Q-value effects on the production of superheavy nuclei

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The formation of superheavy nucleus ^{270}Hs via the 4n evaporation channel of fusion reactions ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra is studied using a two-parameter Smoluchowski equation. The evaporation residual cross sections of the reactions ^{48}Ca + ^{226}Ra and ^{36}S + ^{238}U are obviously enhanced because of their large negative Q values. The enhancement is due to the fact that the excitation energy corresponding to the maximum yield of the evaporation residue depends on the reaction Q value, and the maximum cross section sensitively depends on the increment of this excitation energy relative to the effective threshold energy of which the channel for fission after 4n emission opens.

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For sufficiently heavy systems, automatic fusion will not take place, because the overall length of the saddle-point shape shrinks inside the contact configuration of the colliding nuclei [1]. Hence after contact and the formation of a heavy composite mononucleus, the system finds itself outside the saddle-point barrier, and diffusing over the barrier is necessary for the system to reach the compound nucleus configuration. For the events that fail to diffuse over the saddle-point hill, the composite nucleus then redisintegrates in a fission-like process, which is referred to as quasifission (QF). There is a strong competition between compound-nucleus (CN) formation and quasifission in the fusion process of heavy nuclei. This competition severely hinders the formation of superheavy nuclei. It is well established that the probability of quasifission ($P_{\rm QF}$) strongly depends on the value Z_1Z_2. Moreover, the production cross section depends on mass asymmetry $\eta = (A_2 - A_1)/(A_1 + A_2)$ of the reaction system in the entrance channel. Here $A_1(Z_1)$ and $A_2(Z_2)$ are the mass (charge) numbers of the projectile and target nuclei.

The entrance channel dependence of the formation cross section for superheavy nuclei has been of great concern in recent years. This is because the cross section for the synthesis of superheavy elements (SHE) is so small that it reaches the present experimental limit for the registration of the evaporation residual nuclei. Therefore, it is key to select the optimal reaction and favorable bombarding energy before the experiment to ensure the successful synthesis of SHE. There is a general opinion that it is more favorable to synthesize SHE with large mass asymmetry of the reaction system. This is usually the case, but sometimes it is not because the formation of superheavy nuclei depends not only on the mass asymmetry but also on other factors in the entrance channel. In the present work, we study the entrance channel dependence of the formation cross section of ^{270}Hs through 4n evaporation channels of the reactions ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra. Our results show that in addition to the mass asymmetry, the reaction Q value plays an important role in the formation of superheavy nuclei. In addition, the macroscopic-microscopic approach [2–5] predicts a strong proton-deformed shell at $Z = 108$ to be a partner for the neutron shell at $N = 162$. Thus the nucleus ^{270}Hs is expected to be a relatively strongly bound "double-magic" deformed nucleus. Therefore, it is meaningful to predict the optimal selection of the reaction system and bombarding energy for synthesis of the nucleus ^{270}Hs.

The production cross section of a cold residual nucleus is usually decomposed over partial waves and given by

$$\sigma_{\rm ER}(E) = \pi \lambdabar^2 \sum_{l=0}^{\infty}(2l+1)T_l(E)P_{\rm CN}(E,l)W_{\rm sur}(E,l). \quad (1)$$

Here $T_l(E)$ is the probability that the colliding nuclei penetrate the entrance channel potential barrier and reach the contact point. We calculated $T_l(E)$ by means of an approach proposed by Zagrebaev *et al.* [6,7]. In their approach, the coupling between the relative motion of the nuclei and their dynamic deformations is taken into account in terms of a semiphenomenological barrier distribution function method. This approach was successfully applied to describe the capture cross sections for a number of reactions leading to superheavy nucleus formation.

$P_{\rm CN}$ defines the probability that the system will go from the configuration of two nuclei in contact to the configuration of a compound nucleus. Swiatecki *et al.* [8,9] propose a model to evaluate the probability $P_{\rm CN}$. They assume that the dynamics in the second stage, including statistical fluctuations, can be described by a diffusion process analogous to the one-dimensional Brownian motion of a particle suspended in a viscous fluid at temperature T in the presence of a repulsive parabolic potential. The equation describing the drift and the spreading of the probability distribution $W(x,t)$ is the Smoluchowski partial differential equation. Here x denotes the relative length s between the effective surfaces of the approaching nuclei. As they point out [9] their model has advantages: operationally the scheme is very transparent, and the calculations are sufficiently simple. Based on their model, we make a modification to include the neutron flow in the early stage of evolution, and we assume the diffusion process to be

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described by a two-parameter Smoluchowski equation [10]:

$$\frac{\partial W(x,y,t)}{\partial t} = [L_x(x,y) + \gamma L_y(x,y)]W(x,y,t). \quad (2)$$

The operators L_x and L_y are given by

$$L_x(x,y) = -\frac{\partial}{\partial x}D_x(x,y) + D_{xx}\frac{\partial^2}{\partial x^2}, \quad (3)$$

$$L_y(x,y) = -\frac{\partial}{\partial y}D_y(x,y) + D_{yy}\frac{\partial^2}{\partial y^2}. \quad (4)$$

Here y represents the neutron number N of the light nucleus. We assume that the diffusion coefficients, D_{xx} and D_{yy} are constants, i.e., $D_{xx} = kT/\alpha_x$ and $D_{yy} = kT/\alpha_y$. Here α_x and α_y are proportional to the dissipation acting in the degrees of freedom x and y, respectively. For $t \to \infty$, the final results are independent of them [8,9]. The potential $V_{\mathrm{af}}(x,y)$, which determines the corresponding saddle-point barrier, is calculated with the formulas in Ref. [9] and approximated by a repulsive parabolic potential $V_{\mathrm{af}}(x,y) = -a(y)(x - x_{\max}(y))^2/2$, with $x_{\max}(y)$ locating the maximum value in V_{af}. The mass asymmetry, the neck formation, and the overall length of the composite nucleus are taken into account in the potential calculation. By means of this potential, we get the drift coefficient $D_x = a(y)(x - x_{\max})/\alpha_x$. The drift coefficient D_y is proportional to the y-direction driving force evaluated with the potential governing the neutron flow [10].

In Eq. (2) we introduced parameter γ to indicate the different time scales. It is well established that in low-energy heavy-ion collisions, the N/Z equilibrium happens on a time scale much faster than that of other collective motions, such as the change in the overall length of the configuration. In the limit $\gamma \gg 1$, which is consistent with the assumption that y will decay very rapidly to an equilibrium value y_{eq}, Eq. (2) can be reduced to the Smoluchowski equation with one parameter [10,11],

$$\frac{\partial W(x,t)}{\partial t} = L_{00}W(x,t). \quad (5)$$

The operator $L_{0,0}$ has the form

$$L_{00} = -\frac{\partial}{\partial x}\overline{D}_x(x) + D_{xx}\frac{\partial^2}{\partial x^2}, \quad (6)$$

with

$$\overline{D}_x(x) = \int D_x(x,y)\phi_0(x,y)dy. \quad (7)$$

Here $\phi_0(x,y)$ is the eigenfunction of the operator $L_y(x,y)$ for $n = 0$:

$$\phi_0(x,y) = \sqrt{\frac{b(x)}{2\pi kT}}\exp\left[-\frac{b(x)(y - y_{\mathrm{eq}})^2}{2kT}\right]. \quad (8)$$

The probability to reach the compound nucleus configuration is equal to the area under the distribution's tail in the region $x \leqslant x_{\max}$ [9,10],

$$P_{\mathrm{CN}} = \frac{1}{2}\mathrm{erfc}\sqrt{\overline{\beta}}, \quad (9)$$

where $\overline{\beta} = \overline{B}/kT$ and erfc is the error function complement, equal to (1-erf). At time $t = 0$, the composite nucleus injects the asymmetric fission valley at x_0. According to Eq. (7), the average barrier height to be overcome by the diffusion reads

$$\overline{B} = \int B(y)\phi_0(x_0,y)dy, \quad (10)$$

with $B(y)$ being the barrier height of the saddle point for different y values. Figure 1(a) shows the probability of the CN formation as a function of mass asymmetry for the reactions ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra. The P_{CN} values are evaluated at the energy corresponding to the maximum of evaporation residual excitation function. It can be seen that the probabilities P_{CN} monotonously increase as η. The calculated compound formation cross sections are presented in Fig. 1(b). The CN cross sections are larger for the reaction systems with larger mass asymmetry. Simply, the CN formation is favorable for the systems with larger mass asymmetry.

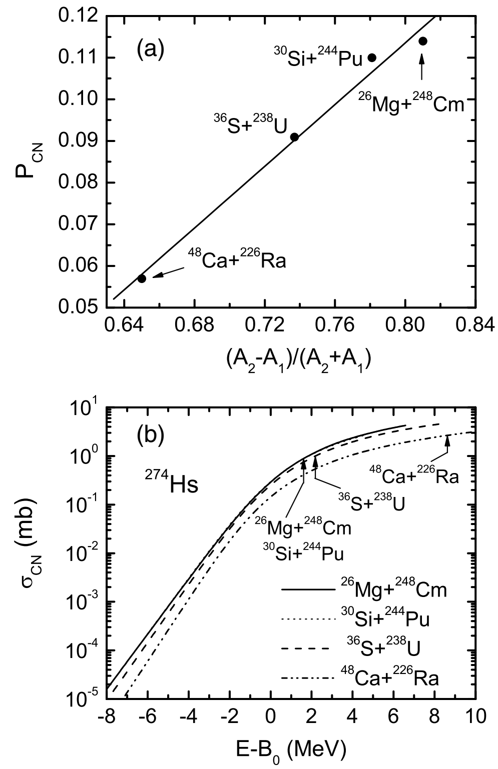

FIG. 1. The probability of CN formation as a function of mass asymmetry (panel a) and the cross sections of the CN formation as a function of $(E - B_0)$ (panel b) for the reaction systems ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra. B_0 is the average height of the entrance barrier. The straight line in (a) represents the least-square fit of the data to guide the eye. The probabilities of the CN formation are calculated at the energy corresponding to the maximum of ER excitation function. The arrows in (b) illustrate the positions of these energies.

Finally, W_{sur} represents the probability that a cold residual nucleus will be produced in the decay of the excited compound nucleus. The survival probability for emitting x neutrons can be written as

$$W_{\mathrm{sur}}(E^*_{\mathrm{CN}}, l) = G_{\mathrm{xn}}(E^*_{\mathrm{CN}}, l)\Pi_k\left[\frac{\Gamma_n(U^{\max}_{k,n})}{\Gamma_f(U^{\max}_{k,f}) + \Gamma_n(U^{\max}_{k,n})}\right]_k, \quad (11)$$

where G_{xn} is the probability of realization of x neutron evaporation [12,13]. The partial width of neutron emission can be expressed as [12,14]

$$\Gamma_n(U_{k,n}^{\max}) = \frac{gm\sigma_n U_{k,n}^{\max}}{\pi^2\hbar^2 a_n}\exp\left(2\sqrt{a_n U_{k,n}^{\max}} - 2\sqrt{a_0 U_0}\right), \quad (12)$$

where m and g are the neutron mass and spin degeneration factor, σ_n is the cross section for the formation of the decaying nucleus with mass number A in the inverse process, and a_0 and a_n are the level density parameters of the parent nucleus and the nucleus after particle emission. The value of the level density parameter is set to $A/12.0$ in our calculations. $U_0 = E_{\rm CN}^* - E_0^{\rm rot} - P_0$ is the thermal excitation energy of the parent nucleus corrected for its pairing energy P_0. The upper limit of the final-state thermal excitation energy for the kth neutron emission is $U_{k,n}^{\max} = E_k^* - S_n(k) - E_k^{\rm rot} - P_k$. The energy E_k^* is the excitation energy after $(k-1)$ neutron emission,

$$E_k^* = E_{\rm CN}^* - \sum_{i=1}^{k-1}(S_n(i) + 2T_i), \quad (13)$$

where $S_n(i)$, and T_i are the neutron separation energy and nuclear temperature of the $(i-1)$th generation daughter nucleus. The rotational energy $E_k^{\rm rot}$ is calculated with the ground-state deformation as predicted in Ref. [15]. In the calculation of Γ_n, we adopt the approximation proposed by Vandenbosch and Huizenga [12], i.e., $(\pi\hbar^2)/(gm\sigma_n) \approx 10\,{\rm MeV}\,A^{-2/3}$. The partial width of fission is given by [12,14]

$$\Gamma_f\left(U_{k,f}^{\max}\right) = \frac{2\sqrt{a_f U_{k,f}^{\max}} - 1}{4\pi a_f}\exp\left(2\sqrt{a_f U_{k,f}^{\max}} - 2\sqrt{a_0 U_0}\right), \quad (14)$$

where a_f is the level density parameter at the saddle point. The ratio of the level density parameters in the fission and neutron evaporation channels is equal to $a_f/a_n = 1.07$ [12]. The $U_{k,f}^{\max}$ denotes the upper limit of the thermal excitation energy at the saddle point for the nucleus after $(k-1)$ neutrons are emitted: $U_{k,f}^{\max} = E_k^* - B_f(k) - E_{k,\rm sd}^{\rm rot} - P_k$. The fission barrier depends on the excitation energy of the nucleus as

$$B_f(k) = B_{\rm LD}(k) - \Delta_{\rm sh}(k)\exp(-E_k^*/E_D), \quad (15)$$

where $E_d = 25$ MeV is the shell-damping energy [16]. The fission barrier B_f contains the macroscopic liquid drop energy part $B_{\rm LD}$ and microscopic shell correction part $\Delta_{\rm sh}$. The values of $B_{\rm LD}$ are estimated from the liquid drop approximation [17], and those of $\Delta_{\rm sh}$ are taken from Ref. [15]. The rotational energy is calculated with the deformation at the saddle point [18].

We present the cross sections of evaporation residue (ER) as a function of the CN excitation energy in Fig. 2 for the reaction systems considered. In contrast to the CN formation, the ER cross sections of the systems ^{48}Ca + ^{226}Ra and ^{36}S + ^{238}U are obviously greater than those of the systems ^{26}Mg + ^{248}Cm and ^{30}Si + ^{244}Pu. This change should be attributed to the reaction Q value because we are dealing with the reactions to forming the same CN ^{270}Hs; hence, the parameters relevant to the exit channels, such as the neutron separation energies S_n and the fission barrier heights B_f, are the same.

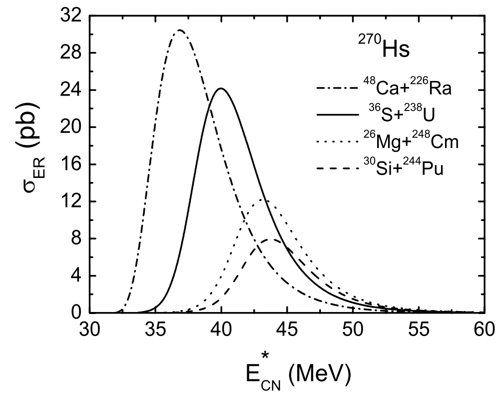

FIG. 2. The evaporation residue cross sections ($\sigma_{\rm ER}$) for the 4n channel of fusion reactions ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra.

There are two reasons for the enhancement of the ER cross sections for the reactions ^{48}Ca + ^{226}Ra and ^{36}S + ^{238}U. First and foremost, the peak position of the ER excitation function depends on the reaction Q value as illustrated in Fig. 3. As formulated in Eq. (1), the ER cross section is the product of three factors: the transmission coefficient T_l, the CN formation probability $P_{\rm CN}$, and the survival probability $W_{\rm sur}$ of the compound nucleus. Among these factors, $W_{\rm sur}$ itself contains two constituents, i.e., G_{xn} and Γ_n/Γ_t, with Γ_t the total decay width. All three factors, T_l, $P_{\rm CN}$, and Γ_n/Γ_t increase with energy, whereas G_{xn} decreases exponentially beyond certain threshold energy $E_{\rm th}$ of which the channel for fission after x neutron emission opens. The dashed, dash-dotted, and solid lines in Fig. 3 represent the reduced ER cross section $\tilde{\sigma}_{\rm ER} = \sigma_{\rm ER}/\pi\bar{\lambda}^2$, the average values $\langle T_l P_{\rm CN}\Gamma_n/\Gamma_t\rangle$, and the survival probability $W_{\rm sur}$ as a function of excitation energy, respectively. For the reaction ^{48}Ca + ^{226}Ra, the center-of-mass energy relevant to the peak position of the ER excitation function, as illustrated by the arrow in Fig. 1(b), locates at well above the Coulomb barrier because of its large negative Q value.

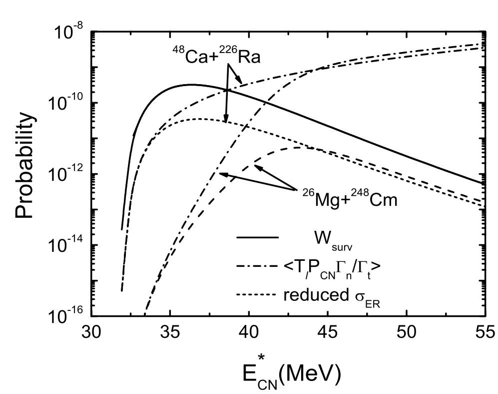

FIG. 3. The reduced ER cross section $\tilde{\sigma}_{\rm ER}$, average value $\langle T_l P_{\rm CN}\Gamma_n/\Gamma_t\rangle$, and survival probability $W_{\rm sur}$ as a function of the CN excitation energy for the reactions ^{26}Mg + ^{248}Cm and ^{48}Ca + ^{226}Ra.

The probability of CN formation $\langle T_l P_{\rm CN}\rangle$ increases relatively slowly as a function of energy at the well-above-barrier-energy region. Hence the increased slope of $\langle T_l P_{\rm CN}\Gamma_n/\Gamma_t\rangle$ is mainly controlled by Γ_n/Γ_t. This results in the maximum of the ER excitation function being at nearly the same excitation energy as the one of the survival probability $W_{\rm sur}$. On the other hand, for the reaction systems ^{26}Mg + ^{248}Cm and ^{30}Si + ^{244}Pu, the maxima of the ER excitation function appear at the energies near the barrier. In this energy region, the increase of the CN formation probability is relatively fast, and the decrease of $G_{\rm xn}$ is somehow balanced by this rapid increase near the threshold excitation energy $E_{\rm th}$, so as to move the peak position of the ER excitation function to higher excitation energy. Because of the exponential decrease of $W_{\rm sur}$, any slight increase of excitation energy above the threshold energy $E_{\rm th}$ will dramatically reduce the ER cross section. This is shown for the reaction ^{26}Mg + ^{248}Cm in Fig. 3. Second, although CN formation is favorable for the systems with larger asymmetry, as shown in Fig. 1(b) the probabilities of CN formation at the energy corresponding to the maximum of ER excitation function for the systems ^{48}Ca + ^{226}Ra and ^{36}S + ^{238}U are still larger than those of the systems ^{26}Mg + ^{248}Cm and ^{30}Si + ^{244}Pu.

In summary, we studied the probabilities of fusion reactions ^{26}Mg + ^{248}Cm, ^{30}Si + ^{244}Pu, ^{36}S + ^{238}U, and ^{48}Ca + ^{226}Ra by using a two-parameter Smoluchowski equation. As expected, the CN formation is favorable for the systems with larger mass asymmetry. However, the ER cross sections for the 4n channel of the reactions ^{48}Ca + ^{226}Ra and ^{36}S + ^{238}U are obviously larger than those of the reaction systems ^{26}Mg + ^{248}Cm and ^{30}Si + ^{244}Pu because of the large negative Q values of the former reactions. The enhancement of ER cross sections is due to the fact that the energy corresponding to the maximum of the ER excitation function depends on the reaction Q value. The maximum ER cross section sensitively depends on the peak position of the ER excitation function relative to the effective threshold energy $E_{\rm th}$. As a final remark, we wish to emphasize that, in addition to the mass asymmetry, the reaction Q value plays an important role in superheavy nucleus formation. This is the reason why the ^{208}Pb and ^{209}Bi targets and the ^{48}Ca projectile are, respectively, used in the cold fusion and hot fusion of superheavy nuclei.

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