

Mass coefficient and Grodzins relation for the ground-state band and γ band

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It is shown that the available experimental data on the energies of the first and the γ -vibrational 2^+ states and the reduced $E2$ transition probabilities from these states to the ground state require for the explanation significantly different values of the mass coefficients for the rotational motion and γ -vibrations.

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I. INTRODUCTION

The Hamiltonian of the phenomenological collective model [1] used for description of the properties of the low-lying collective quadrupole states contains two very important quantities depending on the deformation parameters, namely, the potential energy and mass coefficient. A variation of the potential energy in the transition from spherical to deformed nuclei is well understood. Much less is known about the mass coefficient, however. Frequently [2–8], it is assumed that the mass coefficient is a constant. As a consequence in the case of the axially symmetric well deformed nuclei the same mass coefficient is used for description of the β - and the γ -vibrations and the rotational motion, whose moment of inertia is proportional to the mass coefficient.

On the other side in the microscopic approaches [9,10] in general the mass coefficients are different for different modes of excitation (see, e.g., Appendix A). Thus an analysis of the experimental data aimed at the extraction of the values of the mass coefficients, without assuming that they are equal, is a very interesting task.

In the framework of the Bohr-Mottelson model the mass coefficients for different modes are inversely proportional to the products of the energy of the corresponding collective states and the value of the $E2$ reduced transition probability from this state to the ground state $E(2_i^+)B(E2; 2_i^+ \rightarrow 0_{\text{g.s.}}^+)$ where $i = \text{g.s.}, \beta$ or γ [11,12]. For instance, in the case when the 2^+ state is a vibrational one and there is no anharmonic effect its energy is equal to $\hbar\sqrt{C/B}$ and the $B(E2; 0_1^+ \rightarrow 2^+) \sim \frac{\hbar}{\sqrt{BC}}$. Thus, $E(2^+)B(E2; 0_1^+ \rightarrow 2^+) \sim \frac{1}{B}$. In the case when the first 2^+ state is a rotational one $E(2^+) \sim \frac{\hbar^2}{B\beta_0^2}$ and $B(E2; 0_1^+ \rightarrow 2^+) \sim \beta_0^2$. Again $E(2^+)B(E2; 0_1^+ \rightarrow 2^+) \sim \frac{1}{B}$. Thus, relations between the experimentally observed quantities and the parameters of the collective Hamiltonian are established.

It is the aim of the present paper to analyze the properties of the ground and γ bands in the well-deformed nuclei in which vibrational motion is well separated from the rotational one and basing on the experimental data to determine the relation between the mass coefficients used for description of the ground band rotational motion and γ vibrations.

II. MASS COEFFICIENTS

We are starting with the Bohr Hamiltonian [1] having in mind a possibility to consider the cases when the mass coefficient is effectively different for the rotational motion and γ vibrations [13]. For this aim it is natural to assume that the mass coefficient is a function of the collective variables and since the wave functions of the ground and γ bands have maxima in something different regions of the β - γ sector the mass coefficient will be effectively different for these two bands. Being a function of β and γ the mass coefficient can not only be a scalar but a tensor quantity. However, for simplicity we assume below that it is a scalar. Thus, we start with the Hamiltonian which can be written in the Laboratory frame as

$$H = \frac{1}{4} \left(\sum_{\mu} \pi_{2\mu}^+ \pi_{2\mu} \frac{1}{B(\alpha_2)} + \frac{1}{B(\alpha_2)} \sum_{\mu} \pi_{2\mu}^+ \pi_{2\mu} \right) + V(\alpha_2). \quad (1)$$

Here $\alpha_{2\mu}$ is a collective variable and $\pi_{2\mu}$ is an operator of the conjugate momentum. In the intrinsic frame we obtain from Eq. (1)

$$H = -\frac{\hbar^2}{4B(\beta, \gamma)} \left(\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} - \frac{1}{4\beta^2} \sum_{k=1}^3 \frac{\hat{I}_k^2}{\sin^2(\gamma - \frac{2\pi}{3}k)} \right) - \left(\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} - \frac{1}{4\beta^2} \sum_{k=1}^3 \frac{\hat{I}_k^2}{\sin^2(\gamma - \frac{2\pi}{3}k)} \right) \times \frac{\hbar^2}{4B(\beta, \gamma)} + U(\beta, \gamma). \quad (2)$$

We are considering nuclei near the axial symmetry limit with small amplitudes of the γ -oscillations around $\gamma = 0$ and β -oscillations around $\beta = \beta_0 \neq 0$. It is supposed that the rotational motion, the γ vibrations and β vibrations are well separated from each other as it takes place in the well deformed nuclei. In a correspondence with these assumptions we take a

potential U as a sum of two separate terms corresponding to the potentials for β and γ vibrations

$$U(\beta, \gamma) = \frac{1}{2}C_\beta(\beta - \beta_0)^2 + \frac{1}{2}\frac{C_\gamma}{\beta_0^2}\gamma^2. \quad (3)$$

Looking at the kinetic energy part of the Hamiltonian (2) we see that it consists in three terms each of which is the main one, correspondingly, for the β vibrations, for the γ vibrations and for the rotations. The mass coefficient $B(\beta, \gamma)$ being a function of the collective variables introduces a coupling between different types of collective motion. In addition, the kinetic energy term of γ vibrations includes $\frac{1}{\beta^2}$, and the rotational energy term depends on β and γ through the moments of inertia. In a correspondence with our main assumption we should simplify the kinetic energy part of the Hamiltonian (2) so as to decouple rotational motion, γ vibrations and β vibrations. To separate the collective variables in the Hamiltonian we should put $B(\beta, \gamma)$ to be equal to some number. It is natural to do it in a self-consistent way, i.e., to take instead of the function $B(\beta, \gamma)$ its average value over the wave function of the state under consideration. However, we neglect by this way the effect of the nondiagonal matrix element of the inverted mass coefficient, which is difficult to estimate, however. Having three collective variables in the Hamiltonian, namely, β , γ , and Euler angles we consider three types of states: ground state, γ -vibrational excitation and β -vibrational state. For this reason calculating the wave functions for every of these states we introduces instead of the function $B(\beta, \gamma)$ its averages:

(i) over the ground state

$$\langle \text{g.s.} | B(\beta, \gamma) | \text{g.s.} \rangle \equiv B_{\text{rot}} \quad (4)$$

when we consider ground state rotational band;

(ii) over the γ -vibrational state

$$\langle \gamma | B(\beta, \gamma) | \gamma \rangle \equiv B_\gamma \quad (5)$$

when we consider γ -vibrational state;

(iii) over the β -vibrational state

$$\langle \beta | B(\beta, \gamma) | \beta \rangle \equiv B_\beta \quad (6)$$

when we consider β -vibrational state. The procedure described above assumes a use of the projection operators.

Using Eqs. (3)–(6) and the assumption on a small amplitudes of γ and β vibrations we obtain from Eq. (2) the following approximate Hamiltonian:

$$H = \frac{\hbar^2}{2\langle i | B | i \rangle} \left(-\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} - \frac{\partial^2}{\partial \gamma^2} - \frac{1}{\gamma} \frac{\partial}{\partial \gamma} + \frac{K^2}{4\gamma^2} + \frac{1}{3}(I(I+1) - K^2) \right) + \frac{1}{2}C_\beta(\beta - \beta_0)^2 + \frac{1}{2}\frac{C_\gamma}{\beta_0^2}\gamma^2, \quad (7)$$

where $i = \text{g.s.}, \beta$ or γ band depending what state is considered.

Below we analyse different relations between the observables of the ground and the γ bands which include not only the reduced $E2$ transition probabilities but also the energies of the states. It means that both the stiffness coefficients of

the potential energy and the mass coefficients, are involved in these relations. The eigenfunctions of the states of the ground and the γ bands of the Hamiltonian (7) are

$$\Psi_{K=0}(I) = \sqrt{\frac{2I+1}{8\pi^2}} D_{M0}^I \psi_0(\beta) \left(\frac{2\sqrt{B_{\text{rot}}C_\gamma}}{\hbar} \right)^{1/2} \times \exp\left(-\frac{\sqrt{B_{\text{rot}}C_\gamma}}{2\hbar}\gamma^2\right), \quad (8)$$

$$\Psi_{K=2}(I) = \sqrt{\frac{2I+1}{16\pi^2}} (D_{M2}^I + D_{M-2}^I) \psi'_0(\beta) \left(\frac{2B_\gamma C_\gamma}{\hbar^2} \right)^{1/2} \gamma \times \exp\left(-\frac{\sqrt{B_\gamma C_\gamma}}{2\hbar}\gamma^2\right), \quad (9)$$

where $\psi_0(\beta)$ and $\psi'_0(\beta)$ describes the β oscillations around the value $\beta = \beta_0$. A difference between $\psi_0(\beta)$ and $\psi'_0(\beta)$ is due to a difference between B_{rot} and B_γ . However, in this paper we do not consider the effect of the β motion and therefore do not need in a concrete form of the functions $\psi_0(\beta)$ and $\psi'_0(\beta)$. Above $\Psi_{K=0}(I)$ and $\Psi_{K=2}(I)$ are the wave functions of the ground- and the γ -vibrational bands, respectively, obtained under the assumption that there is no K -mixing. They are orthogonal in spite of different values of the effective mass coefficient used in calculations.

Using the eigenfunctions (8) and (9) we can calculate the reduced matrix elements of the quadrupole moment operator $Q_{2\mu}$, which is taken in the same approximation as the Hamiltonian (7), i.e.,

$$Q_{2\mu} = q \left(D_{\mu 0}^2 \beta_0 + D_{\mu 0}^2 (\beta - \beta_0) + \frac{1}{\sqrt{2}} (D_{\mu 2}^2 + D_{\mu -2}^2) \beta_0 \gamma \right). \quad (10)$$

The results are

$$\langle 2^+, K=0 || Q_2 || 0^+, K=0 \rangle = q\beta_0, \quad (11)$$

$$\langle 2^+, K=2 || Q_2 || 0^+, K=0 \rangle = q\beta_0 \left(\frac{\hbar}{\sqrt{B_\gamma C_\gamma}} \right)^{1/2} \frac{4(B_{\text{rot}}/B_\gamma)^{1/4}}{(1 + \sqrt{B_{\text{rot}}/B_\gamma})^2}, \quad (12)$$

$$\langle 2^+, K=2 || Q_2 || 2^+, K=0 \rangle = \sqrt{\frac{10}{7}} q\beta_0 \left(\frac{\hbar}{\sqrt{B_\gamma C_\gamma}} \right)^{1/2} \frac{4(B_{\text{rot}}/B_\gamma)^{1/4}}{(1 + \sqrt{B_{\text{rot}}/B_\gamma})^2}. \quad (13)$$

Taking the corresponding energy eigenvalues

$$E(2^+, K=0) = \frac{\hbar^2}{B_{\text{rot}}\beta_0^2}, \quad (14)$$

$$E(2^+, K=2) = \frac{\hbar}{\beta_0^2} \sqrt{\frac{C_\gamma}{B_\gamma}} \left(2 - \sqrt{\frac{B_\gamma}{B_{\text{rot}}}} \right), \quad (15)$$

TABLE I. The calculated values of the K -mixing parameter $\tan \varphi$ and of the ratio of the mass coefficients for γ vibrations and rotation in the cases with (B_γ/B_{rot}) and without ($B_\gamma/B_{\text{rot}}(\varphi = 0)$) K mixing.

Nucleus	$\tan \varphi$	B_γ/B_{rot}	$B_\gamma/B_{\text{rot}}(\varphi = 0)$
^{156}Gd	-0.0021	2.18	2.19
^{158}Gd	-0.0049	2.39	2.43
^{160}Gd	-0.0066	2.41	2.46
^{158}Dy	-0.0298	2.06	2.25
^{160}Dy	-0.0075	2.37	2.42
^{164}Dy	-0.0084	2.61	2.67
^{162}Er	-0.0172	2.22	2.32
^{164}Er	-0.0122	2.48	2.54
^{166}Er	-0.0059	2.42	2.45
^{168}Er	-0.0057	2.46	2.50
^{168}Yb	-0.0103	2.40	2.47
^{170}Yb	-0.0043	2.71	2.74

where we did not include a contribution of the zero point energy of the β vibrations, we obtain the following relations:

$$\begin{aligned} & \frac{E(2^+, K=2)\langle 2^+, K=2 \parallel Q_2 \parallel 0^+, K=0 \rangle^2}{E(2^+, K=0)\langle 2^+, K=0 \parallel Q_2 \parallel 0^+, K=0 \rangle^2} \\ &= \frac{B_{\text{rot}}}{B_\gamma} \frac{4(B_{\text{rot}}/B_\gamma)^{1/4}}{(1 + \sqrt{B_{\text{rot}}/B_\gamma})^2} \left(2 - \sqrt{\frac{B_\gamma}{B_{\text{rot}}}} \right), \end{aligned} \quad (16)$$

and

$$0.7 \frac{\langle 2^+, K=2 \parallel Q_2 \parallel 2^+, K=0 \rangle^2}{\langle 2^+, K=2 \parallel Q_2 \parallel 0^+, K=0 \rangle^2} = 1 \quad (17)$$

In the derivation of these relations we have neglected by the terms proportional to $E(2^+, K=0)/E(2^+, K=2)$.

It is interesting to substitute into the relations (16) and (17) the experimental data for the ground and the γ bands respectively. Using the experimental data for a selected set of rare earth nuclei given in Tables I and II we obtain for the

TABLE II. The values of $R_{\gamma/\text{g.s.}}$ determined under the assumption that $B_\gamma = B_{\text{rot}}$ ($R_{\gamma/\text{g.s.}}(\text{cal})$) and the experimental values of $R_{\gamma/\text{g.s.}}$ ($R_{\gamma/\text{g.s.}}(\text{exp})$).

Nucleus	$R_{\gamma/\text{g.s.}}(\text{cal})$	$R_{\gamma/\text{g.s.}}(\text{exp})$
^{156}Gd	0.986	0.325
^{158}Gd	0.962	0.256
^{160}Gd	0.952	0.248
^{158}Dy	0.833	0.308
^{160}Dy	0.950	0.258
^{164}Dy	0.939	0.199
^{162}Er	0.904	0.287
^{164}Er	0.921	0.225
^{166}Er	0.964	0.251
^{168}Er	0.962	0.239
^{168}Yb	0.931	0.246
^{170}Yb	0.962	0.183

ratio (16) the values in the limits 0.2–0.3 which indicate on a significant difference between the two mass coefficients B_γ and B_{rot} . Their ratio is shown in Table I for the case of an absence of the K mixing in the column $B_\gamma/B_{\text{rot}}(\varphi = 0)$. For the branching ratio (17) the values for these nuclei are confined in the interval 1.0–1.4. It is clear that the branching ratio (17) can be strongly influenced by the K -mixing effect. Indeed, a small admixture of the $K = 0$ component to the $|2^+, K = 2\rangle$ state can change strongly the $E2$ decay probabilities from this state to the ground state band. The reason is the large $B(E2)$ values between the states of the ground band. The K mixing can also influence the relation (16), however, not much.

Using the values of the K -mixing parameter $\tan \varphi$ (see Appendix B for definition) and $F = \frac{\langle 2^+, K=2 \parallel Q_2 \parallel 0^+, K=0 \rangle}{\langle 2^+, K=0 \parallel Q_2 \parallel 0^+, K=0 \rangle}$ obtained according the procedure described in Appendix B and the experimental data for

$$R_{\gamma/\text{g.s.}} \equiv \frac{E(2_\gamma^+)B(E2; 0_{\text{g.s.}}^+ \rightarrow 2_\gamma^+)}{E(2_{\text{g.s.}}^+)B(E2; 0_{\text{g.s.}}^+ \rightarrow 2_{\text{g.s.}}^+)} = \frac{E(2_\gamma^+)}{E(2_{\text{g.s.}}^+)} A_{\gamma/\text{g.s.}}, \quad (18)$$

we can find the values of B_γ/B_{rot} corrected by the K -mixing effect. The results are presented in Table I, where we show for comparison the values of B_γ/B_{rot} obtained for the case of the absence of the K -mixing ($\varphi = 0$). The ratio of B_γ and B_{rot} is quite large (2.2–2.7) and the use of two very different values of the mass coefficients is essential for a good description of the data on $R_{\gamma/\text{g.s.}}$. This is shown in Table II where the experimental values of $R_{\gamma/\text{g.s.}}$ are compared with those calculated under the assumption that $B_\gamma = B_{\text{rot}}$.

We note from Table I that the values of $\frac{B_\gamma}{B_{\text{rot}}}$ fluctuate from nucleus to nucleus. The fluctuations of the ratio B_γ/B_{rot} from nucleus to nucleus and the large deviation of this ratio from unity are surprising and need additional comments. In the microscopical approach [9,10] the main contribution to the mass coefficient corresponding to the vibrational motion comes from the single particle states lying near the Fermi surface in the single particle configurational space. For this reason, due to the fluctuations in the single particle spectrum near the Fermi surface with the atomic mass the value of the mass coefficient can fluctuate significantly. As is shown in Appendix A this value can be also quite different for different types of collective motion whose quantum characteristics determines which pairs of the particle-hole (two-quasiparticle) states contribute to the value of the corresponding mass coefficient. Thus, the common use of equal mass coefficients for the ground and the gamma band in phenomenological analyses [2–8] is a simplification, which we show to be not appropriate in general, however.

The anharmonic effects in the vibrational motion can also influence the results for the ratio of the mass coefficients. Indeed, a mixing of the one-phonon γ -vibrational state with the two-phonon (one β -phonon and one γ -phonon) state decreases the energy of the 2_γ^+ state and simultaneously decreases the value of $B(E2; 2_\gamma^+ \rightarrow 0_{\text{g.s.}}^+)$ because the weight of the one-phonon component decreases and the $E2$ transition from the two-phonon component is mainly forbidden. As a result the product $E(2_\gamma^+)B(E2; 2_\gamma^+ \rightarrow 0_{\text{g.s.}}^+)$ decreases also. However,

this effect does not exceed 30% [14] what is not enough to explain large effects shown in Table II.

III. SUMMARY

Analyzing the experimental data on the energies of the first and the γ -vibrational 2^+ states and the reduced $E2$ transition probabilities between them and from these states to the ground state it is shown that although the branching ratio from the γ -vibrational state require for the explanation only a small K -mixing effect, the ratio of the products $E(2_i^+)B(E2; 2_i^+ \rightarrow 0_{\text{g.s.}}^+)$, where $i = \text{g.s.}$ or $i = \gamma$ which for the $2_{\text{g.s.}}^+$ was introduced into consideration by L. Grodzins [15], implies significantly different values of the mass coefficients for the ground band and for the γ -band. Thus, the common use of equal mass coefficients for the ground and the gamma band is a simplification, which is not appropriate in general, however. This conclusion is done for the well-deformed nuclei in which rotational and vibrational motion are rather well separated. The reason for this is the following. In the case of a well separated rotational and vibrational motion the transition matrix elements of the quadrupole operator satisfy the Alaga rules and the intrinsic quadrupole matrix element which is by definition independent from angular momentum can be extracted from the experimental data. This matrix element is used to determine a corresponding mass coefficient. This separation of the angular momentum dependence of the quadrupole transition matrix elements is impossible in the case of transitional nuclei.

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APPENDIX A

In the cranking type approach [9] the following expressions can be derived for the moment of inertia and the mass coefficient of the γ vibrations in deformed nuclei:

$$\mathfrak{S} = 2\hbar^2 \sum_k \frac{|\langle k | j_1 | \text{g.s.} \rangle|^2}{E_k - E_{\text{g.s.}}}, \quad (\text{A1})$$

$$B_\gamma = 2\hbar^2 \sum_k \frac{|\langle k | \frac{\partial}{\partial a_{22}} | \text{g.s.} \rangle|^2}{E_k - E_{\text{g.s.}}}. \quad (\text{A2})$$

In these relations the summation is performed over the excited states. Using the relations

$$\langle k | [H, j_1] | \text{g.s.} \rangle = (E_k - E_{\text{g.s.}}) \langle k | j_1 | \text{g.s.} \rangle, \quad (\text{A3})$$

$$\langle k | \left[H, \frac{\partial}{\partial a_{22}} \right] | \text{g.s.} \rangle = (E_k - E_{\text{g.s.}}) \langle k | \frac{\partial}{\partial a_{22}} | \text{g.s.} \rangle, \quad (\text{A4})$$

the expressions (A1) and (A2) can be rewritten in the following way:

$$\mathfrak{S} = 2\hbar^2 \sum_k \frac{|\langle k | [H, j_1] | \text{g.s.} \rangle|^2}{(E_k - E_{\text{g.s.}})^3}, \quad (\text{A5})$$

$$B_\gamma = 2\hbar^2 \sum_k \frac{|\langle k | \left[H, \frac{\partial}{\partial a_{22}} \right] | \text{g.s.} \rangle|^2}{(E_k - E_{\text{g.s.}})^3}. \quad (\text{A6})$$

Restricting our consideration by the Hamiltonian of independent particles moving in a deformed field

$$H = H_{\text{spher}} - \beta \hbar \omega_0 r^2 Y_{20} - a_{22} \hbar \omega_0 r^2 \frac{1}{\sqrt{2}} (Y_{22} - Y_{2-2}) \quad (\text{A7})$$

we obtain the following expressions for \mathfrak{S} and B_γ :

$$\mathfrak{S} = 3 \left(2\hbar^2 (\hbar \omega_0)^2 \sum_{ss'} \frac{(n_s - n_{s'}) |\langle s | \frac{1}{\sqrt{2}} (q_{21} + q_{2-1}) | s' \rangle|^2}{(\epsilon_s - \epsilon_{s'})^3} \right) \beta^2, \quad (\text{A8})$$

$$B_\gamma = 2\hbar^2 (\hbar \omega_0)^2 \sum_{ss'} \frac{(n_s - n_{s'}) |\langle s | \frac{1}{\sqrt{2}} (q_{22} + q_{2-2}) | s' \rangle|^2}{(\epsilon_s - \epsilon_{s'})^3}. \quad (\text{A9})$$

Here $q_{2\mu}$ is the single-particle quadrupole moment operator. Furthermore s, s' are quantum numbers of the single particle states; n_s is an occupation probability of the single particle state s and ϵ_s is a single particle energy. Comparing Eq. (A8) and the hydrodynamic expression for the moment of inertia $\mathfrak{S} = 3B_{\text{rot}}\beta^2$ we obtain

$$B_{\text{rot}} = 2\hbar^2 (\hbar \omega_0)^2 \sum_{ss'} \frac{(n_s - n_{s'}) |\langle s | \frac{1}{\sqrt{2}} (q_{21} + q_{2-1}) | s' \rangle|^2}{(\epsilon_s - \epsilon_{s'})^3}. \quad (\text{A10})$$

The expression (A9) corresponds to the adiabatic limit of γ vibrations when the frequency of the γ vibrations ω_γ is small compared to the energies of the single-particle transitions. The more general expression is

$$B_\gamma = 2\hbar^2 (\hbar \omega_0)^2 \times \sum_{ss'} \frac{(n_s - n_{s'}) (\epsilon_s - \epsilon_{s'}) |\langle s | \frac{1}{\sqrt{2}} (q_{22} + q_{2-2}) | s' \rangle|^2}{((\epsilon_s - \epsilon_{s'})^2 - \omega_\gamma^2)^2}. \quad (\text{A11})$$

Comparing Eqs. (A10) and (A11) we see that in the general case B_{rot} is not equal to B_γ . Only in the limit when ω_γ is small compared to $(\epsilon_s - \epsilon_{s'})$ and for small deformation B_{rot} and B_γ coincide. Indeed, in the case of deformation going to zero a dependence of the matrix elements $\langle s | \frac{1}{\sqrt{2}} (q_{2\mu} + q_{2-\mu}) | s' \rangle$ on the projection μ contains only in a Clebsch-Gordan coefficient and disappears after summation over the projections of the single-particle angular momenta.

The expressions (A10) and (A11) do not include the effect of pairing. However, the generalization including pairing is well known [16] and does not influence our conclusions.

APPENDIX B

Consider the eigenvectors (8) and (9) of the unmixed Hamiltonian (7) as a basis for the construction of the eigenvectors of the more general Hamiltonian which includes a K -mixing term. We are interested here only in the $I^\pi = 2^+$ states, namely, in the lowest 2^+ state, i.e., $|2_{\text{g.s.}}^+\rangle$ and in the γ -vibrational state— $|2_\gamma^+\rangle$. Of course the lowest 0^+ state is not changed by the K -mixing effect, i.e.,

$$|0_{\text{g.s.}}^+\rangle = |0^+, K = 0\rangle. \quad (\text{B1})$$

The $|2_{\text{g.s.}}^+\rangle$ and $|2_\gamma^+\rangle$ states can be presented in the basis (8) and (9) as

$$|2_{\text{g.s.}}^+\rangle = \cos \varphi |2^+, K = 0\rangle - \sin \varphi |2^+, K = 2\rangle, \quad (\text{B2})$$

$$|2_\gamma^+\rangle = \sin \varphi |2^+, K = 0\rangle + \cos \varphi |2^+, K = 2\rangle. \quad (\text{B3})$$

The total Hamiltonian including the K -mixing term is characterized by the following matrix elements:

$$\langle 0^+, K = 0 | H | 0^+, K = 0 \rangle = 0, \quad (\text{B4})$$

$$\langle 2^+, K = 0 | H | 2^+, K = 0 \rangle = E(2^+, K = 0) = \frac{\hbar^2}{B_{\text{rot}} \beta_0^2}, \quad (\text{B5})$$

$$\begin{aligned} \langle 2^+, K = 2 | H | 2^+, K = 2 \rangle &= E(2^+, K = 2) \\ &= \frac{\hbar}{\beta_0^2} \sqrt{\frac{C_\gamma}{B_\gamma}} \left(2 - \sqrt{\frac{B_\gamma}{B_{\text{rot}}}} \right), \end{aligned} \quad (\text{B6})$$

$$\langle 2^+, K = 2 | H | 2^+, K = 0 \rangle = g_{\text{mix}}. \quad (\text{B7})$$

Using the expressions for the wave functions (B1)–(B3) we can find for the corresponding states the energies and the $E2$ transition matrix elements in terms of the matrix elements

(B4)–(B7). The results are

$$\begin{aligned} \langle 2_{\text{g.s.}}^+ \| Q_2 \| 0_{\text{g.s.}}^+ \rangle &= \cos \varphi \langle 2^+, K = 0 \| Q_2 \| 0^+, K = 0 \rangle \\ &\quad \times (1 - F \tan \varphi), \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} \langle 2_\gamma^+ \| Q_2 \| 0_{\text{g.s.}}^+ \rangle &= \cos \varphi \langle 2^+, K = 0 \| Q_2 \| 0^+, K = 0 \rangle \\ &\quad \times (\tan \varphi + F), \end{aligned} \quad (\text{B9})$$

$$\begin{aligned} \langle 2_\gamma^+ \| Q_2 \| 2_{\text{g.s.}}^+ \rangle &= -\sqrt{\frac{10}{7}} \cos 2\varphi \langle 2^+, K = 0 \| Q_2 \| 0^+, K = 0 \rangle \\ &\quad \times (\tan 2\varphi + F), \end{aligned} \quad (\text{B10})$$

$$E(2_{\text{g.s.}}^+) = E(2^+, K = 2) \frac{GF^2 - \tan^2 \varphi}{1 - \tan^2 \varphi}, \quad (\text{B11})$$

$$E(2_\gamma^+) = E(2^+, K = 2) \frac{1 - GF^2 \tan^2 \varphi}{1 - \tan^2 \varphi}, \quad (\text{B12})$$

where

$$G = \frac{B_\gamma}{B_{\text{rot}}} \frac{(1 + \sqrt{B_{\text{rot}}/B_\gamma})^2}{4(B_{\text{rot}}/B_\gamma)^{1/4} (2 - \sqrt{B_\gamma/B_{\text{rot}}})}, \quad (\text{B13})$$

$$F \equiv \frac{\langle 2^+, K = 2 \| Q_2 \| 0^+, K = 0 \rangle}{\langle 2^+, K = 0 \| Q_2 \| 0^+, K = 0 \rangle} = \left(\frac{\hbar}{\sqrt{B_\gamma C_\gamma}} \right)^{1/2}. \quad (\text{B14})$$

The values of $\tan \varphi$ and F can be found using the experimental data for the following $B(E2)$ ratios:

$$A_{2_\gamma} \equiv 0.7 \frac{B(E2; 2_\gamma^+ \rightarrow 2_{\text{g.s.}}^+)}{B(E2; 2_\gamma^+ \rightarrow 0_{\text{g.s.}}^+)}, \quad (\text{B15})$$

and

$$A_{\gamma/\text{g.s.}} \equiv \frac{B(E2; 2_\gamma^+ \rightarrow 0_{\text{g.s.}}^+)}{B(E2; 2_{\text{g.s.}}^+ \rightarrow 0_{\text{g.s.}}^+)}, \quad (\text{B16})$$

which can be expressed in terms of $\tan \varphi$ and F as

$$A_{2_\gamma} = \left(\frac{\cos 2\varphi (\tan 2\varphi - F)}{\cos \varphi (\tan \varphi + F)} \right)^2 \quad (\text{B17})$$

and

$$A_{\gamma/\text{g.s.}} = \left(\frac{\tan \varphi + F}{1 - F \tan \varphi} \right)^2. \quad (\text{B18})$$

- [1] A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **26**, no. 14, 1 (1952).
 [2] F. Iachello, Phys. Rev. Lett. **85**, 3580 (2000).
 [3] F. Iachello, Phys. Rev. Lett. **87**, 052502 (2001).
 [4] D. Rowe and C. Bahri, J. Phys. A **31**, 4947 (1998).
 [5] D. Bonatsos, D. Lenis, N. Minkov, D. Petrellis, P. P. Raychev, and P. A. Terziev, Phys. Rev. C **70**, 024305 (2004).
 [6] L. Fortunato and A. Vitturi, J. Phys. G **30**, 627 (2004).
 [7] N. Pietralla and O. M. Gorbachenko, Phys. Rev. C **70**, 011304(R) (2004).
 [8] K. Dusling and N. Pietralla, Phys. Rev. C **72**, 011303(R) (2005).

- [9] S. A. Moszkowski, in Encyclopedia of Physics, Structure of Atomic Nuclei, edited by S. Flügge, Vol. xxxix.
 [10] V. A. Khodel and E. E. Saperstein, Phys. Rep. **92**, 183 (1982).
 [11] R. V. Jolos, P. von Brentano, and N. Pietralla, Phys. Rev. C **71**, 044305 (2005).
 [12] R. V. Jolos, P. von Brentano, A. Dewald, and N. Pietralla, Phys. Rev. C **72**, 024310 (2005).
 [13] S. K. Bhardwaj, K. K. Gupta, J. B. Gupta, and D. K. Gupta, Phys. Rev. C **27**, 872 (1983).
 [14] V. O. Nesterenko, Z. Phys. A **335**, 147 (1990).
 [15] L. Grodzins, Phys. Lett. **2**, 88 (1962).
 [16] A. B. Migdal, Nucl. Phys. **13**, 655 (1959).