PHYSICAL REVIEW C **74**, 065804 (2006)

Conversion of a neutron star to a strange star: A two-step process

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(Received 31 August 2006; revised manuscript received 28 October 2006; published 21 December 2006)

The conversion of a neutron star to a strange star is studied. Such a transition may be viewed as a two-step process in which the hadronic matter first gets converted to two-flavor quark matter, which, in turn, converts to strange quark matter in the second step of the process. Relativistic hydrodynamical equations are employed to obtain the velocity of propagation of the first conversion front. The second transition front, arising from the conversion of two-flavor to three-flavor quark matter, is studied by using an appropriate weak interaction rate. The propagation velocity of the first conversion front initially shoots up near the core of the star to eventually saturate to some ultrarelativistic value. The first conversion takes about a millisecond, during which the second conversion front is likely to be generated. The second process takes about a hundred seconds to convert the whole quark star into a strange star.

DOI: 10.1103/PhysRevC.74.065804 PACS number(s): 26.60.+c

I. INTRODUCTION

It has been conjectured that strange quark matter, consisting of almost equal numbers of u, d, and s quarks, may be the true ground state of strongly interacting matter [1,2] at high density and/or temperature. This conjecture is supported by bag model calculations [3] for a certain range of values for the strange quark mass and the strong coupling constant. By considering realistic values for the strange quark mass (150–200 MeV) [4], it may be shown that the strangeness fraction in chemically equilibrated quark matter is close to unity for large baryon densities. Such bulk quark matter would be referred to as "strange quark matter" (SQM) in what follows.

This hypothesis may lead to important consequences both for laboratory experiments as well as for astrophysical observations. Normal nuclear matter at high enough density and/or temperature would be unstable against conversion to two-flavor quark matter. The two-flavor quark matter would be metastable and would eventually decay to SQM in a weak interaction time scale, releasing a finite amount of energy in the process. Such a two-step conversion process may take place in the interior of a neutron star where the densities can be as high as (8–10)ρ_0, with ρ_0 being the nuclear matter density at saturation [5,6]. If Witten's conjecture [1] is correct, the whole neutron star may then convert to a strange star with a significant fraction of strange quarks in it. (Neutron star may also become a hybrid star with a core of SQM in case the entire star is not converted to a strange star. Such a hybrid star would have a mixed-phase region consisting of both quark matter and hadronic matter [7].) Hadron to quark phase transition inside a compact star may also yield observable signatures in the form of quasi-periodic oscillations (QPOs) and γ-ray bursts [8,9].

There are several ways in which conversion may be triggered at the center of the star. A few possible mechanisms for the production of SQM in a neutron star have been discussed by Alcock, Farhi, and Olinto [10]. The conversion from hadron matter to quark matter is expected to start as the star comes in contact with a seed of an external strange quark nugget. Such a seed would then grow by "eating up" baryons in the hadronic matter during its travel to the center of the star, thus converting the neutron star either to a strange star or a hybrid star. Another mechanism for the initiation of the conversion process was given by Glendenning [7], who suggested that a sudden spin down of the star may increase the density at its core, thereby triggering the conversion process spontaneously.

Conversion of neutron matter to strange matter has been studied by several authors. Olinto [11] viewed the conversion process to proceed via weak interactions as a propagating slow-combustion (i.e., a deflagration) front and derived the velocity of such a front. Olesen and Madsen [12] and Heiselberg, Baym, and Pethick [13] estimated the speed of such a conversion front to range between 10 m/s to 100 km/s. The combustive conversion front was assumed to have a microscopic width of a few tens to a few hundreds of femtometers in these calculations.

Collins and Perry [14], however, assumed that the hadronic matter gets converted first to a two-flavor quark matter, which eventually decays to a three-flavor strange matter through weak interactions. Lugones, Benvenuto, and Vucetich [15] argued that the hadron to SQM conversion process may rather proceed as a detonation than as a deflagration even in the case of strangeness production occurring through seeding mechanisms [10].

Horvath and Benvenuto [16] examined the hydrodynamic stability of the combustive conversion in a nonrelativistic

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framework. These authors inferred that a convective instability may increase the velocity of the deflagration front, so that a transition from slow combustion to detonation may occur. They argued that such a detonation may as well be responsible for the type II supernova explosions [17]. In a relativistic framework, Cho, Ng, and Speliotopoulos [18] examined the conservation of the energy-momentum and the baryonic density flux across the conversion front. Using Bethe-Johnson [19] and Fermi-Dirac neutron gas [20] equations of state (EOSs) for the nuclear matter and the bag model for SQM, they found that the conversion process was never a detonation but a slow combustion only for some special cases. Recently, Tokareva *et al.* [21] modeled the hadron to SQM conversion process as a single-step process. They argued that the mode of conversion would vary with the temperature of the SQM and with the value of the bag constant in the bag model EOS. Berezhiani *et al.* [22], Bombaci, Parenti, and Vidana [23], and Drago, Lavagno, and Pagliara [24], however, suggest that the formation of SQM may be delayed if the deconfinement process takes place through a first-order transition [25] so that the purely hadronic star can spend some time as a metastable object.

In this paper, we model the conversion of nuclear matter to SQM in a neutron star as occurring through a two-step process. Deconfinement of nuclear matter to a two- (up and down) flavor quark matter takes place in the first step in a strong interaction time scale. The second step concerns the generation of strange quarks from the excess of down quarks via a weak process. We may add here that this is the first instance where a realistic nuclear matter EOS is used to study the nuclear matter to SQM conversion as a two-step process. Previously, Drago, Lavagno and Pagliara [24] studied the burning of nuclear matter directly to SQM in detail by using the conservation conditions and the compact star models.

To study the conversion of nuclear matter to a two-flavor quark matter, we here consider relativistic EOSs describing the forms of the matter in their respective phases. Along with such EOSs, we would also consider hydrodynamical equations depicting various conservation conditions to examine such conversion processes in a compact neutron star. Development of the conversion front, as it propagates radially through the model star, would be examined. We would next study the conversion of two-flavor quark matter to a three-flavor SQM through a nonleptonic weak interaction process by assuming β equilibrium for the SQM. The paper is organized as follows. In Sec. II, we discuss the EOSs used for the present work. In Sec. III, we discuss the conversion to two-flavor quark matter. Conversion to three-flavor SQM is discussed in Sec. IV. In Sec. V, we summaries the results and also present conclusions that may be drawn from these results regarding the actual conversion process that may take place in a neutron star.

II. THE EQUATION OF STATE

The nuclear matter EOS has been evaluated using the nonlinear Walecka model [26]. The Lagrangian density in this model is given by

$$\begin{aligned}\mathcal{L} = & \sum_i \bar{\psi}_i \big(i\gamma^\mu \partial_\mu - m_i + g_{\sigma i}\sigma + g_{\omega i}\omega_\mu\gamma^\mu \\ & - g_{\rho i}\rho^a_\mu \gamma^\mu T_a \big)\psi_i - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\rho^a_{\mu\nu}\rho^{\mu\nu}_a + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^\mu_a \\ & - \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 - \frac{1}{4}c(g_{\sigma N}\sigma)^4 \\ & + \bar{\psi}_e(i\gamma^\mu\partial_\mu - m_e)\psi_e. \end{aligned} \tag{1}$$

The Lagrangian in Eq. (1) includes nucleons (neutrons and protons), electrons, and isoscalar scalar, isoscalar vector, and isovector vector mesons denoted by ψ_i, ψ_e, σ, ω^μ, and $\rho^{a,\mu}$, respectively. The Lagrangian also includes cubic and quartic self-interaction terms of the σ field. The parameters of the nonlinear Walecka model are meson-baryon coupling constants, meson masses, and the coefficient of the cubic and quartic self-interaction of the σ mesons (b and c, respectively). The meson fields interact with the baryons through linear coupling. The ω and ρ meson masses have been chosen to be their physical masses. The rest of the parameters, namely, the nucleon-meson coupling constants ($\frac{g_\sigma}{m_\sigma}$, $\frac{g_\omega}{m_\omega}$, and $\frac{g_\rho}{m_\rho}$) and the coefficients of cubic and quartic terms of the σ meson self-interaction (b and c, respectively) are determined by fitting the nuclear matter saturation properties, namely, the binding energy/nucleon (-16 MeV), baryon density ($\rho_0 = 0.17$ fm^{-3}), symmetry energy coefficient (32.5 MeV), Landau mass ($0.83\,m_n$), and nuclear matter incompressibility (300 MeV).

In the present paper, we first consider the conversion of nuclear matter, consisting of only nucleons (i.e., without hyperons) to a two-flavor quark matter. The final composition of the quark matter is determined from the nuclear matter EOS by enforcing baryon number conservation during the conversion process. That is, for every neutron, one up and two down quarks are produced and, for every proton, one down and two up quarks are produced, with electron number being same in the two phases. While describing the state of matter for the quark phase we consider a range of values for the bag constant. The nuclear matter EOS is calculated at zero temperature, whereas, the two-flavor quark matter EOS is obtained both at zero temperature as well as at finite temperatures.

III. CONVERSION TO TWO-FLAVOR MATTER

In this section we discuss the conversion of neutron proton (n-p) matter to two-flavor quark matter, consisting of u and d quarks along with electrons for the sake of ensuring charge neutrality. We heuristically assume the existence of a combustive phase transition front. Using the macroscopic conservation conditions, we examine the range of densities for which such a combustion front exists. We next study the outward propagation of this front through the model star by using the hydrodynamic (i.e., Euler) equation of motion and the equation of continuity for the energy density flux [27]. In this study, we consider a nonrotating, spherically symmetric neutron star. The geometry of the problem effectively reduces to a one-dimensional geometry for which radial distance from the center of the model star is the only independent spatial variable of interest.

Let us now consider the physical situation where a combustion front has been generated in the core of the neutron star. This front propagates outward through the neutron star with a certain hydrodynamic velocity, leaving behind a *u-d-e* matter. In the following, we denote all the physical quantities in the hadronic sector by subscript 1 and those in the quark sector by subscript 2.

The condition for the existence of a combustion front is given by [28]

$$\epsilon_2(p, X) < \epsilon_1(p, X), \tag{2}$$

where p is the pressure and $X = (\epsilon + p)/n_B^2$, with n_B being the baryon density. Quantities on opposite sides of the front are related through the energy density, the momentum density, and the baryon number density flux conservation. In the rest frame of the combustion front, these conservation conditions can be written as [21,27,29]

$$\omega_1 v_1^2 \gamma_1^2 + p_1 = \omega_2 v_2^2 \gamma_2^2 + p_2, \tag{3}$$

$$\omega_1 v_1 \gamma_1^2 = \omega_2 v_2 \gamma_2^2, \tag{4}$$

and

$$n_1 v_1 \gamma_1 = n_2 v_2 \gamma_2. \tag{5}$$

In these three conditions v_i $(i = 1, 2)$ is the velocity, p_i is the pressure, $\gamma_i = 1/\sqrt{1 - v_i^2}$ is the Lorentz factor, $\omega_i = \epsilon_i + p_i$ is the specific enthalpy, and ϵ_i is the energy density of the respective phases.

Besides the conservation conditions given in Eqs. (2)–(5), the condition of entropy increase across the front puts an additional constraint on the possibility of the existence of the combustion front. This entropy condition is given by [30]

$$s_1 v_1 \gamma_1 \leqslant s_2 v_2 \gamma_2, \tag{6}$$

with s_i being the entropy density.

The velocities of the matter in the two phases, given by Eqs. (3)–(5), are written as [27]

$$v_1^2 = \frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)} \tag{7}$$

and

$$v_2^2 = \frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}. \tag{8}$$

It is possible to classify the various conversion mechanisms by comparing the velocities of the respective phases with the corresponding velocities of sound, denoted by c_{si}, in these

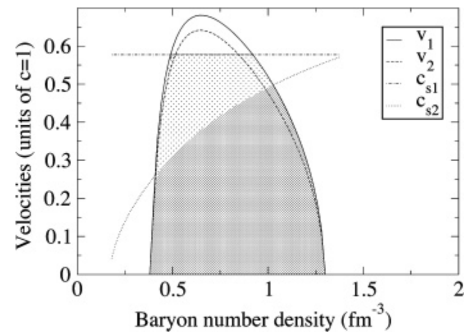

FIG. 1. Variation of different velocities with baryon number density for $T = 0$ MeV, $B^{1/4} = 160$ MeV, and strange quark mass $m_s = 200$ MeV. The dark-shaded region corresponds to deflagration, the light-shaded region corresponds to detonation, and the unshaded region corresponds to supersonic conversion processes.

phases. These conditions are [31]

$$\begin{aligned}
\text{strong detonation}: v_1 &> c_{s1}, & v_2 &< c_{s2},\\
\text{Jouget detonation}: v_1 &> c_{s1}, & v_2 &= c_{s2},\\
\text{supersonic or weak detonation}: v_1 &> c_{s1}, & v_2 &> c_{s2},\\
\text{strong deflagration}: v_1 &< c_{s1}, & v_2 &> c_{s2},\\
\text{Jouget deflagration}: v_1 &< c_{s1}, & v_2 &= c_{s2},\\
\text{weak deflagration}: v_1 &< c_{s1}, & v_2 &< c_{s2}.
\end{aligned}$$

For the conversion to be physically possible, velocities should satisfy an additional condition, namely, $0 \leqslant v_i^2 \leqslant 1$. We here find that the velocity condition, along with the inequality (2), puts a severe constraint on the allowed equations of state.

To examine the nature of the hydrodynamical front arising from the neutron to two-flavor quark matter conversion, we plot, in Fig. 1, the quantities v_1, v_2, c_{s1}, and c_{s2} as functions of the baryon number density (n_B). As mentioned earlier, the u and d quark content in the quark phase is kept the same as that corresponding to the quark content of the nucleons in the hadronic phase. With these fixed densities of the massless u and d quarks and electrons, the EOS of the two-flavor matter has been evaluated using the bag model prescription. We find that both the energy condition [Eq. (2)] and velocity condition ($v_i^2 > 0$) are satisfied only for a small window of $\approx \pm 5.0$ MeV around the bag pressure $B^{1/4} = 160$ MeV. The constraint imposed by these conditions results in the possibility of deflagration, detonation, or a supersonic front as shown in Figs. 1 and 2.

In Fig. 1, we considered both the phases to be at zero temperature. The possibility, however, exists that part of the internal energy can be converted to heat energy, thereby increasing the temperature of the two-flavor quark matter during the exothermic combustive conversion process. Instead of following the prescription for the estimation of temperature as given in Refs. [17,24], we study the changes in the properties of combustion with the temperature of the newly formed two-flavor quark phase in the present paper. In Fig. 2, we plot the variation of velocities with density, with the temperature

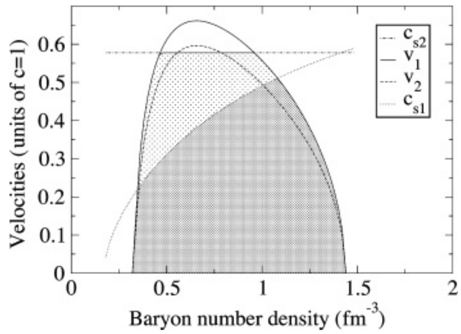

FIG. 2. Variation of velocities with baryon number density for $T = 50$ MeV, $B^{1/4} = 160$ MeV, and strange quark mass $m_s = 200$ MeV. Here the temperature refers to the two-flavor quark phase only; the temperature of the nuclear matter is zero. Different regions correspond to the different modes of conversion as in Fig. 1.

of the two-flavor quark matter being 50 MeV. This figure show that the range of values of baryon density, for which the flow velocities are physical, increases with temperature. Figure 3 shows the variation of velocities with temperature for values of baryon number densities given by $n_B \approx 3\rho_0$ and $7\rho_0$, respectively. In this figure, the difference between velocities v_1 and v_2 increases with temperature of the two-flavor quark matter. In the present paper we have considered only the zero-temperature nuclear matter EOS. However, the two-flavor quark matter has a finite temperature because of the effect of the combustion front.

The preceding discussion is mainly a feasibility study for the possible generation of the combustive phase transition front and its mode of propagation. Having explored such possibilities, we now study the evolution of the hydrodynamical combustion front with position as well as time. This might give us some insight regarding the actual conversion of a hadronic star to a quark star and the time scale involved in such a process. To examine such an evolution, we move to a reference frame in which the nuclear matter is at rest. The speed of the combustion front in such a frame is given by $v_f = -v_1$, with v_1 being the velocity of the nuclear matter in the rest frame of the front.

In the present work, we use the special relativistic formalism to study the evolution of the combustion front as it moves outward in the radial direction inside the model neutron star.

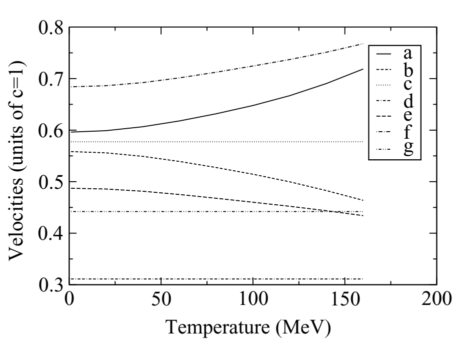

FIG. 3. Variation of velocities with temperature: (a) v_1 for density $3\rho_0$, (b) v_2 for density $3\rho_0$, (c) c_{s1}, (d) c_{s2} for $3\rho_0$, (e) v_2 for $7\rho_0$, (f) v_1 for $7\rho_0$, and (g) c_{s2} for $7\rho_0$.

The relevant equations are the equation of continuity and the Euler's equation, which are given by [21]

$$\frac{1}{\omega}\left(\frac{\partial \epsilon}{\partial \tau} + v\frac{\partial \epsilon}{\partial r}\right) + \frac{1}{W^2}\left(\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial \tau}\right) + 2\frac{v}{r} = 0 \tag{9}$$

and

$$\frac{1}{\omega}\left(\frac{\partial p}{\partial r} + v\frac{\partial p}{\partial \tau}\right) + \frac{1}{W^2}\left(\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r}\right) = 0, \tag{10}$$

where $v = \frac{\partial r}{\partial \tau}$ is the front velocity in the nuclear matter rest frame, $k = \frac{\partial p}{\partial \epsilon}$ is taken as the square of the effective sound speed in the medium, and $W = 1/\gamma_i$ is the inverse of Lorentz factor.

Substituting these expressions for v and k in Eqs. (9) and (10) we get

$$\frac{2v}{\omega}\frac{\partial \epsilon}{\partial r} + \frac{1}{W^2}\frac{\partial v}{\partial r}(1+v^2) + \frac{2v}{r} = 0 \tag{11}$$

and

$$\frac{n}{\omega}\frac{\partial \epsilon}{\partial r}(1+v^2) + \frac{2v}{W^2}\frac{\partial v}{\partial r} = 0. \tag{12}$$

Equations (11) and (12) ultimately yield a single differential equation, which is written as

$$\frac{dv}{dr} = \frac{2vkW^2(1+v^2)}{r[4v^2 - k(1+v^2)^2]}. \tag{13}$$

Equation (13) is integrated, with respect to $r(t)$, starting from the center toward the surface of the star. The nuclear and quark matter EOSs have been used to construct the static configuration of the compact star, for different central densities, by using the standard Tolman-Oppenheimer-Volkoff equations [32]. The velocity at the center of the star should be zero from symmetry considerations. However, the $1/r$ dependence of the $\frac{dv}{dr}$ in Eq. (13) suggests a steep rise in velocity near the center of the star.

Our calculation proceeds as follows. We first construct the density profile of the star for a fixed central density. Equations (7) and (8) then specify the respective flow velocities v_1 and v_2 of the nuclear and quark matter in the rest frame of the front, at a radius infinitesimally close to the center of the star. This would give us the initial velocity of the front ($-v_1$), at that radius, in the nuclear matter rest frame. We next start with Eq. (13) from a point infinitesimally close to the center of the star and integrate it outward along the radius of the star. The solution gives us the variation of the velocity with the position as a function of time of arrival of the front, along the radius of the star. Using this velocity profile, we can calculate the time required to convert the whole star using the relation $v = dr/d\tau$.

In Fig. 4, we show the variation of the velocity for central baryon density values of $3\rho_0$, $4.5\rho_0$, and $7\rho_0$, respectively. The respective initial velocities corresponding to such central densities are taken to be 0.66, 0.65, and 0.47. The figure shows that the velocity of the front, for all the central densities, shoots up near the center and then saturates at a certain velocity for

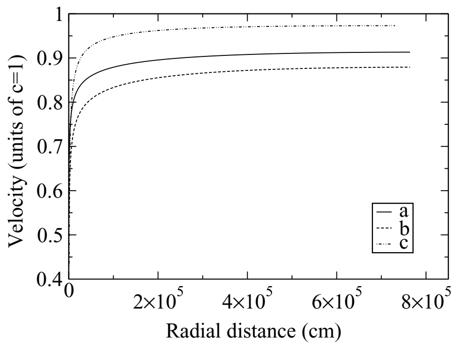

FIG. 4. Variation of velocity of the conversion front with radius of the star, with temperature $T = 0$ MeV for three different values of the central densities, namely, (a) $4.5\rho_0$, (b) $3\rho_0$, and (c) $7\rho_0$, respectively. Here ρ_0 is the nuclear density. The initial velocities for the three cases are 0.66, 0.65, and 0.47, respectively.

higher radius. Such a velocity behavior near the central point is apparent from Eq. (13). The numerically obtained saturation velocity varies from 0.92 for central baryon density $3\rho_0$ to 0.98 for $7\rho_0$. The existence of a saturation velocity, at large r, is apparent from the asymptotic behavior of Eq. (13). A comparison with Fig. 1 shows that, for densities of $3\rho_0$ and $4.5\rho_0$, the conversion starts as weak detonation and stays in the same mode throughout the star. In contrast, for $7\rho_0$, the initial detonation front changes over to weak detonation and the velocity of the front becomes almost 1 as it reaches the outer crust. The corresponding time taken by the combustion front to propagate inside the star is plotted against the radius in Fig. 5. The time taken by the front to travel the full length of the star is of the order of a few milliseconds. According to the present model, the initial neutron star thus becomes a two-flavor quark star in about 10^{-3} s. The results discussed here correspond to the case in which both nuclear and quark matter are at zero temperature. For finite-temperature quark matter, results vary only by a few percent of the front velocities for the quark matter at zero temperature.

We should mention that the equations governing the conversion of nuclear to quark matter presented so far are purely hydrodynamic. There is no dissipative process, so

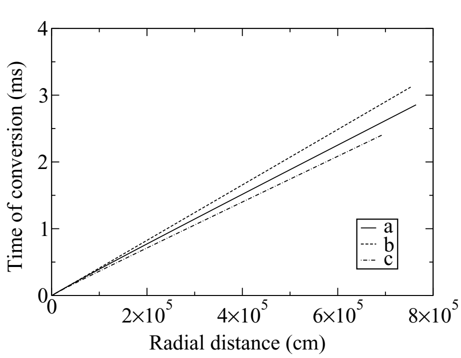

FIG. 5. Variation of the time of arrival of the conversion front at a certain radial distance inside the star as a function of that radial distance from the center of the star for three different central densities. Values of the central density and temperature corresponding to the curves (a), (b), and (c) are the same as shown in Fig. 4.

that the combustion front continues to move with a finite velocity depending on the density profile. Furthermore, there is no reaction rate involved here as the deconfinement process occurs in a strong interaction time scale and hence can be taken to be instantaneous at any particular position of the front inside the star. It should also be noted that two-flavor matter is not stable relative to nuclear matter at zero pressure. But in the absence of any dissipative process, in the present study, the front continues to move radially outward and converts the whole star. The crust may play a role because of its lattice structure; we shall briefly discuss this aspect in the concluding section.

The conversion to two-flavor matter, as described here, is certainly very different from the second-step process, to be discussed in the next section, where the two-flavor matter converts to three-flavor matter. Here, the governing rate equations are weak interaction rates, which play a decisive role in the conversion. By comparing the total time ($\equiv 10^{-3}$ s) taken by the combustion front to travel through the star with the weak interaction time scale (10^{-7}–10^{-8} s), it is evident that the second step may start before the end of the first-step process. In that case, perhaps, one should ideally consider two fronts, separated by a finite distance, moving inside the star. In the present paper, we have taken a much simplified picture and considered the conversion of a chemically equilibrated two-flavor to three-flavor quark matter as the second-step process. Our results may provide us with more information regarding the necessity of considering two fronts.

IV. THE CONVERSION TO THREE-FLAVOR SQM

In this section we discuss the conversion of two-flavor quark matter to three-flavor SQM in a compact star. Similar to the discussion in the previous section, we assume the existence of a conversion front at the core of the star that propagates radially outward leaving behind the SQM as the combustion product. This conversion is governed by weak interactions that take place inside the star.

For a three-flavor quark matter, the charge neutrality and the baryon number conservation conditions yield

$$3n_B = n_u + n_d + n_s, \tag{14}$$

$$2n_u = n_d + n_s + 3n_{e^-}, \tag{15}$$

where n_i is the number density of particle i ($i = u, d, s$, and e^-).

The weak reactions that govern the conversion of excess down quarks to strange quarks can be written as

$$\begin{aligned} d &\rightarrow u + e^- + \nu_{e-}; \\ s &\rightarrow u + e^- + \nu_{e-}; \\ d + u &\rightarrow s + u. \end{aligned} \tag{16}$$

We assume that the neutrinos escape freely from the site of reaction and that the temperature of the star remains constant. The nonleptonic weak interaction in such a case becomes the governing rate equation. The semileptonic weak decays, then, are solely responsible for the chemical equilibration, which can be incorporated through the following relations:

$$\mu_{e^-} = \mu_d - \mu_u; \qquad \mu_d = \mu_s, \tag{17}$$

where μ_i is the chemical potential of the ith particle. The number densities (n_i) of the quarks and electrons are related to their respective chemical potentials by

$$n_i = g_i \int_0^\infty d^3p/(2\pi)^3[f^+ - f^-], \tag{18}$$

where $f^+$and f^- are given by

$$\begin{aligned} f^+ &= \frac{1}{\exp[(E_p - \mu)/T] + 1}, \\ f^- &= \frac{1}{\exp[(E_p + \mu)/T] + 1}. \end{aligned} \tag{19}$$

In Eqs. (18) and (19), g_i is the degeneracy factor and T the temperature. Equations (14)–(19) can be solved self-consistently to calculate the number densities of quarks and electrons.

The conversion to SQM starts at the center ($r = 0$) of the two-flavor star. Assuming that the reaction region is much smaller than the size of the star, we have considered the front to be one dimensional. Moreover, as we are considering spherical static stars only, there is no angular dependence. The combustion front, therefore, moves radially toward the surface of the star. As the front moves outward, excess d quarks get converted to s quarks through the nonleptonic weak process. The procedure employed in the present work is somewhat similar to that of Ref. [11], although the physical boundary conditions are different.

We now define a quantity

$$a(r) = [n_d(r) - n_s(r)]/2n_B \tag{20}$$

such that $a(r = 0) = a_0$ at the core of the star. The quantity a_0 is the number density of the strange quark at the center for which the SQM is stable, and its value lies between 0 and 1. For equal numbers of d and s quarks, $a(r) = 0$. Ideally, at the center of the star a_0 should be zero for strange quark mass $m_s = 0$. Since the s quark has a mass $m_s \sim$ 150–200 MeV, at the center of the star a_0 would be a small finite number, depending on the EOS. The s quark density fraction, however, decreases along with the decrease of the baryon density toward the surface of the star, so that $a(r \to R) \to 1$, with R being the radius of the star. At any point along the radius, say $r = r_1$, the initial $a(r_1)$, before the arrival of the front, is decided by the initial two-flavor quark matter EOS. The final $a(r_1)$, after the conversion, is obtained from the equilibrium SQM EOS at the density corresponding to r_1.

The conversion to SQM occurs via decay of the down quark to the strange quark ($u + d \to s + u$) and the diffusion of the strange quark across the front [11]. The corresponding rate of change of $a(r)$ with time is governed by following two equations:

$$\frac{da}{dt} = -R(a) \tag{21}$$

and

$$\frac{da}{dt} = D\frac{d^2a}{dr^2}, \tag{22}$$

In Eq. (21) $R(a)$ is the rate of conversion of d to s quarks. Equation (22) yields the rate of change of $a(r)$ owing to diffusion of s quarks, with D being the diffusion constant. Following Olinto [11], assuming the one-dimensional steady-state solution, and using Eqs. (21) and (22) we get

$$\mathrm{Da}'' - va' - R(a) = 0, \tag{23}$$

where v is the velocity of the fluid. In Eq. (23) $a' = \frac{da}{dr}$.

Conservation of baryon number flux at any position yields $n_q v_q = n_s v_s$. The subscripts q and s denote the two-flavor quark matter phase and SQM phase, respectively. The baryon flux conservation condition yields the initial boundary condition at any point r along the radius of the star:

$$a'(r) = -\frac{v}{D}[a_i(r) - a_f(r)], \tag{24}$$

where $a_i(r)$ and $a_f(r)$ are the values for the $a(r)$ before and after the combustion, respectively.

The reaction rate for the nonleptonic weak interaction $u + d \to u + s$ is in general a five-dimensional integral for nonzero temperature and m_s [33,34]. Here, instead, we have taken the zero-temperature, small-a limit [11]

$$R(a) \approx \frac{16}{15\pi} G_F^2 \cos^2\theta_c \sin^2\theta_c \mu_u^5 \left(\frac{a}{3}\right)^3, \tag{25}$$

where G_F is the weak coupling constant and θ_c is the Cabibbo angle. This equation can be written in the following form:

$$R(a) \approx \frac{a^3}{\tau}, \tag{26}$$

where

$$\tau = \frac{16}{27 \times 15\pi} G_F^2 \cos^2\theta_c \sin^2\theta_c \mu_u^5$$

depends on the position of the front.

Following the line of arguments given in Ref. [11], we write down the analytic expressions for D and v as

$$D = \frac{\lambda \bar{v}}{3} \approx 10^{-3} \left(\frac{\mu}{T}\right)^2 \mathrm{cm}^2/\mathrm{s}, \tag{27}$$

and

$$v = \sqrt{\frac{D}{\tau} \frac{a_f(r)^4}{2[a_i(r) - a_f(r)]}}. \tag{28}$$

Our calculation proceeds as follows. First, we get the star characteristics for a fixed central baryon density ρ_c. For a given ρ_c, number densities of u, d, and s quarks, in both the two-

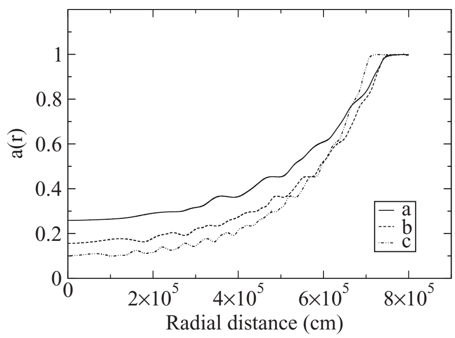

FIG. 6. Variation of *a(r)* (as in the text) with radius of the star for different central densities, for which the central density is (a) $3\rho_0$, (b) $4.5\rho_0$, and (c) $7\rho_0$. Here $B^{1/4} = 160$ MeV, $T = 2$ MeV, the up and down quark mass $m_u = m_d = 0$ MeV, and the strange quark mass $m_s = 200$ MeV.

and three-flavor sectors, are known at any point inside the star. That means $a_i(r)$ and $a_f(r)$ are fixed. Equations (25)–(28) are then used to get the diffusion constant and hence the radial velocity of the front.

The central baryon densities considered here are the same as those of Sec. III. Assuming that the neutrinos leave the star, the temperature is kept constant at some small temperature, $T = 2$ MeV, so that we can use Eq. (25), evaluated in the zero-temperature limit. The variation of $a(r)$ with the radius of the star is given in Fig. 6. The plot shows that $a(r)$ increases radially outward, which corresponds to the fact that, as density decreases radially, the number of excess down quarks that are being converted to strange quarks by the weak interaction also decreases. Hence, it takes less time to reach a stable configuration and hence the front moves faster, as shown in Figs. 7 and 8.

In Fig. 7, we have plotted the variation of velocity along the radius of the star. The velocity shows an increase as it reaches sufficiently low density and then drops to zero near the surface as the $d \rightarrow s$ conversion rate becomes zero. Figure 8 shows the variation of time taken to reach a stable configuration at different radial positions of the star. The total time needed for the conversion of the star, for different central densities, is of the order of 100 s, as can be seen from the figure.

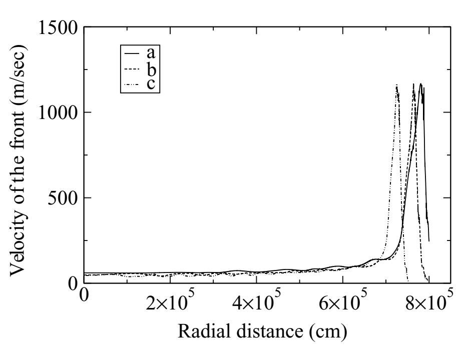

FIG. 7. Variation of velocity of the two- to three-flavor quark conversion front with radius of the star for different central densities, for which the central density is (a) $3\rho_0$, (b) $4.5\rho_0$, and (c) $7\rho_0$; other parameters are the same as in Fig. 6.

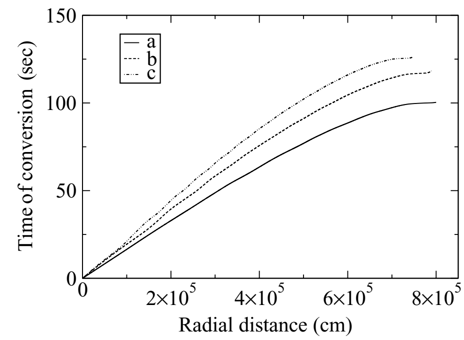

FIG. 8. Variation of time taken for the two- to three-flavor quark conversion front with radius of the star for different central densities, for which the central density is (a) $3\rho_0$, (b) $4.5\rho_0$, and (c) $7\rho_0$; other parameters are the same as in Fig. 7.

V. SUMMARY AND DISCUSSION

We have studied the conversion of a neutron star to a strange star. This conversion takes place in two stages. In the first stage a detonation wave is developed in the hadronic matter (containing neutrons, protons, and electrons). We have described this hadronic matter with a relativistic model. For such an EOS the density profile of the star is obtained by solving the Tolman-Openheimer-Volkoff equations. The corresponding quark matter EOS is obtained by using the bag model. However, this quark matter EOS is not equilibrated and contains two flavors. Matter velocities in the two media, as measured in the rest frame of the front, have been obtained by using conservation conditions. These velocities have been compared with the sound velocity in both phases.

For a particular density inside the star, flow velocities of the matter on the two sides of the front are now fixed. Starting from a point infinitesimally close to the center, hydrodynamic equations are solved radially outward. The solution of the hydrodynamic equations gives the velocity profiles for different central densities. The velocity of the front shoots up very near the core and then saturates at a value close to 1. The mode of combustion is found to be weak detonation for lower central densities. For higher central densities, the initial detonation becomes a weak detonation as the the front moves radially outward inside the star. This result is different from that of Ref. [24], where the conversion process always corresponds to a strong deflagration. The time required for the conversion of the neutron star to a two-flavor quark star is found to be of the order of few milliseconds. After this front passes through, leaving behind a two-flavor matter, a second front is generated. This second front converts the two-flavor matter via weak interaction processes. The velocity of the front varies along the radius of the star. As the front moves out from the core to the crust, its velocity increases, implying faster conversion. The time for the second conversion to take place comes out to be ∼100 s. This is comparable to the time scale obtained in Ref. [11].

The comparison of the time of conversion from neutron star to two-flavor quark star and the weak interaction time scale suggests that, at some time during the passage of the first combustion front, the burning of two-flavor matter to strange matter should start. This means that, at some point of time, there should be two fronts moving inside the star. However, our results show that, inside the model star, the burning of the nuclear matter to two-flavor quark matter takes much less time than the conversion from two-flavor quark matter to SQM. Nonetheless, the consideration of two fronts might provide us with some more information regarding the conversion of a neutron star to a final stable strange star. In the present case we have considered a two-step process, there being only one type of front, inside the star, at any instant of time. Here we would also like to mention that ideally the second step should start with nonequilibrated two-flavor matter [33,35]. Since this is a numerically involved calculation, in the present case we have taken the simplified picture of equilibrated quark matter.

We have mentioned earlier that, although at zero pressure two-flavor matter is not stable, in the present study, the absence of a dissipative process allows the front to move up to the surface, converting the whole star. In a more realistic scenario, the front should get stalled near the crust as the crust of the star is at subnucleonic densities. We plan to investigate these features in our future work. However, if we assume that the first-step conversion stops as the front reaches the crust, then depending on the size of the crust (1–0.2 km for central densities in the range $7\rho_0$–$3\rho_0$) the time scale for the second-step process is reduced by only 2–3%.

Finally, the burning of nuclear matter to two-flavor quark matter is studied using special relativistic hydrodynamic equations. The actual calculation should involve general relativity, taking into account the curvature of the front for the spherical star. Furthermore, the conversion of two- to three-flavor matter should ideally involve hydrodynamical processes. We propose to explore all these detailed features in our subsequent papers.

ACKNOWLEDGMENTS

R.M. would like to thank CSIR, Government of India, for financial support. S.K.G., S.R., and P.S.J. thank DST, Govenrment of India, for support under the IRHPA scheme.

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