Excited states of $^{9}_{\Lambda}$Be and $^{10}_{\Lambda\Lambda}$Be in an α cluster model

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The energies of the degenerate spin-flip doublet ($3^+/2$, $5^+/2$) of $^{9}_{\Lambda}$Be and of the 2^+ state of $^{10}_{\Lambda\Lambda}$Be are analyzed in the α cluster model using a phenomenological dispersive three-body $\Lambda\alpha\alpha$ force that reproduces the ground state energy of $^{9}_{\Lambda}$Be. Two types of phenomenological $\Lambda\alpha$ and $\alpha\alpha$ potentials and a few s-state $\Lambda\Lambda$ potentials are taken as input. The energies of the excited states of the hypernuclei, treated as three- and four-body systems, calculated using the Variational Monte Carlo method, are in good agreement with the experimental values. Our results demonstrate that the existing data are insensitive to whether one employs a dispersive $\Lambda\alpha\alpha$ force along with potentials in the relative angular momentum state $l = 0$ and 2 as in the present work or whether one uses nonlocal $\Lambda\alpha$ potential as in earlier analyses.

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I. INTRODUCTION

Recently, we made a cluster model analysis [1,2] of the ground state binding energies of s- and p-shell $\Lambda\Lambda$ hypernuclei using the Variational Monte Carlo (VMC) method. For clusters restricted to interacting through potentials in the relative angular momentum state $l = 0$ the calculated s-shell hypernuclear binding energies are in very good agreement with those obtained with the non VMC calculations [3–6]. In the last few years a number of cluster model analyses of p-shell hypernuclei using the Faddeev approach [3,5–7] have been performed. In all the above-mentioned analyses a wide range of potentials were used. For a description of the $\alpha\alpha$ system the potentials of Ali and Bodmer [8] and Chien and Brown [9] in $l \geqslant 0$ were used. A wide range of $l = 0$ $\Lambda\alpha$ potentials with ambiguities in the short range behavior of the force was included. Of these, the Isle [3] and Myint, Shinmura, and Akaishi [4] $\Lambda\alpha$ potentials are considered to be realistic and have been widely used in the study of hypernuclei. While the former is phenomenological, the latter has been microscopically calculated using a D2 ΛN potential but tailored to fit B_Λ of $^{5}_{\Lambda}$He. Further, a weak p-wave $\Lambda\alpha$ potential [4] that is motivated from the NSC97e model of ΛN interaction has also been used. The $\Lambda\Lambda$ interaction was described by a sum of three-range Gaussian potentials simulated from various versions of Nijmegen models. The energy calculated in the Faddeev approach [7] for the degenerate spin-flip doublet ($3^+/2$, $5^+/2$) of $^{9}_{\Lambda}$Be in the s- and p-wave model for the $\Lambda\alpha$ potential in Ref. [4] and the Ali and Bodmer $\alpha\alpha$ potential [8] is close to the experimental value. However, for the Chien and Brown $\alpha\alpha$ potential [9], it is marginally overbound. The ground state energy is found to be lower by approximately 10% in comparison with the data. The overbinding was attributed to the dispersive ΛNN force, which has been ignored in the Faddeev method.

Hiyama *et al.* [10] analyzed $S = -1$ and -2 p-shell hypernuclei in the cluster model in the framework of a variational method employing Jacobi-coordinate Gaussian-basis functions. The Λx (cluster x = alpha, triton, deuteron) nonlocal potentials were obtained by folding the three-range Gaussian hyperon-nucleon (YN) potential simulating the G matrix (the YNG interaction) into the density of the x cluster. The YNG interactions between ΛN are derived from the Nijmegen one-boson exchange model ND. The odd/even state ΛN potential [10] was modified to tune the Λx potential so that it fits the ground or excited state energy of the hypernuclei containing clusters. The $\Lambda\alpha$ potential so obtained along with the $\alpha\alpha$ state-independent potential explains the energies of the ground and excited states of a host of p-shell hypernuclei including $^{9}_{\Lambda}$Be and $^{10}_{\Lambda\Lambda}$Be. Thus a modified odd state ΛN potential appears to be crucial in explaining the data. From the analyses [3,5–7,10,11], it seems that the contribution of dispersive ΛNN forces [12,13], which theoretically have been established to be making significant contributions [14–16] to the energy of hypernuclei, can be ignored in the $\Lambda\alpha\alpha$ channel. Here it is worthwhile to remark that microscopically calculated $\Lambda\alpha$ potentials [4,10] may not be free from the inherent uncertainties of the boson exchange models and of the prescription adopted for solving many-body problems. Further, $\Lambda\alpha$ potentials are yet to be tested against the $\Lambda\alpha$ scattering data to explore the realities accommodated by these. However, in all the cluster model analyses referred to above the $\Lambda\alpha$ potentials that fit the binding of $^{5}_{\Lambda}$He may safely be assumed to be simulating the dispersive ΛNN force. But the contribution of dispersive energy $\langle V_{\Lambda\alpha\alpha}\rangle$ between the Λ-α-α triad in the calculations referred to above could not be accounted for. This may be because of 16 pairs of nucleons in the triad $\Lambda N_1 N_2$ and a nucleon coming from each α. The contribution $\langle V_{\Lambda\alpha\alpha}\rangle$ in the energy is quite significant as has been calculated microscopically [14] and neglecting it overbinds the $^{9}_{\Lambda}$Be and $^{10}_{\Lambda\Lambda}$Be. As the major contribution [16] in the $\Lambda\alpha\alpha$ channel arises because of the dispersive ΛNN force, we decided to call it a phenomenological dispersive three-body force. Therefore, in our earlier analyses [1,2] of ground state binding energy of $S = -1$ and -2 p-shell hypernuclei, we proposed a phenomenological three-body $\Lambda\alpha\alpha$ force in analogy with the dispersive ΛNN force. We represented it

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through the simple form

$$V_{\Lambda\alpha\alpha} = W_0 f^2(r_{\Lambda\alpha_1}) f^2(r_{\Lambda\alpha_2}), \quad (1)$$

where radial behavior is assumed to have the form $f^2(r) = \exp(-0.5r)/0.5r$. The potential strengths ($W_0 = 17.0$ MeV for Isle [3] and 13.5 MeV for Myint, Shinmura, and Akaishi [4] $\Lambda\alpha$ potentials) yield B_Λ values of (6.62, 6.60) and (6.53, 6.55) MeV for the $\alpha\alpha$ potentials of Refs. [8] and [9], respectively, close to the experimental value 6.7 MeV of $^9_\Lambda$Be. The $\Lambda\Lambda$ potential, which fits the ground state binding energy of $^6_{\Lambda\Lambda}$He, gives the $B_{\Lambda\Lambda}$ of $^{10}_{\Lambda\Lambda}$Be to be 15.38 MeV for Isle $\Lambda\alpha$ and Ali-Bodmer $\alpha\alpha$ potentials, much lower than the currently accepted experimental value of 17.7 ± 0.7 MeV but in close agreement with the value ($B_{\Lambda\Lambda} = 14.6 \pm 0.4$ MeV) estimated on the premise that a γ-ray must have escaped undetected in the emulsion in the decay of $^9_\Lambda$Be*. A similar prediction was made in an earlier microscopic work [10]. In view of the success of using either dispersive ΛNN force or odd state ΛN potential in explaining the data, one can advance an argument that odd state potential [10] adjusted to fit B_Λ of $^9_\Lambda$Be may be simulating dispersive $\Lambda\alpha\alpha$ force and on the other hand the dispersive $\Lambda\alpha\alpha$ force in Refs. [1,2] may be assumed to be simulating the effect of the odd state $\Lambda\alpha$ potential. Consequently, both the models successfully explain the data.

In the last few years, a number of VMC calculations [2,12–16] mainly for ground state of s- and p-shell hypernuclei in the few-body framework in terms of baryons degrees of freedom or of cluster model have been performed. Here we analyze the experimental binding energy $B_\Lambda = 3.66$ MeV of the degenerate doublet ($3^+/2, 5^+/2$) of $^9_\Lambda$Be [17,18] and $B_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV of the excited 2^+ state of $^{10}_{\Lambda\Lambda}$Be [10,19] in Λ and α cluster models using dispersive $\Lambda\alpha\alpha$ force in a VMC approach. To our knowledge this is the first calculation that uses the VMC method. We also calculate the quadrupole moments and root mean square (rms) radii of hadron pairs to gain further insight into the structure of hypernuclei.

The article is organized as follows: In the next section we describe the Hamiltonians, wave functions, potentials model, and quadrupole moments in the cluster model for the excited states of $^9_\Lambda$Be and $^{10}_{\Lambda\Lambda}$Be. The results and discussion are presented in Sec. III along with the calculation of quadrupole moments. We present the summary of our study in the last section.

II. EXCITED STATES OF P-SHELL HYPERNUCLEI, WAVE FUNCTIONS, AND QUADRUPOLE MOMENTS IN α CLUSTER MODEL

A. Hamiltonian and wave functions of p-shell hypernuclei

The energies of the degenerate doublet ($3^+/2, 5^+/2$) of $^9_\Lambda$Be and the 2^+ excited state of $^{10}_{\Lambda\Lambda}$Be have been calculated in the α cluster model. These states are considered to be built on the first excited state $J = 2^+$ of the ^{8}Be core nucleus, which is believed have $L = 2, S = 0$ structure. The coupling of $0s\Lambda$ particle to the ^{8}Be core in $^9_\Lambda$Be gives rise to a spin-flip doublet. The measured energy spacing of $\sim$0.03 MeV of the doublet is attributed to a very weak spin-orbit force that is ignored here. The excited state 2^+ of $^{10}_{\Lambda\Lambda}$Be is the antisymmetric pair of Λ coupled to $J = 2^+$ of the ^{8}Be core.

The $^{10}_{\Lambda\Lambda}$Be is treated as a four-body system in the $\Lambda\Lambda + \alpha\alpha$ model. For the calculation of energy of the 2^+ excited state, we consider $\alpha\alpha$ particles moving in the relative $l_{\alpha\alpha} = 2$ state and other cluster-cluster pairs have relative angular motion in the s state. The Hamiltonian is given by

$$\begin{aligned} H^{\alpha\alpha}_{\Lambda\Lambda} = & \sum_{i=1}^{2}(K_\alpha(i) + K_\Lambda(i+2)) + V^{(2)}_{\alpha\alpha}(r_{12}) \\ & + \sum_{i<j\; i=1,2,j=3,4} V^{(0)}_{\Lambda\alpha}(r_{ij}) \\ & + \sum_{i=3}^{4} V_{\Lambda\alpha\alpha}(r_{i1}, r_{i2}) + V^{(0)}_{\Lambda\Lambda}(r_{34}), \end{aligned} \quad (2)$$

where labels 1,2 specify the two α clusters and labels 3,4 the two Λ particles. $V^{(l_{xy})}_{xy}$ denotes the potential for the pair $xy(= \alpha\alpha, \Lambda\alpha$, and $\Lambda\Lambda)$ in the relative angular momentum l_{xy} state and constrained by experimental data pertaining to the relevant pair. The $V_{\Lambda\alpha\alpha}$ is the phenomenological dispersive three-body Yukawa shape potential. The hypernucleus $^9_\Lambda$Be is analyzed in terms of three-body $\Lambda + \alpha\alpha$ clusters. The degenerate doublet ($3^+/2, 5^+/2$) is assumed to be built on the relative $l_{\alpha\alpha} = 2$ of two Bose particles with $l_{\Lambda\alpha} = 0$. On suppressing one Λ index and making $V^{(0)}_{\Lambda\Lambda}(r_{34}) = 0$, Eq. (1) reduces to the Hamiltonian for $^9_\Lambda$Be. The contribution of $\langle V_{\Lambda\alpha\alpha}\rangle$ to the energy is quite significant as shown in Refs. [12–16], neglecting it among the $\Lambda\alpha\alpha$ clusters overbinds the $^9_\Lambda$Be and $^{10}_{\Lambda\Lambda}$Be. It may be noted that $V^{(0)}_{\Lambda\alpha}$, which reproduces the experimental energy of the $^5_\Lambda$He hypernucleus, simulates the dispersive energy and other smaller effects.

The trial wave functions for hypernuclei in the state (J, J_z) are the product of two-body correlation functions $f^{(l)}_{xy}$ in the relative angular momentum l state and the functions $\xi^J_{J_z}$ and $\zeta^J_{J_z}$:

(i) $\Lambda\alpha\alpha$ system in the state $(J, J_z) = (3^+/2, 3/2)$ and $(5^+/2, 5/2)$

$$\Psi^{\alpha\alpha}_\Lambda(J, J_z) = \left[\prod_{i=1}^{2} f^{(0)}_{\Lambda\alpha}(r_{i3})\right] f^{(2)}_{\alpha\alpha}(r_{12})\xi^J_{J_z} \quad (3)$$

(ii) $\Lambda\Lambda\alpha\alpha$ system in the state $J = 2, J_z = 2$

$$\begin{aligned} \Psi^{\alpha\alpha}_{\Lambda\Lambda}(J, J_z) = & \left[\prod_{i=1}^{2}\prod_{j=3}^{4} f^{(0)}_{\Lambda\alpha}(r_{ij})\right] \\ & \times f^{(0)}_{\Lambda\Lambda}(r_{12}) f^{(2)}_{\alpha\alpha}(r_{34})\zeta^J_{J_z}. \end{aligned} \quad (4)$$

The function ξ (or ζ) is obtained by coupling the spherical harmonic $Y_{2m_l}(\Omega_{12})$ and the appropriate spin function χ_{sm_s}. The correlation functions $f^{(l)}_{xy}(r)$, as usual, are obtained from the solution of the Schrödinger-type equation for the relative angular momentum state l.

B. Potentials models

The phenomenological $V^{(l)}_{hh}(hh = \Lambda\alpha, \alpha\alpha)$ interaction [3,4,8] in the angular momentum state l is given as

$$\begin{aligned} V^{(l)}_{\rm hh}(r) = & \ V^l_{\rm rep} \exp\left(-\left(r/\beta^l_{\rm rep}\right)^2\right) \\ & - V^l_{\rm att} \exp\left(-\left(r/\beta^l_{\rm att}\right)^2\right) + V_{\rm Coul}(r), \end{aligned} \quad (5)$$

TABLE I. The parameters of the potentials in state l used in the present work. The $\Lambda\alpha$ potentials from Refs. [3] and [4] are designated as Isle and MSA, respectively, and the $\alpha\alpha$ potential from Ref. [8] is abbreviated as AB.

Potentials	Angular momentum l	$V^l_{\rm rep}$ (MeV)	$\beta^l_{\rm rep}$ (fm)	$V^l_{\rm att}$ (MeV)	$\beta^l_{\rm att}$ (fm)
$\Lambda\alpha$					
Isle	0	450.4	1.25	404.9	1.41
MSA	0	91.0	1.30	95.0	1.70
$\alpha\alpha$					
AB	2	20.0	1.53	30.18	2.85
Chien and Brown [9]	2	176.5	1./0.620	85.0	1.0/1.35

where V^l_i and β^l_i are the strength and the range parameter in the relative l state, respectively, for i = rep(att), and $V_{\rm Coul}(r)$ is the finite size Coulomb $\alpha\alpha$ potential, which is zero for the $\Lambda\alpha$ pair. The potential parameters for $hh = \Lambda\alpha$ for $l = 0$ are taken from Refs. [3,4] and reproduce the experimental data on $^{5}_{\Lambda}$He. The $\alpha\alpha$ potentials of Ali and Bodmer [8] and Chien and Brown [9] were chosen in $l = 2$ and the results for the binding energies in the two cases do not differ by more than 0.11 MeV as has been observed in earlier work [7]. We refer the reader to Ref. [9] for the radial form factor of the Chien and Brown potential. The potentials' parameters are listed in Table I.

We have chosen three-range Gaussian simulated $\Lambda\Lambda$ potential [3] that is phase shift equivalent to the Nijmegen realistic interaction NSC97e. This has the form

$$V^{(0)}_{\Lambda\Lambda}(r) = \sum_{i=1}^{3} v_i \exp\left(-r^2/\alpha_i^2\right), \tag{6}$$

where symbols have meaning as given in Ref. [3]. The other effective potential $V^{(0)}_{\Lambda\Lambda}(r)$ (denoted by NAGSIM) was taken from Ref. [5] and has been constrained to reproduce $B_{\Lambda\Lambda}$ of $^{6}_{\Lambda\Lambda}$He. The two potentials differ in attractive strengths of the midrange part of $V^{(0)}_{\Lambda\Lambda}$. We have also chosen the $\Lambda\Lambda$ single channel effective potential [4] $V^{e1}_{\Lambda\Lambda}$ that is constructed from the Nijmegen soft-core NSC97e potential and reproduces $B_{\Lambda\Lambda}$ of $^{6}_{\Lambda\Lambda}$He. An Urbana type $\Lambda\Lambda$ potential has been successfully used in the past [2,12,15] to explain the binding of $S = -2$ s- and p-shell hypernuclei. Therefore, in the present analysis an Urbana type potential with strength $V_0^{\Lambda\Lambda} = 5.65$ MeV, consistent with $B_{\Lambda\Lambda}$ of $^{6}_{\Lambda\Lambda}$He, is also included:

$$V^{(0)}_{\Lambda\Lambda}(r) = V_c(r) - V_0^{\Lambda\Lambda} T_\pi^2(r), \tag{7}$$

where

$$V_c(r) = \frac{2137}{1 + \exp\frac{(r-0.5)}{0.2}},$$
$$T_\pi(r) = (1 + 3/x + 3/x^2)(\exp(-x)/x)(1 - \exp(-2r^2))^2,$$
$$x = 0.7r,$$

and other symbols have the same meaning as in Refs. [12–16].

Because the phenomenological dispersive $\Lambda\alpha\alpha$ force in Eq. (1) successfully explains the ground state binding energies of $^{9}_{\Lambda}$Be and $^{10}_{\Lambda\Lambda}$Be, we use it to analyze their spectra as well.

C. Energy and quadrupole moment

The energy $-B_{\Lambda\Lambda}$(or $-B_\Lambda$) for the $S = -2$ (or -1) system in the cluster model in the state (J, J_z) is evaluated using the relation

$$-B_\lambda(J, J_z) = \frac{\left\langle \Psi^{\rm N}_\lambda(J, J_z)\middle| H^{\rm N}_\lambda \middle| \Psi^{\rm N}_\lambda(J, J_z)\right\rangle}{\left\langle \Psi^{\rm N}_\lambda(J, J_z)\middle|\Psi^{\rm N}_\lambda(J, J_z)\right\rangle}, \tag{8}$$

where symbols λ and N have the following definitions:

(i) $\Lambda\alpha\alpha$ model of $^{9}_{\Lambda}$Be $\lambda = \Lambda$ and N $= \alpha\alpha$
(ii) $\Lambda\Lambda\alpha\alpha$ model of $^{10}_{\Lambda\Lambda}$Be $\lambda = \Lambda\Lambda$ and N $= \alpha\alpha$.

The VMC estimates of the energy were made for 100,000 points. The statistical error in the energies are of the order of half a percent.

We calculate the quadrupole moments in the unit of electronic charge e using the expression

$$\langle Q\rangle_{J,J} = \left\langle \Psi^{\rm N}_\lambda(J, J_z)\middle| Q \middle| \Psi^{\rm N}_\lambda(J, J_z)\right\rangle_{J_z=J}, \tag{9}$$

where the quadrupole moment's operator is given by

$$Q = \sum_{i=1}^{2} 2\left(3z_i^2 - r_i^2\right), \tag{10}$$

with the summation index i running over coordinates of two α's treated as point particles. The distances are measured from the c.m. of the two α's.

III. RESULTS AND DISCUSSION

A. Spin-flip doublet ($3^+/2, 5^+/2$) of $^{9}_{\Lambda}$Be

A detailed study of the ground state energy of $^{9}_{\Lambda}$Be in the $\Lambda\alpha\alpha$ model was performed in our earlier work [2] using s-state hh potentials along with the dispersive $\Lambda\alpha\alpha$ force given in Eq. (1). In the same spirit we calculated the excitation energies of the degenerate doublet ($3^+/2, 5^+/2$) for chosen $\alpha\alpha$ and $\Lambda\alpha$ potentials. The $l_{\alpha\alpha} = 2$ $\alpha\alpha$ potential of Ali and Bodmer [8] and the Isle $\Lambda\alpha$ potentail for $l = 0$ were employed. The binding energy of the spin-flip doublet as shown in Table II for a combinations of potentials turns out to be close to the experimental value.

Our results demonstrate that the difference in energy for Ali and Bodmer [8] and Chien and Brown [9] $\alpha\alpha$ potentials for a given $\Lambda\alpha$ potential is small. On the other hand use of the $\Lambda\alpha$ potentials of Isle [7] and Myint, Shinmura, and Akaishi [4] makes a difference of about 3% in the energy for a given $\alpha\alpha$ potential. Such a behavior is also observed in Ref. [20]. To compare our calculations with those of Cravo, Fonseca, and Koike [20] we also used the phenomenological $\Lambda\alpha$ potential of Maeda and Schmid [21] that uses a sum of two Woods-Saxon terms. Such potentials induce weak correlation. This potential along with the Ali-Bodmer $\alpha\alpha$ potential gives the binding energy 6.52 MeV close to the experimental value but excluding either dispersive three-body $\Lambda\alpha\alpha$ or explicit use of $\Lambda\alpha$ and $\alpha\alpha$ potentials in higher partial waves. Despite that, the VMC binding energy is marginally lower by about 3% than the one calculated by Cravo, Fonseca, and Koike [20] who have used same $\Lambda\alpha$ potential for higher partial waves. The plausible reason for this difference is given in Ref. [7]. We note that

TABLE II. Degenerate spin-flip doublet $(3^+/2, 5^+/2)$ of ${}^{9}_{\Lambda}$Be in the $\alpha\alpha\Lambda$ model. The spin quantum numbers of the excited states, the quadrupole moments, and the contribution of the dispersive three-body $\langle V_{\Lambda\alpha\alpha}\rangle$ for the $\alpha\Lambda$ potentials listed in the first column are given in the second, third, and fourth columns, respectively. The binding energies (subtract $\alpha\alpha$ resonance energy ≈ 0.1 MeV) of the excited states; $R_{\alpha\alpha}$, rms distance between the two α's; and $R_{(\alpha\alpha)\Lambda}$, rms distance between the c.m. of the two α's and hyperon are given in the fifth, sixth, and seventh columns, respectively. The results within square brackets are for Chien and Brown [9] and those outside the brackets are for Ali and Bodmer [8]. Experimental $B_\Lambda(3^+/2, 5^+/2) = 3.66$ MeV [17,18].

$\alpha\Lambda$ potential	${}^{9}_{\Lambda}$Be (J, J) state	$-\langle Q\rangle_{J,J}$ (e fm^2)	$\langle V_{\Lambda\alpha\alpha}\rangle$ (MeV)	B_Λ (MeV)	$R_{\alpha\alpha}$ (fm)	$R_{(\alpha\alpha)\Lambda}$ (fm)
Isle	$(3^+/2, 3/2)$	6.41[6.35]	1.37[1.36]	3.54[3.45]	4.00[3.98]	2.75[2.77]
MSA		6.66[6.65]	1.31[1.29]	3.55[3.45]	4.09[4.08]	2.86[2.87]
Isle	$(5^+/2, 5/2)$	9.02[8.84]	1.46[1.45]	3.54[3.45]	3.97[3.94]	2.73[2.72]
MSA		9.63[9.51]	1.31[1.29]	3.55[3.44]	4.09[4.07]	2.87[2.86]

our result is close to the s-wave model Faddeev calculation [7] for $l_{\alpha\Lambda} \leqslant 2$ and $l_{\alpha\alpha} \leqslant 4$. The Woods-Saxon potential for the spin-flip doublet produces a binding energy of 3.79 MeV, which is about 2.7 MeV above the ground state. Furthermore, it is to be noted that the sophisticated calculation in Ref. [10] underestimates the separation energy of the excited state of ${}^{9}_{\Lambda}$Be by about 7%. We may remark here that it is satisfying to note the success of our simple prescription of using the dispersive $\Lambda\alpha\alpha$ force along with s-state $\Lambda\alpha$ and d-state $\alpha\alpha$ potentials in explaining the binding of the degenerate doublet of ${}^{9}_{\Lambda}$Be viz-a-viz the other calculations [7,20] that employed $\Lambda\alpha$ and $\alpha\alpha$ potentials for $l \geqslant 0$ and the calculation of Hiyama *et al.* [10] who employed odd state ΛN potentials adjusted to ${}^{9}_{\Lambda}$Be ground state binding energy.

To further explore the structure of the excited state ${}^{9}_{\Lambda}$Be we calculated the rms radii of the relevant pairs and the quadrupole moments in the (J, J_z) states $(3^+/2, 3/2)$ and $(5^+/2, 5/2)$ for all the potentials included in the present work (see Table II). No experimental information is available on these quantities except for the theoretically calculated values [20] for the quadrupole moment in the $\Lambda\alpha\alpha$ model with which we compare our result. For both of the states, the calculated values for the quadrupole moments are negative but $\approx$25% higher in magnitude than the ones listed in Ref. [20]. The difference seems to arise from the fact that spherical harmonics for $l > 2$, which are excluded in our wave function, are included in the work of Ref. [20]. Because $l = 2$ makes a dominant contribution, our analysis confirms the finding of earlier work [20] that ${}^{9}_{\Lambda}$Be has an oblate shape. Further listed in Table II are $R_{\alpha\alpha}$, rms distance between two α's, and $R_{(\alpha\alpha)\Lambda}$, rms distance between the c.m. of the two α's and the hyperon. $R_{\alpha\alpha}$ is marginally larger than that for the ground state found in Ref. [2]. This is not unexpected as the $\alpha\alpha$ pair is stretched out because of the centrifugal barrier in the relative $l = 2$ state.

B. 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be

The binding energy of ${}^{10}_{\Lambda\Lambda}$Be for the 2^+ state is calculated in the $\Lambda\Lambda\alpha\alpha$ model using the dispersive $\Lambda\alpha\alpha$ force in combination with $\Lambda\alpha$ and $\alpha\alpha$ potentials as used here for ${}^{9}_{\Lambda}$Be. The calculated binding energy for a range of $\Lambda\Lambda$ potentials is given in Table III. We note from Table III that the calculated $B_{\Lambda\Lambda}$ for all but the NSC97e $\Lambda\Lambda$ potential is within the bound of experimental values and, therefore, these potentials seem to be almost equivalent. The agreement between the $B_{\Lambda\Lambda}$ value of the Demachi-Yanagi event [10] and the value calculated in our model is remarkably good. A similar result was obtained by Hiyama *et al.* [10] but for a nonlocal $\Lambda\alpha$ potential adjusted to the B_Λ of ${}^{9}_{\Lambda}$Be. Thus explicit use of dispersive force in their work was ruled out. However, we feel that the nonlocal $\Lambda\alpha$ potential may have simulated it. On the contrary from the present work it appears that phenomenological dispersive force is adequate to explain the data but it may be simulating the effect of an l dependent $\Lambda\alpha$ potential.

Further, the value of the quadrupole moment of ${}^{10}_{\Lambda\Lambda}$Be in the $(2^+, 2)$ state, calculated for the first time, turns out to be around $-7.0\,\mathrm{e\,fm^2}$ for all the pairs of interactions considered in the present work. Thus the presence of not only single but also two Λ's in the Be nucleus makes the system oblate. The relevant rms separations for various pairs are listed in Table III. $R_{\alpha\alpha}$ and $R_{(\alpha\alpha)\Lambda}$ are, as expected, smaller in comparison with the values found for ${}^{9}_{\Lambda}$Be. The presence of additional Λ in ${}^{10}_{\Lambda\Lambda}$Be further compresses the system ${}^{9}_{\Lambda}$Be.

TABLE III. The 2^+ state of ${}^{10}_{\Lambda\Lambda}$Be in the $\alpha\alpha\Lambda\Lambda$ model. $B_{\Lambda\Lambda}$ values for the $\Lambda\Lambda$ potentials are listed in the first column. The results for combinations of $\Lambda\alpha + \alpha\alpha$ potentials are labeled by using abbreviations of authors' last names. $R_{(\alpha\alpha)\Lambda}$, rms distance between the c.m. of $\alpha\alpha$ and Λ. The other quantities are the same as in the preceding tables. Experimental $B_{\Lambda\Lambda}(2^+) = 12.33^{+0.35}_{-0.21}$ MeV [10,19].

$\Lambda\Lambda$ potential	$-\langle Q\rangle_{J,J}$ (e fm^2)	$\langle V_{\Lambda\alpha\alpha}\rangle$ (MeV)	$B_{\Lambda\Lambda}$ (MeV)	$R_{\alpha\alpha}$ (fm)	$R_{\Lambda\Lambda}$ (fm)	$R_{(\alpha\alpha)\Lambda}$ (fm)
		Isle + AB				
NSC97e	7.23	3.29	11.74	3.56	3.42	2.59
NAGSIM	7.06	3.45	12.29	3.53	3.34	2.57
Urbana	7.09	3.39	12.27	3.54	3.40	2.58
$V^{e1}_{\Lambda\Lambda}$	7.11	3.43	12.33	3.54	3.33	2.58
		MSA + AB				
NSC97e	7.06	3.71	11.70	3.52	3.28	2.53
NAGSIM	7.14	3.68	12.27	3.55	3.23	2.53
Urbana	7.20	3.71	12.27	3.56	3.26	2.53
$V^{e1}_{\Lambda\Lambda}$	7.02	3.76	12.35	3.52	3.23	2.52

It may be worth recalling here that in earlier analyses [2,12–15] the role of both dispersive ΛNN and space-exchange ΛN force has been explored on the binding energies of hypernuclei. Further it is to be noted that space-exchange ΛN force is equivalent to l dependent force in the case of hypernuclei. In our work [16] we have concluded that extraction of a unique combination of dispersive and space-exchange forces is impossible as the determination of one masks the other or one can simulate the other. Thus our VMC calculation for the energy of the excited state of Be hypernuclei demonstrates that dispersive force is adequate to explain the data. In the light of these remarks we note that the energy of the excited states of Be hypernuclei in the cluster model calculations is explained using a dispersive force or nonlocal $\Lambda\alpha$ potential [10]. Faddeev calculations [7] based on ${}^{9}_{\Lambda}$Be data indicate that a dispersive force along with the $l = 0, 1\,\Lambda\alpha$ potentials are likely to be required for explanation. The good agreement of VMC calculated binding energies with the data indicates that our model for explaining the ground [1,2] and excited states of two p-shell hypernuclei seems to be quite satisfactory.

IV. SUMMARY

In the present work we have carried out three- and four-body cluster model VMC calculations for the excited states and quadrupole moments of two Be hypernuclei using a variety of $\Lambda\alpha$, $\alpha\alpha$, and $\Lambda\Lambda$ potentials combined with dispersive $\Lambda\alpha\alpha$ force. This is the first cluster model VMC calculation reported by us. The close agreement of the calculated energies of the excited states with the data strongly supports the prescription of dispersive $\Lambda\alpha\alpha$ force in the description of Be hypernuclei. The result of non VMC calculation of $B_{\Lambda\Lambda}$ for $\Lambda\alpha$ nonlocal potential that depends on odd state ΛN potential agrees with ours. We note that from the present work it is not obvious whether effective dispersive or odd state ΛN force alone or an appropriate combination of the $l = 0, 1\,\Lambda\alpha$ potential and dispersive ΛNN force as is indicated in the Faddeev approach is required to explain the data of ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be. Finally, we note that both the hypernuclei are deformed and have oblate shape.

From the work reported here we conclude that there is a need not only to accurately measure the ground and excited state energies of existing p-shell hypernuclei but also to add many more new species along with the other data in the existing list to settle the ambiguity raised above.

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