

**Electromagnetic decay of  $\Lambda(1520)$** 

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The electromagnetic decay processes of excited hyperon states are a very sensitive probe of the structure of hyperons. We will argue that the recent measurements of electromagnetic decay rates indicate that the wave functions of the hyperon ground states should contain sizable components of excited quark states (configuration mixing). Flavor-SU(3) is a broken symmetry and it appears that the hyperon wave functions should preferably be written in a  $uds$ -basis, where only the light  $u$  and  $d$ -quarks are symmetrized, and not in the usual SU(6)-basis.

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**I. INTRODUCTION**

The electromagnetic transition rates of excited baryons to their respective ground states offer a stringent test on the quark model dynamics, since electromagnetic transitions provide a relatively clean probe of the wave functions of the initial and final baryon states. It is therefore highly desirable to measure the electromagnetic decay rates from excited baryon states in order to refine the quark model description of the baryons. To date very few electromagnetic transition rates have been measured for the excited baryon resonances. For a detailed discussion of the experimental and theoretical status of the excited baryons and their electromagnetic decays, see the review by Landsberg [1]. The radiative decays of excited hyperon states have very small branching ratios, and the experimental determination of the  $\Lambda(1520)$  radiative decay rate is quite an accomplishment. One therefore naturally expects these difficult measurements of the radiative transition rates to have relatively large experimental uncertainties. Fortunately, despite the large uncertainties, these rates can be very decisive in discriminating among the different quark model predictions. Recently the CLAS Collaboration at Jefferson Lab published a new measurement of the radiative decay widths of the excited hyperons  $\Sigma^0(1385)$  and  $\Lambda(1520)$  [2]. The measured decay rate for  $\Lambda(1520) \rightarrow \gamma \Lambda(1116)$ ,  $\Gamma_\gamma = 167 \pm 43^{+26}_{-12}$  keV [2], confirms the recent result of Antipov *et al.* [3] and the rate measured a long time ago by Mast *et al.* [4]. These three experiments establish the decay rate to be  $\Gamma_\gamma > 100$  keV, which is in contradiction with  $\Gamma_\gamma < 50$  keV found by Bertini *et al.* [5,6]. As will be argued, despite the large errors, the measured rate gives valuable clues regarding the structure of the hyperon wave functions.

In this short note we concentrate the discussion on the measured  $\Lambda(1520) \rightarrow \gamma \Lambda(1116)$  decay rate  $\Gamma_\gamma$ . In the theoretical evaluation of the electromagnetic transition rates the usual assumption is that the excited hyperon states are three-quark states. It has been argued by many that the lowest lying excited hyperon states, e.g.,  $\Lambda(1405)$ , could contain a dominant  $\bar{K}N$  component (a “ $\bar{q}q^4$ ”-admixture) due to the nearby  $\bar{K}N$  threshold, see, e.g., Ref. [7]. A  $\bar{q}q^4$ -admixture in the excited hyperon wave function would modify the predicted

electromagnetic transition rates. In the discussion below we will not consider this complication but we will comment on this aspect at the end of this paper. We will assume that  $\Lambda(1520)$  is predominantly a three-quark state. The experimental value of  $\Gamma_\gamma$  [2–4] is consistent with the predictions of a particular nonrelativistic quark model (NRQM) and a relativized version of the NRQM [8–10]. The measured rate differs markedly from, for example, the various bag-model evaluations of  $\Gamma_\gamma$  [10–12]. As will be argued, there are two principal reasons why all bag models fail to explain the large observed  $\Gamma_\gamma$  value. Below we will also discuss the reason why some potential model evaluations of  $\Gamma_\gamma$  give too small a value for this rate, and comment on some recent model calculations of  $\Gamma_\gamma$  [13–15]. We will show that the experiments [2–4] give a clear indication that the ground state  $\Lambda(1116)$  contains configuration mixing from excited quark-states.

**II. THE HYPERON WAVE FUNCTIONS**

In order to discuss the electromagnetic transition  $\Lambda(1520) \rightarrow \gamma \Lambda(1116)$ , we first present the wave functions of these two hyperon states. The ground state,  $\Lambda(1116)$ , is often assumed to be the simple SU(6) spin-flavor spatially-symmetric state,

$$|\Lambda(1116)\rangle = |8, {}^2S_S\rangle, \quad (1)$$

i.e., the three quarks are all in the lowest energy  $s$ -state forming a spatially symmetric ( $S$ ), flavor SU(3)-octet (**8**), spin doublet state.

The NRQM of Isgur and Karl [16,17] generated a  $\Lambda(1116)$  three-quark ground state wave function in the  $uds$ -basis instead of the common SU(6)-basis. The rationale is that the heavy  $s$ -quark mass breaks flavor SU(3) symmetry ( $m_u = m_d \neq m_s$ ). In the  $uds$ -basis only the equal mass  $u$  and  $d$  quarks are symmetrized. Isgur and Karl assumed a spin-independent harmonic oscillator (HO) confining potential, and the two HO frequencies associated with the two Jacobi coordinates,  $\rho$  and  $\lambda$ , are different since  $m_s > m_d = m_u$ , see also Ref. [18]. The spatial wave function for the  $s$ -quark, which is associated with the  $\lambda$  coordinate, is therefore not the same as the  $u - d$  wave function characterized by the  $\rho$  coordinate. Furthermore, Isgur and Karl showed that in the NRQM the

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effective color hyperfine interaction, where they included only a local spin-spin and a tensor interaction among pairs of quarks, couples the quark ground state to higher energy HO-states,  $S'$  and  $D$ , and modifies the  $\Lambda(1116)$  state, Eq. (1). This generates a more refined description of the  $\Lambda(1116)$  state wave function and these extra wave function components give important contributions to the value of  $\Gamma_\gamma$ . The modified  $\Lambda(1116)$  wave function is (see, e.g., Ref. [19]):

$$|\Lambda(1116)'\rangle = 0.93|\mathbf{8},^2S_S\rangle - 0.30|\mathbf{8},^2S'_S\rangle - 0.20|\mathbf{8},^2S_M\rangle \\ - 0.05|\mathbf{1},^2S_M\rangle - 0.03|\mathbf{8},^4D_M\rangle. \quad (2)$$

Here  $\mathbf{8}$  denotes a flavor-octet state, the notation  $^{(2S+1)}L_\sigma$  specifies the spin-multiplet  $2S + 1$  and the total orbital angular momentum  $L$  in a spatial symmetry-state  $\sigma$  with total angular momentum and parity  $J^P = |\vec{L} + \vec{S}|^P = 1/2^+$ . The symmetries of the spatial wave function is either a symmetric,  $\sigma = S$ , or a mixed symmetry wave function,  $\sigma = M$ . Thus, according to NRQM of Isgur and Karl,  $|\Lambda(1116)'\rangle$  is predominantly a flavor-octet,  $\mathbf{8}$ , spatially symmetric state with smaller admixtures from the excited HO ( $L = 0$ )  $S'$  and ( $L = 2$ )  $D$ -states. The configuration admixtures in Eq. (2) affect strongly the calculated rate  $\Gamma_\gamma$  as we will discuss.

The first excited hyperon three-quark states,  $\Lambda^*$ , have one quark in a  $p$ -state and the two others in the lowest-energy  $s$ -state. When the spatial quark wave functions are Gaussians, e.g., Ref. [17], the separation of the center-of-mass (c.m.) coordinate and the baryon's internal Jacobi coordinates appears trivially. In the HO model of Ref. [17] this internal wave function of the three-quark excited hyperon describes a pair of quarks in relative  $s$ -state with the third quark "orbiting" the two in a  $p$ -state. As explicitly shown by Isgur and Karl [17], the usual SU(6)-states for excited baryons are only recovered in the limit of equal quark masses, i.e.,  $m_s \rightarrow m_d = m_u$ .<sup>1</sup> The usual assumption is that the negative parity  $\Lambda^*$  states belong to the **70**-plet of the spin-flavor SU(6) symmetry where the wave functions of the  $\Lambda^*$  states have mixed spatial symmetry and contain substantial SU(3)-multiplet mixing. In the SU(6)-basis, the three-quark  $\Lambda^*$  states combine the flavor-singlet  $\mathbf{1}$  state with a spin-doublet,  $2S + 1 = 2$ , state whereas the flavor octet  $\mathbf{8}$  can be in either a spin doublet or a spin-quartet,  $2S + 1 = 4$ , state. This assumption leads to the following generic wave function for the  $\Lambda(1520)$   $J^P = 3/2^-$  state:

$$|\Lambda^*(J^P)\rangle = a|^2\mathbf{1}_J\rangle + b|^4\mathbf{8}_J\rangle + c|^2\mathbf{8}_J\rangle. \quad (3)$$

As discussed below, the different quark models result in different values for the coefficients  $a$ ,  $b$ , and  $c$ . A few comments regarding the exclusion of the SU(6) **56**-plet are necessary for the discussion below. Historically, the spatially symmetric wave function of two quarks in  $s$ -states and one in a  $p$ -state is considered to be the translational mode of the

simple baryon ground state, e.g.,  $\Lambda(1116)$  Eq. (1), where all three quarks are in the lowest  $s$ -state, see, e.g., Ref. [20]. The spatially symmetric three-quark wave function belongs to the SU(6) **56**-plet, and one therefore assumes that the negative parity  $\Lambda^*$  states are described by the spatially mixed-symmetry three-quark wave functions of the **70**-plet. The assumption that the excited negative-parity baryons states belong to the SU(6) **70**-plet is adopted in many quark models. However, the argument that the **56**-plet describes the translation of the ground state has to be reconsidered if the ground state baryons contain other spatial configurations, see e.g. Eq. (2). Using the MIT-bag model Rebbi [21] investigated the argument that the **56**-plet could describe the c.m. motion of a baryon and, despite the fact that in the static cavity approximation the bag is not a momentum eigenstate, Rebbi found that this argument is reasonable. (This argument was re-examined in the cloudy bag [22] and found to be questionable.)

### III. QUARK MODELS

The quark wave functions of the excited baryon states are determined using a *spin-independent* harmonic oscillator (HO) potential as the confining potential in the NRQM. The NRQM also includes a spin-dependent potential which is derived from the one-gluon-exchange (OGE) between quarks. This OGE potential naturally contains a combination of spin-spin and tensor terms. One typically neglects the spin-orbit ( $LS$ ) potential which appears in a nonrelativistic reduction of the OGE amplitude. The pragmatic reason is that the mass spectrum of the excited baryons show no strong evidence for a strong  $LS$  potential between quarks. As a possibility Isgur and Karl [17] argued that the  $LS$ -potential from the effective OGE could be canceled by the  $LS$ -potential from a Lorentz-scalar confining interaction of the quarks. This cancellation is shown to occur in a chiral (cloudy) bag-model description of the negative parity  $N^*$  and  $\Delta^*$  states [23], see erratum in Ref. [22]. The *spin-independent* confining HO potential, implies that the two  $p$ -states with  $j = 1/2$  and  $3/2$  are degenerate in energy. The OGE spin-dependent potential contributes to the SU(3)-multiplet mixing in the excited baryon states. Isgur and Karl observed that most physical states are closer to pure  $uds$ -basis states (characterized by the two Jacobi coordinates and their corresponding HO frequencies) than to pure SU(3) eigenstates [17].

The Bonn group [13] calculates the electromagnetic decay of hyperon resonances within a relativistic quark model where the propagators of the three quarks are given by the two Jacobi momenta. In their evaluations they propose a confinement potential  $V_{\text{conf}}$  with a Dirac structure such that  $V_{\text{conf}}$  reduces to a *spin-independent* linear confinement potential in the nonrelativistic limit. In addition, Ref. [13] includes an instanton induced two-quark potential instead of the effective OGE potential. These two potentials are included in the Bethe-Salpeter equation, which is solved in the instantaneous approximations.

In bag models the (Lorentz scalar) confinement condition introduces a strong  $LS$  splitting of the quark eigenstates. The quark  $p$ -state with  $j = 3/2$  ( $p_{3/2}$ ) has a lower energy than

<sup>1</sup>This means both Eqs. (1) and (2) are approximations of the  $uds$  basis, see, e.g., Sec. III in the second paper in Ref. [17]. This reference also says that most (excited) physical states are closer to pure  $\rho$  or  $\lambda$  states than to pure SU(3) eigenstates.

TABLE I. The value of  $\Gamma_\gamma$  from various publications are given in the last column. The coefficients  $a$ ,  $b$  and  $c$  of the  $\Lambda(1520)$  wave functions, Eq. (3) used in the various publications are given. The version of the ground state wave function  $\Lambda(1116)$ , Eq. (1) or (2), used in the different evaluations of  $\Gamma_\gamma$  is indicated in the next to last column. The “dash” means that values cannot be ascertained. See text for details and for the significance of the \*.

Models	References	$a$	$b$	$c$	$\Lambda(1116)$	$\Gamma_\gamma$ (keV)
NRQM	[8]	0.91	0.01	0.40	Eq. (1)	96
NRQM (SU(6)-basis)	[10]	0.91	0.01	0.40	—	98
$\chi$ QM	[15]	0.91	0.01	-0.40	Eq. (1)*	85
$\chi$ QM	[25]	0.91	0.01	-0.40	Eq. (2)*	134
NRQM ( $uds$ -basis)	[10]	—	—	—	Eq. (2)	154
MIT bag	[10]	0.86	0.34	-0.37	—	46
Chiral/cloudy bag	[11,12]	-0.95	-0.09	0.29	Eq. (1)	32
RCQM	[9]	0.91	0.01	0.40	Eq. (2)	215
Bonn-CQM	[13]	—	—	—	—	258

the  $j = 1/2$  ( $p1/2$ ) state which affect the two corresponding eigenfunctions. Furthermore, this effective  $LS$  component introduces an SU(3) multiplet mixing in the nonperturbed wave functions of the excited baryon in addition to the mixing due to the effective OGE discussed above. The pion and kaon clouds in the chiral (cloudy) bag model mix the SU(3) multiplets as well. As mentioned above, in the bag model the OGE automatically contains an effective  $LS$  force of opposite sign and about the same magnitude compared to the effective  $LS$  of the bag confinement condition [22,23]. In bag models the  $u$ - and  $d$ -quarks are massless whereas the  $s$ -quark is massive, i.e., the explicit breaking of flavor-SU(3) is built into the excited baryon wave functions. This is however not the  $uds$ -basis discussed by Isgur and Karl. Unlike the NRQM it is highly non-trivial to introduce the Jacobi coordinates for the three quark wave functions of bag models which are described by a product of Dirac spinors. In the bag model the  $\Lambda(1116)$  translational mode is projected out following the procedure of DeGrand *et al.* [24]. In this procedure one transforms the three-quark  $jj$  coupled states to the  $LS$ -basis and assumes that the  $SU(6) \times O(3)$   $L = 1$ ,  $\mathbf{56}$ -plet is the translational mode of the hyperon ground state,  $\Lambda(1116)$ . The baryon mass spectrum appears to be well described in (cloudy/chiral) bag models despite the strong  $LS$  confinement component, see Refs. [22,23].

#### IV. THE ELECTROMAGNETIC DECAY RATE

The various theoretical models generate very different values for  $\Gamma_\gamma$ , which is very sensitive to the amount of admixture of higher configuration states in the hyperon ground state. Another important factor, which can change the value of  $\Gamma_\gamma$ , is the amount of SU(3)- octet admixtures in  $\Lambda(1520)$ , i.e. the magnitude of the coefficients  $b$  and  $c$  in Eq. (3), see, e.g., the discussion in Ref. [10]. In Table I the first column indicate the quark model used in calculating  $\Gamma_\gamma$  and the second column gives the corresponding reference. The values of the coefficients in Eq. (3) used in the different model calculations

are presented in the next three columns. The sixth column gives which  $\Lambda(1116)$  wave function is used, Eq. (1) or (2), and the last column gives the value for the decay rate evaluated in the different references. As is evident from Table I, most calculations adopt the values for  $a$ ,  $b$  and  $c$  as determined by Isgur and Karl [17],  $a = 0.91$ ,  $b = 0.01$ , and  $c = 0.40$ . The bag model results for the values of the coefficients,  $a$ ,  $b$  and  $c$ , differ from the NRQM of Isgur and Karl mainly due to the bag confinement condition. The three coefficients in the row labeled “MIT bag” in Table I are determined from the expression in Eq. (4) of Ref. [10] using the definitions Eqs. (4.6) and (B.4) in Ref. [23]. In the chiral/cloudy bag model evaluations the meson cloud contribute to further change the values of the three coefficients.<sup>2</sup>

#### V. DISCUSSION AND CONCLUSIONS

Table I shows how the calculated values of  $\Gamma_\gamma$  varies according to which wave function for  $\Lambda(1116)$  is used. The standard SU(6)-basis evaluations of Refs. [8,10,15], in the first three rows are very similar and they find  $\Gamma_\gamma \approx 100$  keV, confirming the early evaluation of  $\Gamma_\gamma = 100$  keV by Copley *et al.* [26]. It should be noted that Refs. [15,25] “dress” the constituent quark wave function with Goldstone bosons (denoted by an \* in Table I). In these three first rows the hyperon ground state wave function appears to be given by Eq. (1). All the three values of  $\Gamma_\gamma$  are small compared to the experimental value  $\sim 160$  keV.

The influence of the configuration mixing in the hyperon ground state, Eq. (2), due to the effective color hyperfine interaction [17] is evident if we compare the values of  $\Gamma_\gamma$  in

<sup>2</sup>The bag model values of the three coefficients are extracted using the procedure of Ref. [24]: After a recoupling from the relativistic  $jj$ -basis to the  $LS$ -basis, the  $\mathbf{56}$ -plet is projected out. It is then possible to determine the multiplet mixing coefficients,  $a$ ,  $b$  and  $c$ , in Eq. (3) given in Table I.

the three first rows with the fourth row  $\Gamma_\gamma$  from Ref. [25], the published version of Ref. [15]. In the fifth row  $\Gamma_\gamma$  is evaluated in the  $uds$  basis by Kaxiras *et al.* [10] where Eq. (2) enters naturally. In other words, when configuration mixing in both  $\Lambda(1520)$  and  $\Lambda(1116)$  are included, the calculated values of  $\Gamma_\gamma$  appear to be close to the experimental value of  $\sim 160$  keV.

The importance of configuration mixing in  $\Lambda(1116)$  is also found by Warns *et al.* [9]. References [9] and [13] evaluate  $\Gamma_\gamma$  using a relativistic approach. Warns *et al.* [9] employ a relativistic constituent quark model (RCQM) and quote a (lowest order) nonrelativistic result  $\Gamma_\gamma = 48$  keV. As a relativistic correction Ref. [9] considers the configuration mixing generated by the color hyperfine interaction (OGE) of Isgur and Karl, which results in a configuration mixing of  $\Lambda(1116)$ , Eq. (2). Furthermore, the center-of-mass recoil effect is also counted as a relativistic correction by Ref. [9]. The relativistic “full” RCQM result of Ref. [9] is shown in Table I. As mentioned, Van Cauteren *et al.* [13] use the Bethe-Salpeter equation to account for relativity in evaluating, e.g.  $\Gamma_\gamma$ . The recoil boost of  $\Lambda(1116)$  is an integral part in this evaluation. Unfortunately, these two references [9,13] do not indicate the magnitudes of the different relativistic corrections. It is not clear why these two publications, Refs. [9,13], find values so much larger than the other model evaluations of  $\Gamma_\gamma$ .

The bag models clearly predict too small decay rates. One reason for this could be that the quark energy differences  $E_{p3/2} - E_{s1/2} \approx 230$  MeV and  $E_{p1/2} - E_{s1/2} \approx 350$  MeV (bag radius  $\approx 1$  fm) are both smaller than the photon energy  $E_\gamma \approx 400$  MeV. These two energy differences reflect the strong spin-orbit splitting of the bag confinement condition and are built into the quark wave functions. The electromagnetic transition of a quark from one of the two  $p$ -states to the ground state,  $s1/2$  is evaluated using the relativistic bag wave functions. To project out the translational mode the SU(6) argument, which was discussed earlier and found to be questionable, was implemented. In addition, Refs. [11,12] used Eq. (1) to describe  $\Lambda(1116)$  and furthermore the boost of the ground state  $\Lambda(1116)$  (recoil momentum  $p_\Lambda \approx 400$  MeV/c) was neglected. The small difference between the MIT bag and the chiral (cloudy) bag model results are mainly due to two factors. In the chiral/cloudy bag all OGE contributions involving the  $p3/2$  quark state are included and an additional small contribution to the decay rate [12] comes from the meson cloud (pions and kaons). This small meson cloud contribution to  $\Gamma_\gamma$  is confirmed by Ref. [14], which uses a unitary extension of chiral perturbation theory to evaluate  $\Gamma_\gamma$  in a meson-baryon coupled channels approach.

As presented above the electromagnetic decay rates discriminate easily between the different model descriptions

of the hyperons. In order to make further progress in our understanding of the structure of the baryon states, it is necessary to experimentally determine several other radiative decay rates from excited baryon states. There also exists one serious theoretical problem for quark models, namely their prediction that the lightest  $J^P = 1/2^-$ -state,  $\Lambda(1405)$  and the lightest  $J^P = 3/2^-$ -state,  $\Lambda(1520)$  are almost degenerate states, see, e.g., Refs. [11,13,17,19,20,22]. This might indicate that  $\Lambda(1405)$  is a special state containing a dominant  $\bar{q}q^4$  (or meson-baryon) component. As originally proposed [7], a full coupled channel treatment of a meson-baryon system coupled to the “bare”  $q^3$  states with the quantum numbers of both  $\Lambda(1405)$  and  $\Lambda(1520)$  could possibly resolve this (almost) mass-degeneracy found in quark model calculations.

To summarize the  $uds$ -basis for the NRQM as advocated by Isgur and Karl [17] supplemented by an effective color hyperfine interaction, predicts an important configuration mixing of  $\Lambda(1116)$  in addition to the mixing of the flavor components of  $\Lambda(1520)$ . It appears that these non-trivial mixings are needed to explain the observed large value of  $\Gamma_\gamma$ . The evaluations of  $\Gamma_\gamma$  in Refs. [10] and [25] include the mixed states of Isgur and Karl in both the initial and final hyperons and their values for  $\Gamma_\gamma$  are consistent with the experimental value, see Refs. [2–4]. Reference [9] also includes the configuration mixing in both hyperons, and find a result that lies within the experimental errors of the CLAS Collaboration result [2].

One conclusion to be drawn is that flavor SU(3) breaking as well as higher quark states configuration mixing of the ground state baryons have to be an integral part of the wave function of the baryons, i.e., we should use the  $uds$ -basis to evaluate the three-quark wave functions of the baryons. The SU(3) breaking is most easily “visualized” in the excited hyperon states: given that the  $s$ -quark is heavy compared to the  $u$ - and  $d$ -quarks (this mass difference is dramatic in bag models), a scenario emerges where the pair of light quarks “orbits” the heavy  $s$ -quark in the excited hyperons. Another tentative conclusion, which is based on the bag model results, is that the confinement mechanism should not have a strong spin-orbit component. Furthermore, in some models the procedure for removing the center-of-mass coordinate has to be re-examined, as discussed in, e.g., Ref. [22]. Flavor SU(3) is broken and therefore the SU(6) **56**-multiplet is not representative of the translational mode of the baryon ground state.

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