Deuteron tensor polarization component $T_{20}(Q^2)$ as a crucial test for deuteron wave functions

A. F. Krutov*
Samara State University, RU-443011 Samara, Russia

V. E. Troitsky†
D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, RU-119992, Russia
(Received 17 July 2006; published 10 January 2007)

The deuteron tensor polarization component $T_{20}(Q^2)$ is calculated by the relativistic Hamiltonian dynamics approach. It is shown that in the range of momentum transfers available with current experiments, relativistic effects, meson exchange currents, and the choice of nucleon electromagnetic form factors almost do not influence the value of $T_{20}(Q^2)$. However, this value depends strongly on the actual form of the deuteron wave function, that is, on the model of the NN interaction in the deuteron. So the existing data for $T_{20}(Q^2)$ provide a crucial test for deuteron wave functions.

DOI: 10.1103/PhysRevC.75.014001 PACS number(s): 21.45.+v, 24.10.Jv, 24.70.+s

I. INTRODUCTION

For a long time within the framework of nonrelativistic approaches, deuteron tensor polarization has been considered as an important tool for probing the nucleon-nucleon (NN) interaction at short distances (see, e.g., Refs. [1,2]), making it possible to choose among different model deuteron wave functions, that is, among different models of NN interaction. During the past few years great success has been achieved in polarization experiments on elastic electron-deuteron scattering [3–9]. Now accessible values of Q^2 are so large that the relativistic theory is needed. Unfortunately, further progress in these measurements is questionable because no reasonable technique exists to extend polarization measurements to higher Q^2 [10,11]. So, in the immediate future one does not expect any new experimental information on the subject in question.

In the present work we analyze the existing data on polarization *ed* scattering in connection with different NN interaction models on the basis of an essentially relativistic approach. We show that the existing data provide the crucial test of the deuteron wave function even in the relativistic theory. This possibility is based on the following results obtained in the paper:

(i) The relativistic corrections to $T_{20}(Q^2)$ are small up to $Q^2 \simeq 3\,(\mathrm{GeV}/c)^2$.
(ii) The quantity $T_{20}(Q^2)$ is almost independent of the actual nucleon electromagnetic form factors [although the deuteron structure functions $A(Q^2)$ and $B(Q^2)$ depend on them strongly].
(iii) The contribution of meson exchange currents (MEC) to $T_{20}(Q^2)$ are small.
(iv) The quantity $T_{20}(Q^2)$ depends strongly on the choice of the deuteron wave function, that is, on the model of NN interaction.

We consider the most popular model deuteron wave functions to obtain the best description of polarization data.

The analysis is performed in the framework of the variant of the instant form of the relativistic Hamiltonian dynamics developed by the authors [12–15]. (Relativistic Hamiltonian dynamics is sometimes called Poincaré invariant quantum mechanics (see, e.g., Ref. [16]).) The main features of our approach to deuteron are the following. First, the form of the dynamics is close to the nonrelativistic case. Second, our method of construction of the matrix element of the electroweak current operator makes it possible to formulate relativistic impulse approximation in such a way that the Lorentz covariance of the current is ensured. In our approach it is possible to use the Siegert theorem [17,18] to estimate the contribution of MEC to the deuteron electromagnetic structure. Our estimation of the role of different contributions—nucleon dynamics, relativistic effects, MEC, and nucleon internal structure—demonstrates that one can use the function $T_{20}(Q^2)$ to discriminate different model deuteron wave functions and to choose the most adequate models of nucleon-nucleon interaction. Our calculation shows that the most popular model wave functions [19–21] do not give adequate description of $T_{20}(Q^2)$ and should be rejected in favor of those obtained in the dispersion potential-less inverse scattering approach with no adjustable parameters [22,23] and giving the best description.

The paper is organized as follows. In Sec. II we formulate the problem of obtaining the best deuteron wave function using the data on $T_{20}(Q^2)$. Section III contains a brief review of the instant form of relativistic Hamiltonian dynamics. In Sec. IV the electromagnetic deuteron form factors are calculated. The deuteron tensor polarization $T_{20}(Q^2)$ is given in terms of these form factors; the relativistic effects and the effect of MEC are estimated in Sec. V. Section VI presents the conclusions. In Appendix A and B the equations for relativistic and nonrelativistic free form factors, respectively, for two nucleons in the $^3S_1$-$^3D_1$ channel are given. These form factors enter into Eqs. (12) and (13) of the main text.

*Electronic address: krutov@ssu.samara.ru
†Electronic address: troitsky@theory.sinp.msu.ru

II. WAVE FUNCTIONS FROM DEUTERON EXPERIMENTS

The problem of obtaining the most adequate wave function from deuteron experimental data, in general, can be correctly formulated only in the conventional nonrelativistic nuclear model. In the framework of this model all existing nucleon-nucleon interaction potentials have the correctly fixed long-distance part defined by the one-pion exchange and the intermediate and the short-distance part of strongly pronounced model character, which is different in different models. That is why the deuteron wave functions in coordinate representation for different approaches usually coincide at $r \geqslant 1.5$ fm and differ essentially at $r \leqslant 0.5$ fm. This quite obvious formulation of the problem in the conventional nonrelativistic nuclear model, however, is not valid beyond of the framework of the approach. This takes place, for example, when taking into account relativistic effects, MEC (interaction currents), or different effects in the deuteron electromagnetic structure caused by quark degrees of freedom. These effects are usually strongly model dependent and contain a kind of arbitrariness, so that they "mask" effectively the dependence of observables on the choice of the dynamics of the NN interaction. For example, the experimental data on $T_{20}(Q^2)$ in ed scattering were described well by the relativistic approach of Ref. [24] because of functional arbitrariness in the definition of the nucleon electromagnetic current. Another example is represented by the calculation of the contribution of MEC to deuteron electromagnetic from factors, where in fact an arbitrariness is contained in the $\rho\pi\gamma$ form factor [25].

Now one understands clearly that it is impossible to neglect relativistic effects in the deuteron so that one needs a consistent relativistic formulation of the deuteron problem, particularly at large momentum transfer [11,26]. As was noted in Ref. [11], today there are two main classes of relativistic schemas of the description of deuteron. The first class is based on field-theoretical concepts (following Ref. [11]—propagator dynamics). This class contains the Bethe-Salpeter equation and quasipotential approaches (and the approach of Ref. [24] as well). The second class—relativistic Hamiltonian dynamics (RHD)—is based on the realization of the Poincaré algebra on the set of dynamical observables of a system with a finite number of degrees of freedom. One can find the description of the RHD method in the reviews of Ref. [27] (see also Ref. [28]) and especially the case of the deuteron in the reviews of Refs. [11,29]. As is noted in Ref. [11], the connection between the propagator dynamics and RHD is ambiguous. Each of the approaches has its own advantages as well as difficulties. One should mention in addition the dispersion methods of describing composite systems; these methods deal with, in fact, finite numbers of degrees of freedom, as RHD does [30–33].

In all these relativistic approaches the process of constructing the operator of the Lorentz covariant conserved electromagnetic current is connected to the relativistic nucleon-nucleon dynamics used in the approach. That is why the problem of obtaining the information about the dynamics itself from the deuteron data is not, in general, correctly formulated.

Is it possible in principle to formulate correctly the problem of obtaining the most adequate deuteron wave functions from the deuteron experiments? Our opinion is that it is possible if the following requirements are satisfied:

(i) It is necessary to find an approach that is relativistic from the very beginning with the dynamics close to nonrelativistic Schrödinger dynamics, that is, with relativistic deuteron wave functions that are close to the nonrelativistic ones.
(ii) It is necessary to find such a measurable quantity that, in the chosen approach, is almost independent of relativistic corrections, MEC, and the internal structure of nucleons.

In this paper we propose such a relativistic approach—a variant of the instant form of RHD developed by the authors in Refs. [12–15]. In this approach the adequate observable is the component $T_{20}(Q^2)$ of the deuteron polarization tensor in elastic electron-deuteron scattering.

Let us review briefly the dynamics in our relativistic approach.

III. DYNAMICS OF THE INSTANT FORM OF RELATIVISTIC HAMILTONIAN DYNAMICS

We use the so-called instant form of relativistic Hamiltonian dynamics (IF RHD) [34]. In this form the kinematic subgroup of Poincaré algebra contains the generators of the group of rotations and translations in the three-dimensional Euclidean space (interaction-independent generators):

$$\hat{\vec{J}}\,,\quad \hat{\vec{P}}. \tag{1}$$

The remaining generators of the time translation and Lorentz boosts are (interaction-dependent) Hamiltonians:

$$\hat{P}^0\,,\quad \hat{\vec{N}}. \tag{2}$$

The additive inclusion of interaction into the mass square operator (Bakamjian-Thomas procedure; see, e.g., Ref. [27] for details) presents one of the possible technical ways to include interaction in the algebra of the Poincaré group:

$$\hat{M}_0^2 \to \hat{M}_I^2 = \hat{M}_0^2 + \hat{U}. \tag{3}$$

Here $\hat{M}_0$ is the operator of invariant mass for the free system and $\hat{M}_I$ is that for the system with interaction. The interaction operator $\hat{U}$ must satisfy the following commutation relations:

$$[\hat{\vec{P}},\, \hat{U}] = [\hat{\vec{J}},\, \hat{U}] = [\vec{\nabla}_P,\, \hat{U}] = 0. \tag{4}$$

These constraints (4) ensure that the algebraic relations of the Poincaré group are fulfilled for the interacting system. The relations (4) mean that the interaction potential does not depend on the total momentum of the system nor on the projection of the total angular momentum. The conditions (3) and (4) can be considered only as the model ones. There are other approaches with potential depending on the total momentum, but these are outside the scope of this paper.

In RHD the wave function of the system of interacting particles is the eigenfunction of a complete set of commuting operators. In IF this set is

$$\hat{M}_I^2\,,\quad \hat{J}^2\,,\quad \hat{J}_3\,,\quad \hat{\vec{P}}. \tag{5}$$

$\hat{J}^2$ is the operator of the square of the total angular momentum. In IF the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ coincide with those for the free system. So, in the system (5) only the operator $\hat{M}_I^2$ depends on the interaction.

To find the eigenfunctions of the system (5) one first has to construct the adequate basis in the state space of the composite system. In the case of a two-particle system (e.g., a two-nucleon system) the Hilbert space in RHD is the direct product of two one-particle Hilbert spaces: $\mathcal{H}_{NN} \equiv \mathcal{H}_N \otimes \mathcal{H}_N$.

As a basis in $\mathcal{H}_{NN}$ one can choose the following set of two-particle state vectors, where the motion of the two-particle center of mass is separated and where three operators of the set (5) are diagonal:

$$|\vec{P},\ \sqrt{s},\ J,\ l,\ S,\ m_J\rangle, \tag{6}$$

where $P_\mu = (p_1 + p_2)_\mu$, $p_1^2 = p_2^2 = M^2$, M is nucleon mass, $P_\mu^2 = s$, $\sqrt{s}$ is the invariant mass of the two-particle system, l is the orbital angular momentum in the center-of-mass (c.m.) frame, $\vec{S}^{\,2} = (\vec{S}_1 + \vec{S}_2)^2 = S(S+1)$, S is the total spin in the c.m. frame, J is the total angular momentum with the projection m_J, and the parameters S and l play the role of invariant parameters of degeneracy. Because in the basis (6) the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ in system (5) are diagonal, one needs to diagonalize only the operator $\hat{M}_I^2$ to obtain the system wave functions.

The eigenvalue problem for the operator $\hat{M}_I^2$ in the basis (6) coincides with the nonrelativistic Schrödinger equation within the following difference between corresponding eigenvalues (see, e.g., Refs. [26,27]):

$$\left(\frac{M_d^2}{4\,M} - M\right) - (M_d - 2M) = \frac{(M_d - 2M)^2}{4\,M} = \epsilon_d^2/4M. \tag{7}$$

Here M_d is the deuteron mass and ϵ_d is the deuteron binding energy.

The difference (7) is negligible for most problems. Let us note that if one uses the deuteron bound-state wave number instead of the binding energy, then the equation for $\hat{M}_I^2$ coincides with the nonrelativistic Schrödinger equation exactly.

The corresponding composite-particle wave function has the form

$$\langle \vec{P}', \sqrt{s'}, J', l', S', m_J' \,|\, p_c\rangle = N_C\,\delta(\vec{P}' - \vec{p}_c)\delta_{JJ'}\delta_{m_J m_J'}\varphi_{l'S'}^{J'}(k'), \tag{8}$$

where $|p_c\rangle$ is an eigenvector of the set (5), $J(J+1)$ and m_J are the eigenvalues of $\hat{J}^2$ and $\hat{J}_3$, respectively, and N_C is the normalization constant.

We use the normalization with the relativistic density of states:

$$k^2 dk \to \frac{k^2 dk}{2\sqrt{(k^2 + M^2)}}. \tag{9}$$

This gives the following two-particle wave function of relative motion for equal masses and total angular momentum and total spin fixed:

$$\varphi_{lS}^J(k(s)) = \sqrt[4]{s}\,u_l(k)k, \tag{10}$$

with the normalization condition

$$\sum_l \int u_l^2(k)k^2 dk = 1. \tag{11}$$

Functions $u_l(k)$, $l = 0,\ 2$ coincide with the model nonrelativistic deuteron wave functions within the difference (7). The wave function (10) coincides with that obtained by "minimal relativization" in Ref. [35].

So, in our approach the wave functions in the RHD sense are close to the corresponding nonrelativistic wave functions and the dynamical equation is close to the nonrelativistic Schrödinger equation.

Let us emphasize that our formalism enables one to use any model wave functions obtained as the solution of the Schrödinger equation.

In this paper we consider the following models of NN interaction: the Paris potential [19], versions I, II, and 93 of the Nijmegen model [20], and the charge-dependent version of the Bonn potential [21]. The deuteron wave functions for these potentials give the results for deuteron electromagnetic properties that differ essentially from one another. It is a difficult task to give preference to any one of them. Quite different kind of results are presented for the deuteron wave functions (MT) [22] obtained in potential less approach to the inverse scattering problem (see for the details [23]).

Now let us calculate the deuteron electromagnetic form factors.

IV. DEUTERON ELECTROMAGNETIC FORM FACTORS

The main point of our approach is a construction of the matrix element of the electroweak current operator. In our method the electroweak current matrix element satisfies the relativistic covariance conditions and in the case of electromagnetic current also the conservation law automatically. The properties of the system as well as the approximations are formulated in terms of form factors. The approach makes it possible to formulate a relativistic impulse approximation in such a way that Lorentz covariance of the current is ensured. In the electromagnetic case the current conservation law is also ensured.

Usually it is supposed that MEC must be taken into account to provide gauge invariance and current conservation [27]. However, today constructing the relativistic impulse approximation without breaking the relativistic covariance and current conservation law is a common trend of different approaches [11,13,28,36,37]. In our approach this is realized by making use of the Wigner-Eckart theorem for the Poincaré group. It enables one (for given current matrix element) to separate the reduced matrix elements (form factors), which are invariant under the Poincaré group action. The matrix element of a given operator is represented as a sum of terms, each one of them being a covariant part multiplied by an invariant part. In such a representation the covariant part describes the transformation properties of the matrix element. The conservation law is satisfied explicitly because the vector of the covariant part is orthogonal to the vector Q_μ. All the dynamical information on the transition is contained in the invariant part (form factors). In our variant of the impulse approximation (modified impulse approximation) the reduced matrix elements are calculated with no change of the covariant part (see Ref. [13] for the details) although MEC are neglected. The correct transformation properties are thus guaranteed.

The charge, quadrupole, and magnetic form factors of the deuteron in our approach have the form [14]

$$G_C(Q^2) = \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0C}^{ll'}(s, Q^2, s') \varphi^{l'}(s'),$$
$$G_Q(Q^2) = \frac{2M_{\rm d}^2}{Q^2} \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0Q}^{ll'}(s, Q^2, s') \varphi^{l'}(s'),$$
$$G_M(Q^2) = -M_D \sum_{l,l'} \int d\sqrt{s} d\sqrt{s'} \varphi^l(s) g_{0M}^{ll'}(s, Q^2, s') \varphi^{l'}(s'). \quad (12)$$

Here $g_{0i}^{ll'}(s, Q^2, s')$, $i = C, Q, M$ are the free charge, quadrupole, and magnetic two-particle form factors, that is, the form factors describing electromagnetic properties of the system of proton and neutron without interaction, the system having deuteron quantum numbers $l, l' = 0, 2$ (orbital moments) and wave functions $\varphi^l(s)$ in the sense of RHD.

Free two-particle form factors for a system of two fermions with total momentum 1 (without taking into account of D state) were obtained in Ref. [14]. Corresponding equations for the neutron-proton system with deuteron quantum numbers are given in Appendix A. Free two-particle charge (only) form factors of the proton-neutron system without interaction in the deuteron quantum numbers channel are given also in Ref. [38].

For the deuteron electromagnetic form factors (12) the correspondence principle is valid. The nonrelativistic limit ($M \to \infty$) of Eqs. (12) gives the standard equations for deuteron form factors in the nonrelativistic impulse approximation in terms of wave functions in the momentum representation (see, e.g., Refs. [39,40]):

$$G_C^{\rm NR}(Q^2) = \sum_{l,l'} \int k^2 dk k'^2 dk' u^l(k) \tilde{g}_{0C}^{ll'}(k, Q^2, k') u^{l'}(k'),$$
$$G_Q^{\rm NR}(Q^2) = \frac{2M_D^2}{Q^2} \sum_{l,l'} \int k^2 dk k'^2 dk' u^l(k) \times \tilde{g}_{0Q}^{ll'}(k, Q^2, k') u^{l'}(k'), \quad (13)$$
$$G_M^{\rm NR}(Q^2) = -M_D \sum_{l,l'} \int k^2 dk k'^2 dk' u^l(k) \times \tilde{g}_{0M}^{ll'}(k, Q^2, k') u^{l'}(k').$$

Free charge, quadrupole, and magnetic two-particle form factors $\tilde{g}_{0i}^{ll'}(k, Q^2, k')$, $i = C, Q, M$ can be calculated as nonrelativistic limits of relativistic two-particle form factors given in Appendix A. The explicit forms of the free nonrelativistic two-particle form factors are given in Appendix B.

So, to solve the problem in question we propose the essentially relativistic approach which makes it possible to calculate the deuteron electromagnetic form factors and takes into account the relativistic covariance and the conservation law for the electromagnetic current. The efficiency of our approach was demonstrated in a number of calculations [12–15]. In particular, the values of the neutron charge form factor extracted from the deuteron charge form factor [15] are in good accordance with the values of other authors.

Now let us apply our formalism to polarization *ed* scattering.

V. POLARIZED *ed* SCATTERING

The component $T_{20}(Q^2)$ of the deuteron polarization tensor in elastic *ed* scattering can be written in terms of deuteron form factors (12) in the following form [11]:

$$T_{20}(Q^2) = -\sqrt{2} \frac{Y(Y+2) + X}{1 + 2Y^2 + 4X}, \quad (14)$$

where

$$Y = \frac{2}{3} \eta \frac{G_Q(Q^2)}{G_C(Q^2)}, \quad X = \frac{1}{6} \eta \frac{G_M^2(Q^2)}{G_C^2(Q^2)} f(\theta),$$
$$f(\theta) = 1 + 2(1 + \eta) \tan^2 \frac{\theta}{2}, \quad \eta = \frac{Q^2}{4M_d^2},$$

where θ is the scattering angle in the laboratory frame.

In the range of existing experiments one can neglect X so that Eq. (14) takes the form

$$T_{20}(Q^2) = -\sqrt{2} \frac{Y(Y+2)}{1 + 2Y^2}. \quad (15)$$

To elucidate the role of relativistic effects in $T_{20}(Q^2)$ let us calculate the quantity

$$\Delta(Q^2) = T_{20}^R(Q^2) - T_{20}^{\rm NR}(Q^2). \quad (16)$$

Here $T_{20}^R(Q^2)$ is the relativistic value of $T_{20}(Q^2)$, calculated according to Eqs. (14) and (12), and $T_{20}^{\rm NR}(Q^2)$ is the corresponding nonrelativistic value given by Eqs. (14) and (13).

The dependence of relativistic effects on the choice of the interaction model is shown in Fig. 1. The calculation was made using nucleon form factors [41] and different model wave functions. One can see from Fig. 1 that the relativistic effects are small for $Q^2 \simeq 3$ GeV^2 for all of wave functions. So, in the region available for current experiments for $T_{20}(Q^2)$

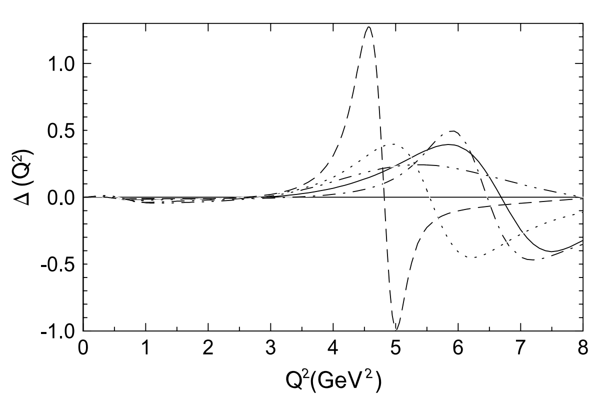

FIG. 1. $\Delta(Q^2)$ calculated following Eq. (16) with nucleon form factors [41] and different wave functions. Solid line—N-II [20], dashed line [22], dotted line [19], dot-dashed line—N-I [20], dashed double-dotted line [21].

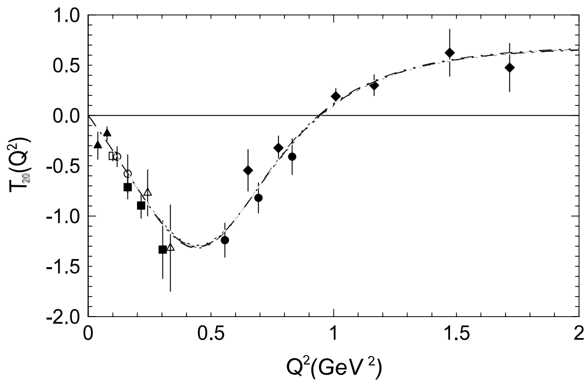

FIG. 2. $T_{20}(Q^2)$ calculated with the MT wave functions [22] and different nucleon form factors: solid line [43], dashed line [41], dotted line [44], dot-dashed line [11], dashed double-dotted line [45]. The lines are almost indistinguishable. Experimental data: open circles [3], open squares [7], open triangles [5], filled circles [6], filled squares [8], filled diamonds [9], filled triangles [4].

the relativistic corrections calculated in our approach are small and almost independent of the model wave functions. At $Q^2 \geqslant 3.5\ \mathrm{GeV}^2$ the corrections become larger and depend upon the model.

Let us discuss the role of the nucleon structure. To estimate this role we have calculated $T_{20}(Q^2)$ for different fits for nucleon form factors. Let us note that one of the fits is that given in Ref. [11], taking into account recent data for the ratio of the charge to magnetic form factors for proton G_E^p/G_M^p, obtained in the JLab experiment (see, e.g., Ref. [42]).

The relativistic $T_{20}(Q^2)$ calculated with the use of different nucleon form factors and with the MT wave functions obtained through the potential-less approach to the inverse scattering problem in Ref. [22] (see also Ref. [23]) is shown in Fig. 2. From Fig. 2, one can see, as one would expect (see, e.g., the discussion in Ref. [11]), that the dependence on the fit for nucleon form factors is weak. Note that this result does not depend on the form of wave functions used in the calculation. So, $T_{20}(Q^2)$ depends weakly on the nucleon structure.

Let us discuss the possible contributions of two-particle MEC to $T_{20}(Q^2)$.

It is accepted generally that one has to take MEC into account in a way compatible with the basic principles of the chosen approach. So, the value of MEC corrections is different for different approaches. We hope that we can neglect MEC in our approach when the relativistic corrections are small. The base for this is given by the following theorem (Siegert [17]; see especially the deuteron case in Ref. [18]). If the electromagnetic current satisfies the conservation law in its differential form and if the dynamics of the two-particle system is of nonrelativistic type then the charge density of the exchange current (the null component) is zero independently of the kind of potential. So, in the energy range where nonrelativistic dynamics is valid (the continuity equation is valid everywhere) the exchange current contributions to the charge and quadrupole form factors are zero. We suppose that when the nonrelativistic dynamics is valid approximately then the MEC contributions to $T_{20}(Q^2)$ are small.

In the experimental range of Q^2 the approximate equation (15) is valid for $T_{20}(Q^2)$, so that this quantity is a function of charge and quadrupole form factors only. This means that the MEC contribution to $T_{20}(Q^2)$ is small.

It is interesting to discuss the compatibility of the Siegert theorem that we use here with the condition of relativistic covariance of the electromagnetic current operator. Let us consider, for example, the commutator

$$[\hat{K}^i, [\hat{K}^i, \hat{\rho}]] = -\hat{\rho}, \tag{17}$$

where $\hat{\rho}$ is the charge density operator and $\hat{K}^i$ is the Lorentz boosts generator. This equation follows from the covariance condition for the electromagnetic current considered as four-vector. In IF RHD the boost generators depend on the interaction, so in the standard impulse approximation, when instead of the current with interaction one uses the free current (i.e., when $\hat{\rho}$ is interaction independent), the implementation of Eq. (17) seems to be impossible. In our variant of the impulse approximation, as was mentioned, it is possible to construct an explicitly covariant current operator without using MEC. In terms of commutators (17) this means that the charge density operator $\hat{\rho}$ depends on the interaction (through the covariant part of the current operator) as the null component of the current. So, in our variant of the impulse approximation the Siegert theorem, that is, the explicit absence of MEC in the charge density operator, does not contradict the condition of current covariance in the form (17).

So, in our approach, the quantity $T_{20}(Q^2)$ depends weakly on relativistic effects, on meson exchange currents, and on nucleon internal structure. This quantity is defined mainly by the choice of the deuteron wave function, so that polarization experiments really could be the test experiments for these wave functions. One can use the experimental data for $T_{20}(Q^2)$ to choose the most adequate deuteron wave functions. In fact, we have made calculations using different model wave functions to compare the predictions with the experiment. Figure 3 presents the results of our calculation of $T_{20}(Q^2)$ with the use of different wave functions [19–22] and nucleon form factors from Ref. [11] as well as the experimental points from the Refs. [3–9].

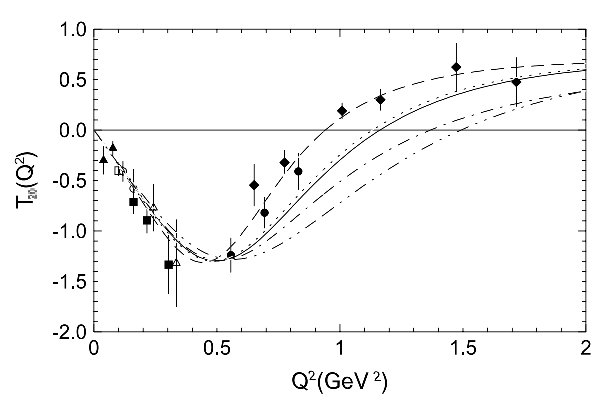

FIG. 3. Experimental data (legend as in Fig. 2) and $T_{20}(Q^2)$ calculated with nucleon form factor [41] and different wave functions (legend as in Fig. 1).

The calculation was made with the use of nucleon form factors obtained in Ref. [41]. One can see that the results are strongly model dependent and the best description of the experimental data is obtained with the wave functions obtained by the potential-less approach to the inverse scattering problem [22]. The results for different models coincide only at $Q^2 \leqslant 0.5$ GeV^2. The recent experimental data [9] unambiguously choose the MT wave functions [22] in comparison with the model wave functions [19–21], which are the most widely used in today's nuclear calculations.

The important feature of MT wave functions is the fact that they are "almost model independent": No form of the NN interaction Hamiltonian is used. However, the MT wave functions are given by the dispersion type integral directly in terms of the experimental scattering phases and the mixing parameter for NN scattering in the $^3S_1$-$^3D_1$ channel. Regge analysis of experimental data on NN scattering was used to describe the phase shifts at large energy.

It is worth noticing that the MT wave functions were obtained by using quite general assumptions about analytical properties of quantum amplitudes such as the validity of the Mandelstam representation for the deuteron electrodisintegration amplitude. This is just the requirement given by analytical properties that select the given wave function from the set of possible phase-equivalent models described in Ref. [46]. These wave functions have no fitting parameters and can be altered only with the improvement of the NN scattering phase analysis. The MT wave functions were used in the nonrelativistic calculation of the deuteron form factors [47] and for the relativistic deuteron structure in Ref. [48].

The construction of these wave functions is closely related to the equations obtained in the framework of the dispersion approach based on the analytic properties of the scattering amplitudes [30–33] (see also Ref. [13] and especially the detailed version in Ref. [49]). In fact, this approach is a kind of dispersion technique using integrals over composite-system masses. Let us note that the MT wave functions have been obtained long in advance for the polarization experiments and contain no parameters to be fitted from deuteron properties.

So, in our approach the problem of determination of the behavior of deuteron wave functions at small distances from polarization experiments is solved. Note that in other approaches with different dynamics a good description of $T_{20}(Q^2)$ also can be achieved. However, in those approaches it seems impossible to separate the contributions to $T_{20}(Q^2)$ of the dynamics itself, of relativistic effects generated by the current operator construction, and of the effects of nuclear structure. This concerns, for example, the light-front RHD calculations [50]. In the approach of Ref. [50] quite different dynamics is used, which gives a 16-component deuteron wave function, and a good description of $T_{20}(Q^2)$ is achieved because of relativistic corrections.

VI. CONCLUSION

In this paper the deuteron tensor polarization $T_{20}(Q^2)$ is calculated through a relativistic Hamiltonian dynamics approach. It is shown that the experimental data for the $T_{20}(Q^2)$ component of deuteron polarization tensor in elastic electron-deuteron scattering up to $Q^2 \approx 2\,(\mathrm{GeV}/c)^2$ can be described in terms of nonrelativistic theory without accounting for relativistic effects and meson exchange currents. These data for $T_{20}(Q^2)$ could be a touchstone for nonrelativistic deuteron wave functions, with the results of calculations depending crucially on the choice of wave functions. It is also shown that the wave functions obtained by the dispersion method of the potential-less inverse scattering problem give the best results for $T_{20}(Q^2)$.

APPENDIX A: RELATIVISTIC FREE TWO-NUCLEON FORM FACTORS IN THE $^3S_1$-$^3D_1$ CHANNEL

The relativistic two-particle form factors of the free (without interaction) np system in the $^3S_1$-$^3D_1$ channel are 2×2 matrices. The elements of three corresponding matrices are given in the following.

The free two-particle charge form factor is

$$\begin{aligned}
g_{0C}^{ll'}(s,Q^2,s') &= R(s,Q^2,s')Q^2\Big[(s+s'+Q^2)\\
&\quad\times\big(G_E^p(Q^2)+G_E^n(Q^2)\big)\,g_{\mathrm{CE}}^{ll'}\\
&\quad+\frac{1}{M}\xi(s,Q^2,s')\big(G_M^p(Q^2)\\
&\quad+G_M^n(Q^2)\big)\,g_{\mathrm{CM}}^{ll'}\Big],\\
g_{\mathrm{CE}}^{00} &= \left(\frac12\cos\omega_1\cos\omega_2+\frac16\sin\omega_1\sin\omega_2\right),\\
g_{\mathrm{CM}}^{00} &= \left(\frac12\cos\omega_1\sin\omega_2-\frac16\sin\omega_1\cos\omega_2\right),\\
g_{\mathrm{CE}}^{02} &= -\frac{1}{6\sqrt2}\left(P'_{22}+2P'_{20}\right)\sin\omega_1\sin\omega_2,\\
g_{\mathrm{CM}}^{02} &= \frac{1}{6\sqrt2}\left(P'_{22}+2P'_{20}\right)\sin\omega_1\cos\omega_2,\\
g_{\mathrm{CE}}^{22} &= \Big[\frac12 L_1\cos\omega_1\cos\omega_2+\frac{1}{24}L_2\sin(\omega_2-\omega_1)\\
&\quad+\frac{1}{12}L_3\sin\omega_1\sin\omega_2\Big],\\
g_{\mathrm{CM}}^{22} &= -\Big[-\frac12 L_1\cos\omega_1\sin\omega_2\\
&\quad+\frac{1}{24}L_2\cos(\omega_2-\omega_1)\\
&\quad+\frac{1}{12}L_3\sin\omega_1\cos\omega_2\Big].
\end{aligned}\tag{A1}$$

The quadrupole two-particle charge form factor is

$$\begin{aligned}
g_{0Q}^{ll'}(s,Q^2,s') &= \frac12 R(s,Q^2,s')Q^2\Big[(s+s'+Q^2)\big(G_E^p(Q^2)\\
&\quad+G_E^n(Q^2)\big)\,g_{\mathrm{QE}}^{ll'}+\frac1M\xi(s,Q^2,s')\\
&\quad\times\big(G_M^p(Q^2)+G_M^n(Q^2)\big)\,g_{\mathrm{QM}}^{ll'}\Big],
\end{aligned}$$

$$
\begin{aligned}
g_{\mathrm{QE}}^{00} &= \sin\omega_1 \sin\omega_2, \quad g_{\mathrm{QM}}^{00} = -\sin\omega_1 \cos\omega_2,\\
g_{\mathrm{QE}}^{02} &= -\frac{3}{2\sqrt{2}}\Big\{2P'_{20}\cos\omega_1\cos\omega_2 - P'_{21}\sin(\omega_1-\omega_2) + \frac{1}{3}\left(4P'_{20}-P'_{22}\right) \times \sin\omega_1\sin\omega_2\Big\},\\
g_{\mathrm{QM}}^{02} &= \frac{3}{2\sqrt{2}}\Big\{-2P'_{20}\cos\omega_1\sin\omega_2 + P'_{21}\cos(\omega_1-\omega_2) + \frac{1}{3}\left(4P'_{20}-P'_{22}\right) \times \sin\omega_1\cos\omega_2\Big\},\\
g_{\mathrm{QE}}^{22} &= \frac{3}{2}\Big\{L_4\cos\omega_1\cos\omega_2 - \frac{1}{12}L_5\sin(\omega_1-\omega_2) + \frac{1}{6}L_6\sin\omega_1\sin\omega_2\Big\},\\
g_{\mathrm{QM}}^{22} &= -\frac{3}{2}\Big\{-L_4\cos\omega_1\sin\omega_2 + \frac{1}{12}L_5\cos(\omega_1-\omega_2) + \frac{1}{6}L_6\sin\omega_1\cos\omega_2\Big\}.
\end{aligned}
\tag{A2}
$$

The magnetic two-particle charge form factor is

$$
\begin{aligned}
g_{0M}^{ll'}(s,Q^2,s') &= -R(s,Q^2,s')\big[\xi(s,Q^2,s')\big(G_E^p(Q^2) + G_E^n(Q^2)\big)g_{\mathrm{ME}}^{ll'} + \big(G_M^p(Q^2) + G_M^n(Q^2)\big)g_{\mathrm{MM}}^{ll'}\big],\\
g_{\mathrm{ME}}^{00} &= \sin(\omega_1-\omega_2),\\
g_{\mathrm{MM}}^{00} &= \frac{1}{2M}\Big\{\Big[\gamma_1 - \frac{1}{2}(\gamma_3(s,Q^2,s') + \gamma_3(s',Q^2,s))\Big]\cos\omega_1\cos\omega_2 + \frac{1}{4}\left(\gamma_2(s,Q^2,s')+\gamma_2(s',Q^2,s)\right) \times \cos\omega_1\sin\omega_2 + \frac{1}{2}\gamma_1\sin\omega_1\sin\omega_2\Big\},\\
g_{\mathrm{ME}}^{02} &= -\frac{1}{4\sqrt{2}}(P'_{22}+2P'_{20})\sin(\omega_1-\omega_2),\\
g_{\mathrm{MM}}^{02} &= \frac{1}{8\sqrt{2}M}\Big\{-\Big[2P'_{20}\gamma_1 + P'_{21}\gamma_2 + (P'_{22}-2P'_{20})\gamma_3\Big]\cos\omega_1\cos\omega_2\\
&\quad + \Big[P'_{21}\gamma_1 + \frac{1}{2}(P'_{22}-2P'_{20})\gamma_2 - 2P'_{21}\gamma_3\Big] \times \cos\omega_1\sin\omega_2\\
&\quad + \Big[P'_{21}\gamma_1 - \frac{1}{2}(P'_{22}+6P'_{20})\gamma_2 - 2P'_{21}\gamma_3\Big] \times \sin\omega_1\cos\omega_2\\
&\quad + [2P'_{20}\gamma_1 + P'_{21}\gamma_2 - (P'_{22}+6P'_{20})\gamma_3] \times \sin\omega_1\sin\omega_2\Big\},\\
g_{\mathrm{ME}}^{22} &= -\frac{1}{4}\Big\{\frac{1}{2}L_2\cos(\omega_1-\omega_2) + L_3\sin(\omega_1-\omega_2)\Big\},\\
g_{\mathrm{MM}}^{22} &= \frac{1}{8M}\Big\{\Big[-L_7\gamma_1 - \frac{1}{8}L_8(\gamma_2(s,Q^2,s') - \gamma_2(s',Q^2,s)) + \frac{1}{2}L_9(\gamma_3(s,Q^2,s) + \gamma_3(s',Q^2,s))\Big]\cos\omega_1\cos\omega_2\\
&\quad + \frac{1}{4}[(L_{10}(s,Q^2,s') + L_{10}(s',Q^2,s))\gamma_1 - L_9(\gamma_2(s,Q^2,s') + \gamma_2(s',Q^2,s)) - L_8(\gamma_3(s,Q^2,s') - \gamma_3(s',Q^2,s))]\\
&\quad \times \cos\omega_1\sin\omega_2 + \frac{1}{4}[8L_{11}\gamma_1 + L_{12} \times (\gamma_2(s,Q^2,s') - \gamma_2(s',Q^2,s)) + L_{13}(\gamma_3(s,Q^2,s') + \gamma_3(s',Q^2,s))]\\
&\quad \times \sin\omega_1\cos\omega_2 + \frac{1}{2}\Big[(L_{14}(s,Q^2,s') + L_{14}(s',Q^2,s))\gamma_1 - \frac{1}{4}L_{13}(\gamma_2(s,Q^2,s') + \gamma_2(s',Q^2,s)) + L_{12}(\gamma_3(s,Q^2,s') - \gamma_3(s',Q^2,s))\Big]\\
&\quad \times \sin\omega_1\sin\omega_2\Big\}.
\end{aligned}
\tag{A3}
$$

The following equation is valid for form factors:

$$g_{0i}^{ll'}(s,Q^2,s') = g_{0i}^{l'l}(s',Q^2,s), \quad i = C, Q, M.$$

The previous equations use the following notation:

$$
\begin{aligned}
R(s,Q^2,s') &= \frac{(s+s'+Q^2)}{\sqrt{(s-4M^2)(s'-4M^2)}}\frac{\vartheta(s,Q^2,s')}{[\lambda(s,-Q^2,s')]^{3/2}} \times \frac{1}{\sqrt{1+Q^2/4M^2}},\\
\xi(s,Q^2,s') &= \sqrt{ss'Q^2 - M^2\lambda(s,-Q^2,s')},\\
L_1 &= L_1(s,Q^2,s') = P_{20}P'_{20} + \frac{1}{3}P_{21}P'_{21} + \frac{1}{12}P_{22}P'_{22},\\
L_2 &= L_2(s,Q^2,s') = P_{21}(P'_{22}-6P'_{20}) - P'_{21}(P_{22}-6P_{20}),\\
L_3 &= L_3(s,Q^2,s') = 2P_{21}P'_{21} + 4P_{20}P'_{20} - P_{20}P'_{22} - P_{22}P'_{20},\\
L_4 &= L_4(s,Q^2,s') = P_{20}P'_{20} + \frac{1}{6}P_{21}P'_{21} - \frac{1}{12}P_{22}P'_{22},\\
L_5 &= L_5(s,Q^2,s') = P'_{21}(P_{22}+6P_{20}) - P_{21}(P'_{22}+6P'_{20}),\\
L_6 &= L_6(s,Q^2,s') = 8P_{20}P'_{20} + P_{21}P'_{21} + P_{20}P'_{22} + P_{22}P'_{20},\\
L_7 &= L_7(s,Q^2,s') = P_{21}P'_{21} + 4P_{20}P'_{20},\\
L_8 &= L_8(s,Q^2,s') = P_{21}(P'_{22}+2P'_{20}) + P'_{21}(P_{22}+2P_{20}),
\end{aligned}
$$

$$L_9 = L_9(s, Q^2, s') = P_{20}P'_{22} + P_{22}P'_{20} + 4P_{20}P'_{20},$$
$$L_{10} = L_{10}(s, Q^2, s') = P_{22}P'_{21} + 4P_{21}P'_{20} - 2P_{20}P'_{21},$$
$$L_{11} = L_{11}(s, Q^2, s') = P'_{21}P_{20} - P'_{20}P_{21},$$
$$L_{12} = L_{12}(s, Q^2, s') = P_{20}P'_{22} - P_{22}P'_{20},$$
$$L_{13} = L_{13}(s, Q^2, s') = P_{21}(P'_{22} + 2P'_{20}) - P'_{21}(P_{22} + 2P_{20}),$$
$$L_{14} = L_{14}(s, Q^2, s') = P_{22}P'_{20} - P_{21}P'_{21} - 2P_{20}P'_{20},$$
$$\gamma_1 = \gamma_1(s, Q^2, s') = (s + Q^2 + s')Q^2,$$
$$\gamma_2 = \gamma_2(s, Q^2, s') = \xi(s, Q^2, s') \times \frac{(s - s' + Q^2)(\sqrt{s'} + 2M) + (s' - s + Q^2)\sqrt{s'}}{\sqrt{s'}(\sqrt{s'} + 2M)},$$
$$\gamma_3 = \gamma_3(s, Q^2, s') = \frac{\xi^2(s, Q^2, s')}{\sqrt{s'}(\sqrt{s'} + 2M)};$$

ω_1 and ω_2 are the Wigner rotation parameters,

$$\omega_1 = \arctan \frac{\xi(s, Q^2, s')}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})}, \tag{A4}$$
$$\omega_2 = \arctan \frac{\alpha(s, s')\xi(s, Q^2, s')}{M(s + s' + Q^2)\alpha(s, s') + \sqrt{ss'}(4M^2 + Q^2)},$$

where $\alpha(s, s') = 2M + \sqrt{s} + \sqrt{s'}$; $P_{2i} = P_{2i}(z)$, $P'_{2i} = P_{2i}(z')$ are the adjoint Legendre functions given by

$$P_{20}(z) = \tfrac{1}{2}(3z^2 - 1), \quad P_{21}(z) = 3z\sqrt{1 - z^2}, \quad P_{22}(z) = 3(1 - z^2); \tag{A5}$$

z, z' are the arguments of the Legendre functions,

$$z = z(s, Q^2, s') = \frac{\sqrt{s}(s' - s - Q^2)}{\sqrt{\lambda(s, -Q^2, s')(s - 4M^2)}},$$
$$z' = z'(s, Q^2, s') = -z(s', Q^2, s);$$

and $\vartheta(s, Q^2, s') = \theta(s' - s_1) - \theta(s' - s_2)$, with θ the step function and

$$s_{1,2} = 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2) \mp \frac{1}{2M^2}\sqrt{Q^2(Q^2 + 4M^2)s(s - 4M^2)}.$$

The functions $s_{1,2}(s, Q^2)$ give the kinematically available region in the plane (s, s'). They are obtained in Ref. [13]. $G^{p,n}_{E,M}(Q^2)$ are Sachs form factors for the proton and neutron.

APPENDIX B: NONRELATIVISTIC FREE TWO-NUCLEON FORM FACTORS IN THE $^3S_1$-$^3D_1$ CHANNEL

The nonrelativistic charge two-particle free form factor is

$$\tilde g^{ll'}_{0C}(k, Q^2, k') = g(k, Q^2, k')\left(G^p_E(Q^2) + G^n_E(Q^2)\right)\tilde g^{ll'}_{\rm CE},$$
$$\tilde g^{00}_{\rm CE} = 1, \quad \tilde g^{02}_{\rm CE} = \tilde g^{20}_{\rm CE} = 0, \tag{B1}$$
$$\tilde g^{22}_{\rm CE} = \tilde P_{20}\tilde P'_{20} + \tfrac{1}{3}\tilde P_{21}\tilde P'_{21} + \tfrac{1}{12}\tilde P_{22}\tilde P'_{22}.$$

The nonrelativistic quadrupole two-particle free form factor is

$$\tilde g^{ll'}_{0Q}(k, Q^2, k') = \tfrac{3}{2}g(k, Q^2, k')\left(G^p_E(Q^2) + G^n_E(Q^2)\right)\tilde g^{ll'}_{\rm QE},$$
$$\tilde g^{00}_{\rm QE} = 0, \quad \tilde g^{02}_{\rm QE} = -\sqrt{2}\tilde P'_{20}, \quad \tilde g^{20}_{\rm QE} = -\sqrt{2}\tilde P_{20},$$
$$\tilde g^{22}_{\rm QE} = \tilde P_{20}\tilde P'_{20} + \tfrac{1}{6}\tilde P_{21}\tilde P'_{21} - \tfrac{1}{12}\tilde P_{22}\tilde P'_{22}. \tag{B2}$$

The nonrelativistic magnetic two-particle free form factor is

$$\tilde g^{ll'}_{0M}(k, Q^2, k') = -\frac{1}{4\sqrt{2}M}g(k, Q^2, k') \times \left[\left(G^p_E(Q^2) + G^n_E(Q^2)\right)\tilde g^{ll'}_{\rm ME} + \left(G^p_M(Q^2) + G^n_M(Q^2)\right)\tilde g^{ll'}_{\rm MM}\right],$$
$$\tilde g^{00}_{\rm ME} = \tilde g^{02}_{\rm ME} = \tilde g^{20}_{\rm ME} = 0, \tag{B3}$$
$$\tilde g^{00}_{\rm MM} = 4\sqrt{2}, \quad g^{02}_{\rm MM} = -2\tilde P'_{20}, \quad g^{20}_{\rm MM} = -2\tilde P_{20},$$
$$\tilde g^{22}_{\rm ME} = \gamma[\tilde P_{21}(\tilde P'_{22} - 6\tilde P'_{20}) - \tilde P'_{21}(\tilde P_{22} - 6\tilde P_{20})]$$
$$g^{22}_{\rm MM} = -\sqrt{2}[\tilde P_{21}\tilde P'_{21} + 4\tilde P'_{20}\tilde P_{20}].$$

Here

$$g(k, Q^2, k') = \frac{1}{kk'Q}\left[\theta\left(k' - \left|k - \frac{Q}{2}\right|\right) - \theta\left(k' - k - \frac{Q}{2}\right)\right];$$

$\tilde P_{2i} = P_{2i}(y)$, $\tilde P'_{2i} = P_{2i}(y')$ are the adjoint Legendre functions (A5); and y, y' are the arguments of the Legendre functions,

$$y = y(k, Q^2, k') = -\frac{4(k^2 - k'^2) + Q^2}{4kQ},$$
$$y' = y'(k, Q^2, k') = -y(k', Q^2, k),$$
$$\gamma = -\frac{1}{2}\left(\frac{2k'^2(1 - y'^2)}{Q^2}\right)^{1/2} = -\frac{1}{2}\left(\frac{2k^2(1 - y^2)}{Q^2}\right)^{1/2}.$$

[1] J. S. Levinger, Acta. Phys. **33**, 135 (1973); T. J. Brady, E. L. Tomusiak, and J. S. Levinger, Bull. Am. Phys. Soc. **17**, 438 (1972).
[2] M. J. Moravcsik and P. Ghosh, Phys. Rev. Lett. **32**, 321 (1974).
[3] M. E. Schulze *et al.*, Phys. Rev. Lett. **52**, 597 (1984).
[4] V. F. Dmitriev *et al.*, Phys. Lett. **B157**, 143 (1985).
[5] R. Gilman *et al.*, Phys. Rev. Lett. **65**, 1733 (1990).
[6] I. The *et al.*, Phys. Rev. Lett. **67**, 173 (1991).
[7] M. Ferro-Luzzi *et al.*, Phys. Rev. Lett. **77**, 2630 (1996).
[8] M. Bouwhuis *et al.*, Phys. Rev. Lett. **82**, 3755 (1999).
[9] D. Abbott *et al.*, Phys. Rev. Lett. **84**, 5053 (2000).
[10] R. Gilman, Nucl. Phys. **A737**, 156 (2004).
[11] R. Gilman and F. Gross, J. Phys. G **28**, R37 (2002).
[12] E. V. Balandina, A. F. Krutov, and V. E. Troitsky, J. Phys. G **22**, 1585 (1996); A. F. Krutov and V. E. Troitsky, J. High Energy Phys. 10 (1999) 028; A. F. Krutov, O. I. Shro, and V. E. Troitsky, Phys. Lett. **B502**, 140 (2001); A. F. Krutov and V. E. Troitsky, Teor. Mat. Fiz. **143**, 258 (2005) [Theor. Math. Phys. **143**, 704 (2005)] .
[13] A. F. Krutov and V. E. Troitsky, Phys. Rev. C **65**, 045501 (2002).

[14] A. F. Krutov and V. E. Troitsky, Phys. Rev. C **68**, 018501 (2003).
[15] A. F. Krutov and V. E. Troitsky, Eur. Phys. J. A **16**, 285 (2003).
[16] B. D. Keister and W. N. Polyzou, Phys. Rev. C **73**, 014005 (2006).
[17] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).
[18] H. Baier, Fortschr. Phys. **27**, 209 (1979).
[19] M. Lacomb, B. Loiseau, R. Vinh Mau, J. Coté, P. Pirés, and R. de Tourreil, Phys. Lett. **B101**, 139 (1981).
[20] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
[21] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[22] V. M. Muzafarov and V. E. Troitsky, Yad. Fiz. **33**, 1461 (1981) [Sov. J. Nucl. Phys. **33**, 783 (1981)].
[23] V. E. Troitsky, in *Proceedings of Quantum Inversion Theory and Applications*, Germany, 1993, edited by H. V. von Geramb, Lecture Notes in Physics **467** (Springer-Verlag, Berlin, 1994), p. 50.
[24] J. W. Van Orden, N. Devine, and F. Gross, Phys. Rev. Lett. **75**, 4369 (1995).
[25] M. Gari and H. Hyuga, Nucl. Phys. **A264**, 409 (1976).
[26] P. L. Chung, F. Coester, B. D. Keister, and W. N. Polyzou, Phys. Rev. C **37**, 2000 (1988).
[27] B. D. Keister and W. Polyzou, Adv. Nucl. Phys. **20**, 225 (1991).
[28] W. H. Klink, Phys. Rev. C **58**, 3587 (1998).
[29] M. Garcon and J. W. Van Orden, Adv. Nucl. Phys. **26**, 293 (2001).
[30] V. E. Troitsky and Yu. M. Shirokov, Theor. Math. Fiz. **1**, 213 (1969).
[31] A. I. Kirillov, V. E. Troitsky, S. V. Trubnikov, and Yu. M. Shirokov, Fiz. Elem. Chastits At. Yadra **6**, 3 (1975) [Sov. J. Part. Nuclei **6**, 3 (1975)].
[32] V. V. Anisovich, M. N. Kobrinsky, D. I. Melikhov, and A. V. Sarantsev, Nucl. Phys. **A544**, 747 (1992).
[33] V. Anisovich, D. Melikhov, and V. Nikonov, Phys. Rev. D **52**, 5295 (1995).
[34] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[35] L. L. Frankfurt, I. L. Grach, L. A. Kondratyuk, and M. I. Strikman, Phys. Rev. Lett. **62**, 387 (1989).
[36] F. M. Lev, E. Pace, and G. Salmé, Phys. Rev. C **62**, 064004 (2000).
[37] T. W. Allen, W. H. Klink, and W. N. Polyzou, Phys. Rev. C **63**, 034002 (2001).
[38] A. V. Afanasev, V. D. Afanas’ev, and S. V. Trubnikov, Preprint TJNAF. JLAB-THY-98-01 (Newport News, 1998).
[39] G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).
[40] L. Mathelitsch and H. F. K. Zingl, Nuovo Cimento **44**, 81 (1978).
[41] S. Galster, H. Klein, J. Moritz, K. H. Schmidt, and D. Wegener, Nucl. Phys. **B32**, 221 (1971).
[42] M. Jones *et al.*, Phys. Rev. Lett. **84**, 1398 (2000).
[43] M. Gari and W. Krümpelman, Z. Phys. A **322**, 689 (1985).
[44] P. Mergell, U.-G. Meissner, and D. Drechsel, Nucl. Phys. **A596**, 367 (1996).
[45] E. L. Lomon, Phys. Rev. C **64**, 035204 (2001).
[46] H. Ekstein, Phys. Rev., **117**, 1590 (1960).
[47] I. I. Belyantsev, V. K. Mitryushkin, P. K. Rashidov, and S. V. Trubnikov, J. Phys. G **9**, 871 (1983).
[48] R. G. Arnold *et al.*, Phys. Rev. Lett. **58**, 1723 (1987).
[49] A. F. Krutov and V. E. Troitsky, hep-ph/0101327.
[50] J. Carbonell and V. A. Karmanov, Eur. Phys. J. A **6**, 9 (1999).