\pi}{2}$$

is necessary. This choice results in

$$P_z^{(\mathrm{EDM})}(t) = \pm \frac{\mathcal{E}_0''}{\Omega'} \sin(\Omega' t), \qquad P_z^{(\mathrm{tensor})}(t) = \pm \frac{\mathcal{E}_0(\omega_0 - \omega)}{\omega'^2} \sin^2(\omega' t). \quad (61)$$

Since the quantities $\mathcal{E}_0''$, $\mathcal{E}_0$ are very small, $\Omega' \approx \omega' \approx |\omega_0 - \omega|$. When we analyze only the ratio of amplitudes in Eq. (61), which is approximately equal to $2\mathcal{E}_0''/\mathcal{E}_0$, we find that the values of α_T in Refs. [24–26] correspond to the false EDM moments of $|d| = 3 \times 10^{-29}$, 3×10^{-29}, and 2×10^{-29} e cm, respectively. However, the EDM contribution to P_z grows linearly with time when $\Omega' t \ll 1$, while the tensor-electric-polarizability contribution is negligible in this case. Therefore, keeping the frequency and phase of the coherent longitudinal oscillations almost equal to the frequency and phase of the spin rotations makes it possible to cancel the effect of the tensor electric polarizability in the framework of the deuteron

EDM experiment. The same conclusion was recently drawn in Ref. [27]. Nevertheless, the tensor electric polarizability of deuteron should be taken into account. To check the possible existence of the deuteron EDM, one can also use other possibilities for separating the EDM and Baryshevsky effects listed below.

(i) The spin dynamics caused by first-order interactions (including the EDM effect) and second-order interactions (including the Baryshevsky effect) is defined by the operator equations of spin motion

$$\frac{d\boldsymbol{S}}{dt} = A\boldsymbol{\Omega}\times\boldsymbol{S} \tag{62}$$

and

$$\frac{dS_i}{dt} = \beta_{ijk}S_jS_k, \tag{63}$$

respectively. Therefore, the EDM effect reverses the sign when the beam polarization is reversed, while the sign of the Baryshevsky effect remains unchanged.

(ii) Since both the EDM and Baryshevsky effects depend on the difference $\psi-\varphi$, reversing the beam polarization ($\psi\to\psi+\pi$) is equivalent to the transition to the opposite phase ($\varphi\to\varphi+\pi$). Naturally, such a transition is technically simpler.

If two measurements fulfilled according to point (1) or point (2) give the values P_{z1} and P_{z2} for the BVP, then the EDM and Baryshevsky effects are characterized by the values $(P_{z1}-P_{z2})/2$ and $(P_{z1}+P_{z2})/2$, respectively.

(iii) In the particle rest frame, the EDM and Baryshevsky effects are linear and quadratic in the electric field, respectively. The experimental dependence can be determined by changing the amplitude of the field in the resonators.

(iv) The frequency of BVP caused by the Baryshevsky effect is approximately twice as large as that conditioned by the EDM.

(v) The use of a tensor polarized deuteron beam even at the angular frequency $\omega\approx\omega_0$ cancels the EDM effect and main systematic errors. The evolution of beam polarization is given by

$$P_z^{(\mathrm{tensor})}(t) = -\frac{a_2}{2\omega'}\left\{\frac{\omega_0-\omega}{\omega'}\cos[2(\psi-\varphi)][1 - \cos(2\omega' t)] + \sin[2(\psi-\varphi)]\sin(2\omega' t)\right\}, \tag{64}$$

if the initial polarization is defined by Eq. (47).

Thus, the EDM and Baryshevsky effects can be effectively differentiated and the latter effect can be canceled in the framework of the deuteron EDM experiment.

IX. DISCUSSION AND SUMMARY

The spin dynamics conditioned by the tensor electric polarizability of deuteron has been calculated for the first time in Refs. [1–3]. To compare our results with those obtained in [1–3], it is helpful to introduce the effective field defined by

$$E_{\mathrm{eff}}^2 = \beta^2\gamma B_z^2. \tag{65}$$

In Refs. [1–3], the effect has been described to within first-order terms in β. In this approximation, the squared effective field is equal to

$$E_{\mathrm{eff}}^2 = \left(E_{\mathrm{eff}}^{(0)}\right)^2 + 2B_z^2\beta_0\cdot\Delta\beta_0\cos(\omega t+\varphi) + \tfrac{1}{2}B_z^2(\Delta\beta_0)^2\cos[2(\omega t+\varphi)]. \tag{66}$$

The evolution of the polarization vector is given by Eqs. (24) and (29) in Ref. [2] and Eqs. (44) and (49) in Ref. [3]. The final equation has the form

$$\frac{dP_z}{dt} = -\frac{1}{2}\Delta\Omega_T\cos(2\Omega_f t + 2\varphi_f) \times\left[P_{\rho\phi}(0)\cos(2\Omega t) - \frac{P_{\rho\rho}(0)-P_{\phi\phi}(0)}{2}\sin(2\Omega t)\right], \tag{67}$$

where

$$\Delta\Omega_T = -\tfrac{2}{3}\alpha_T B_z^2(\Delta\beta_0)^2, \tag{68}$$

Ω corresponds to our designation ω_0, and $\varphi_f=\varphi+\pi/2$. The spin rotation is supposed to be clockwise in Refs. [1–3] and counter-clockwise ($\omega_0>0$) in the present work. Therefore, $\Omega_f\approx-\Omega$, and averaging Eq. (67) over time with allowance for Eq. (68) results in

$$\frac{dP_z}{dt} = -\tfrac{1}{12}\alpha_T B_z^2(\Delta\beta_0)^2\{2P_{\rho\phi}(0)\cos(2\varphi) - [P_{\rho\rho}(0)-P_{\phi\phi}(0)]\sin(2\varphi)\}. \tag{69}$$

This equation fully agrees with Eq. (50).

Agreement of the results obtained in Refs. [2,3] and in the present work confirms their validity. The method used in Refs. [2,3] is less convenient for calculating the spin dynamics in oscillatory external fields than in static ones. In the theory of magnetic resonance, the transition to a rotating frame is commonly used [20]. Evidently, the transition to the rotating frame is necessary to determine the spin dynamics with the method developed in Refs. [2,3] when $\omega\neq\omega_0(\Omega\neq|\Omega_f|)$.

The calculated effect of the BVP caused by the tensor electric polarizability of deuteron is an exciting example of new spin physics brought by tensor interactions. In the considered case, the deuteron spin is governed by the electromagnetic interaction. The similar effect investigated in works by Baryshevsky [28,29] is affected by the strong interaction of the deuteron with nuclear matter. These effects stimulated by tensor interactions result in the transformation of tensor polarization into the vector one and the other way round.

Equation (37) shows that the spin-up state is converted into the spin-down state and the other way round. This property is caused by the nondiagonal terms in Hamiltonian (36). As a result, the vertical component of the polarization vector oscillates. This phenomenon is similar to light birefringence in crystals [28,29].

A similar behavior of spin takes place at the NMR when a nucleus is placed into a resonant horizontal magnetic field.

As is well known, the NMR also consists of an oscillation of P_z. However, there is an essential difference between two effects. The Baryshevsky effect exists even when the beam is tensor polarized, while the NMR does not change the beam polarization in this case.

The calculation shows that the Baryshevsky effect can be observed in storage rings. Performing the measurements with the use of resonance $\omega \approx 2\omega_0$ offers an opportunity to measure the deuteron's tensor electric polarizability with the accuracy of $10^{-45} - 10^{-44}$ cm^3 ($10^{-6} - 10^{-5}$ fm^3). It is also possible to use low-energy deuterons in a Penning trap.

The problem of the influence of the tensor electric polarizability on spin dynamics in a deuteron EDM experiment in storage rings has been investigated. The EDM and Baryshevsky effects can be effectively differentiated, and the latter effect can be canceled in the framework of this experiment.

The present work has derived general formulas describing the BVP conditioned by the tensor electric polarizability. The calculated formulas agree with the previous results [1–3] obtained in a more particular case. The method based on the use of the Hamiltonian approach and spin wave functions happens to be very convenient for investigating the effect.

ACKNOWLEDGMENT

The author would like to thank V. G. Baryshevsky for helpful discussions and acknowledge financial support by BRFFR (grant No. Φ06D-002).

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