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# The $qqqq\bar{q}$ components and hidden flavor contributions to the baryon magnetic moments

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The contributions from the $qqqq\bar{q}$ components to the magnetic moments of the octet as well as the $\Delta^{++}$ and $\Omega^-$ decuplet baryons are calculated for the configurations that are expected to have the lowest energy if the hyperfine interaction depends on both spin and flavor. The contributions from the $u\bar{u}$, $d\bar{d}$, and the $s\bar{s}$ components are given separately. It is shown that addition of $qqqq\bar{q}$ admixtures to the ground state baryons can improve the overall description of the magnetic moments of the baryon octet and decuplet in the quark model without SU(3) flavor symmetry breaking, beyond that of the different constituent masses of the strange and light-flavor quarks. The explicit flavor (and spin) wave functions for all the possible configurations of the $qqqq\bar{q}$ components with light and strange $q\bar{q}$ pairs are given for the baryon octet and decuplet. Admixtures of ~10% of the $qqqq\bar{q}$ configuration where the flavor-spin symmetry is $[4]_{FS}[22]_F[22]_S$, which is likely to have the lowest energy, in particular reduces the deviation from the empirical values of the magnetic moments $\Sigma^-$ and the $\Xi^0$ compared with the static $qqq$ quark model.

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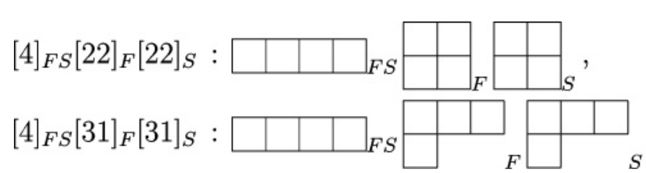
## I. INTRODUCTION

Recent measurements of the $\bar{d}/\bar{u}$ asymmetry in the nucleon sea indicate a considerable isospin symmetry breaking in the light quark sea of nucleon. This indicates that the nucleon contains notable $qqqq\bar{q}$ components, if not more exotic components, besides the conventional $qqq$ component [1–4]. The experiments on parity violation in electron-proton scattering moreover indicate that $s\bar{s}$ quark pairs lead to nonzero contributions to the magnetic moment of the proton [5–11].

Here the contributions to the baryon magnetic moments from wave function components with at most one $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$ sea quark pair are calculated in the nonrelativistic quark model. The conventional $qqq$ constituent quark model by itself provides a qualitative description of the magnetic moments of the baryon octet. Quantitatively the description is, however, not better than ~15%, the largest differences from the experimental values being the magnetic moments of the $\Sigma^-$ and the $\Xi^0$. While covariant reformulation of the $qqq$ model does not change this situation [12], the overall description may be improved somewhat by adding effects of meson exchange and orbital excitations to the $qqq$ model wave functions [13–15]. This of course also suggests that explicit $q\bar{q}$ terms should be included in the quark model for an improved description. Such explicit $qqqq\bar{q}$ components with light $q\bar{q}$ pairs have been shown to improve significantly the agreement between the calculated and the measured the decays of low-lying baryon resonances [16–18].

The experimental results on the strangeness magnetic moments can be described, at least qualitatively, by $uuds\bar{s}$ configurations in the proton, where the $\bar{s}$ antiquark is in the $S$-state [19–21]. Since configurations with the antiquark in the $S$-state cannot be represented by long range pion or kaon loop fluctuations, this motivates to a systematic extension of the $qqq$ quark model to include the $qqqq\bar{q}$ configurations explicitly.

Here the explicit flavor and spin wave functions for all the possible configurations of the $qqqq\bar{q}$ system in the baryon octet and $\Delta^{++}$ and $\Omega^-$ decuplet baryons, with totally symmetric spin-flavor symmetry are derived, when both light and strange $q\bar{q}$ pairs are taken into account. Finally the contributions to the magnetic moments of the octet and decuplet baryons are evaluated for the mixed symmetry configurations $[4]_{FS}[22]_F[22]_S$ and $[4]_{FS}[31]_F[31]_S$, which are likely to have the lowest energy in the case of the octet and decuplet baryons, respectively. This notation is a shorthand for the Young tableaux decomposition:




(1)

An interesting result is that this extension of the quark model allows to a notable reduction of the overall disagreement with the empirical magnetic moments when the $qqqq\bar{q}$ admixture is of the order of ~10% or more and if the $qqqq\bar{q}$ component is more compact than the $qqq$ component. The improvement is particularly notable in the case of the $\Sigma^-$ and $\Xi^0$ hyperons.

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Explicit expressions for the magnetic moments from the $u\bar{u}, d\bar{d}$ as well as the $s\bar{s}$ seaquark pairs are given. From these the "strangeness magnetic" moments, i.e., the magnetic moment contributions from $s\bar{s}$ pairs, of all the baryons can be read off. In addition the light flavor seaquark contributions as, e.g., the contributions of $u\bar{u}$ and $d\bar{d}$ pairs to the magnetic moment of the $\Omega^-$ are given explicitly.

The present paper is organized in the following way. In Sec. II the explicit flavor wave functions of the $qqqq\bar{q}$ components in the baryon octet and decuplet from the SU(3) symmetry are given. Section III contains the expressions and the corresponding numerical results for the baryon octet. The corresponding expressions for the two decuplet magnetic moments are given in Sec. IV. The strangeness magnetic moments are considered in Sec. V and the light flavor seaquark magnetic moments in Sec. VI. Finally Sec. VII contains a concluding discussion.

## II. THE FLAVOR WAVE FUNCTIONS OF THE $qqqq\bar{q}$ COMPONENTS

The color symmetry of the $qqqq$ subsystem in a $qqqq\bar{q}$ component in a baryon is limited to $[211]_C$ by the requirement that it form a color singlet in combination with the antiquark. The Pauli principle then requires that the corresponding orbital-flavor-spin states have the mixed symmetry $[31]_{XFS}$ in order to combine with the color state $[211]_C$ to the required completely antisymmetric $4q$ state [1111]. Since the intrinsic parity is positive for a quark and negative for an antiquark, the $qqqq\bar{q}$ components, which have positive parity, require that either the $\bar{q}$ is in the P-state and that the $qqqq$ subsystem is in the spatially symmetric ground state ($[4]_X$) or that one of the quarks is in the P-state so that the $qqqq$ subsystem has mixed spatial symmetry $[31]_X$ and that the $\bar{q}$ is in its ground state. The extant experimental data on the strangeness magnetic moment of the proton suggests that the $qqqq$ subsystem has mixed spatial symmetry $[31]_X$ if one of the quarks is strange [19–21].

The possible spin symmetry states of four quarks are $[4]_S$, $[31]_S$, and $[22]_S$. There are four possible flavor symmetry configurations for the $qqqq$ subsystem, which can combined with the spatial and spin symmetry to form the orbital-flavor-spin symmetry $[31]_{XFS}$: $[4]_F$, $[31]_F$, $[22]_F$, and $[211]_F$ in the Weyl tableaux of the group SU(3) [24,25]. Combination of these flavor states with the antiquark with flavor $[1]^*_F$ leads to the following $qqqq\bar{q}$ multiplet representations of SU(3):

$$[4]_F \otimes [1]^* = \mathbf{10} \oplus \mathbf{35}, \tag{2}$$
$$[31]_F \otimes [1]^* = \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{27}, \tag{3}$$
$$[22]_F \otimes [1]^* = \mathbf{8} \oplus \overline{\mathbf{10}}, \tag{4}$$
$$[211]_F \otimes [1]^* = \mathbf{1} \oplus \mathbf{8}. \tag{5}$$

Here the numbers in boldface on the right-hand side of the equations indicate the dimensions of the pentaquark representations. As an example the possible $\theta^+$-pentaquark may belong to the baryon antidecuplet $\overline{\mathbf{10}}$ representation [22]. It's obvious that the $qqqq\bar{q}$ components of the baryon decuplet belong to the **10** representations, and that of the baryon octet belongs to the **8** representations. The combination of the flavor states with the spin states gives rise to several flavor-spin states, which can be split by the spin-dependent hyperfine interaction between the quarks.

If the hyperfine interaction between the quarks depends on spin- and flavor [23] the $qqqq$ systems with the mixed spatial symmetry $[31]_X$ are expected to be the configurations with the lowest energy, and therefore most likely to form appreciable components of the proton. Consequently the flavor-spin state of the $qqqq$ system is most likely totally symmetric: $[4]_{FS}$. Moreover in the case of the baryon octet the flavor-spin state $[4]_{FS}[22]_F[22]_S$, with one quark in its first orbitally excited state, is likely to have the lowest energy [26]. For the baryon decuplet the flavor symmetry $[22]_F$ is, however, not possible for the $qqqq$ subsystem, and the $[4]_{FS}[31]_F[31]_S$ symmetry configuration is expected to have the lowest energy [26].

A baryon wave function that includes $qqqq\bar{q}$ components in addition to the conventional $qqq$ components may be written in the following general form:

$$|B\rangle = \sqrt{P_{3q}}|qqq\rangle + \sqrt{P_{5q}}\sum_i A_i|qqqq_i\bar{q}_i\rangle. \tag{6}$$

Here $P_{3q}$ and $P_{5q}$ are the probabilities of the $qqq$ and $qqqq\bar{q}$ components, respectively; the sum over $i$ runs over all the possible $qqqq_i\bar{q}_i$ components, and $A_i$ denotes the amplitude of the corresponding $5q$ component. The wave functions of the $qqq$ components are the conventional $SU(6)_{SF}$ ones. Here the flavor (and spin) wave functions of every $qqqq_i\bar{q}_i$ component is constructed along with a calculation of the corresponding amplitudes $A_i$. Note that the baryons still have the SU(3) flavor symmetry when the $qqqq\bar{q}$ components have been taken into account.

The weight diagram method [24] of the SU(3) group will be employed here for the explicit construction of the wave functions of baryon multiplet in $qqqq_i\bar{q}_i$ configurations. These wave functions and amplitudes for the baryon octet and decuplet, which have complete flavor-spin symmetry $[4]_{FS}$ are listed in Tables I, II, and III.

TABLE I. The $qqqq\bar{q}$ components, in which the $qqqq$ subsystem has the flavor symmetry $[4]_F$.

| $(\frac{3}{2}^+, Y, I, I_3)$ | $qqqq\bar{q}$ component |
|---|---|
| $(1, 3/2, +3/2)$ | $\sqrt{\frac{2}{3}}\|uuuu\bar{u}\rangle + \sqrt{\frac{1}{6}}\|uuud\bar{d}\rangle + \sqrt{\frac{1}{6}}\|uuus\bar{s}\rangle$ |
| $(1, 3/2, +1/2)$ | $\sqrt{\frac{1}{2}}\|uudu\bar{u}\rangle + \sqrt{\frac{1}{3}}\|uudd\bar{d}\rangle + \sqrt{\frac{1}{6}}\|uuds\bar{s}\rangle$ |
| $(1, 3/2, -1/2)$ | $\sqrt{\frac{1}{3}}\|uddu\bar{u}\rangle + \sqrt{\frac{1}{2}}\|uddd\bar{d}\rangle + \sqrt{\frac{1}{6}}\|udds\bar{s}\rangle$ |
| $(1, 3/2, -3/2)$ | $\sqrt{\frac{1}{6}}\|dddu\bar{u}\rangle + \sqrt{\frac{2}{3}}\|dddd\bar{d}\rangle + \sqrt{\frac{1}{6}}\|ddds\bar{s}\rangle$ |
| $(0, 1, +1)$ | $\sqrt{\frac{1}{2}}\|uusu\bar{u}\rangle + \sqrt{\frac{1}{6}}\|uusd\bar{d}\rangle + \sqrt{\frac{1}{3}}\|uuss\bar{s}\rangle$ |
| $(0, 1, 0)$ | $\sqrt{\frac{1}{3}}\|udsu\bar{u}\rangle + \sqrt{\frac{1}{3}}\|udsd\bar{d}\rangle + \sqrt{\frac{1}{3}}\|udss\bar{s}\rangle$ |
| $(0, 1, -1)$ | $\sqrt{\frac{1}{6}}\|ddsu\bar{u}\rangle + \sqrt{\frac{1}{2}}\|ddsd\bar{d}\rangle + \sqrt{\frac{1}{3}}\|ddss\bar{s}\rangle$ |
| $(-1, 1/2, +1/2)$ | $\sqrt{\frac{1}{3}}\|ussu\bar{u}\rangle + \sqrt{\frac{1}{6}}\|ussd\bar{d}\rangle + \sqrt{\frac{1}{2}}\|usss\bar{s}\rangle$ |
| $(-1, 1/2, -1/2)$ | $\sqrt{\frac{1}{6}}\|dssu\bar{u}\rangle + \sqrt{\frac{1}{3}}\|dssd\bar{d}\rangle + \sqrt{\frac{1}{2}}\|dsss\bar{s}\rangle$ |
| $(-2, 0, 0)$ | $\sqrt{\frac{1}{6}}\|sssu\bar{u}\rangle + \sqrt{\frac{1}{6}}\|sssd\bar{d}\rangle + \sqrt{\frac{2}{3}}\|ssss\bar{s}\rangle$ |

TABLE II. The $qqqq\bar{q}$ components, in which the $qqqq$ subsystem has the flavor symmetry $[31]_F$.

| $(\frac{3}{2}^+, Y, I, I_3)$ | $qqqq\bar{q}$ component | $(\frac{1}{2}^+, Y, I, I_3)$ | $qqqq\bar{q}$ component |
|---|---|---|---|
| (1, 3/2, +3/2) | $-\sqrt{\frac{1}{2}}(\|uuud\bar{d}\rangle + \|uuus\bar{s}\rangle)$ | | |
| (1, 3/2, +1/2) | $\sqrt{\frac{1}{6}}\|uudu\bar{u}\rangle - \sqrt{\frac{1}{3}}\|uudd\bar{d}\rangle - \sqrt{\frac{1}{2}}\|uuds\bar{s}\rangle$ | (1, 1/2, +1/2) | $-(\sqrt{\frac{8}{15}}\|uudu\bar{u}\rangle + \sqrt{\frac{4}{15}}\|uudd\bar{d}\rangle + \sqrt{\frac{3}{15}}\|uuds\bar{s}\rangle)$ |
| (1, 3/2, −1/2) | $\sqrt{\frac{1}{3}}\|uddu\bar{u}\rangle - \sqrt{\frac{1}{6}}\|uddd\bar{d}\rangle - \sqrt{\frac{1}{2}}\|udds\bar{s}\rangle$ | (1, 1/2, −1/2) | $-(\sqrt{\frac{4}{15}}\|uddu\bar{u}\rangle + \sqrt{\frac{8}{15}}\|uddd\bar{d}\rangle + \sqrt{\frac{3}{15}}\|udds\bar{s}\rangle)$ |
| (1, 3/2, −3/2) | $\sqrt{\frac{1}{2}}\|dddu\bar{u}\rangle - \sqrt{\frac{1}{2}}\|ddds\bar{s}\rangle$ | | |
| (0, 1, +1) | $\sqrt{\frac{1}{6}}\|uusu\bar{u}\rangle - \sqrt{\frac{1}{2}}\|uusd\bar{d}\rangle - \sqrt{\frac{1}{3}}\|uuss\bar{s}\rangle$ | (0, 1, +1) | $-(\sqrt{\frac{8}{15}}\|uusu\bar{u}\rangle + \sqrt{\frac{3}{15}}\|uusd\bar{d}\rangle + \sqrt{\frac{4}{15}}\|uuss\bar{s}\rangle)$ |
| (0, 1, 0) | $\sqrt{\frac{1}{3}}\|udsu\bar{u}\rangle - \sqrt{\frac{1}{3}}\|udsd\bar{d}\rangle - \sqrt{\frac{1}{3}}\|udss\bar{s}\rangle$ | (0, 1, 0) | $\sqrt{\frac{11}{30}}\|udsu\bar{u}\rangle - \sqrt{\frac{11}{30}}\|udsd\bar{d}\rangle - \sqrt{\frac{4}{15}}\|udss\bar{s}\rangle$ |
| (0, 1, −1) | $\sqrt{\frac{1}{2}}\|ddsu\bar{u}\rangle + \sqrt{\frac{1}{6}}\|ddsd\bar{d}\rangle - \sqrt{\frac{1}{3}}\|ddss\bar{s}\rangle$ | (0, 1, −1) | $\sqrt{\frac{3}{15}}\|ddsu\bar{u}\rangle - \sqrt{\frac{8}{15}}\|ddsd\bar{d}\rangle - \sqrt{\frac{4}{15}}\|ddss\bar{s}\rangle$ |
| (−1, 1/2, +1/2) | $\sqrt{\frac{1}{3}}\|ussu\bar{u}\rangle - \sqrt{\frac{1}{2}}\|ussd\bar{d}\rangle - \sqrt{\frac{1}{6}}\|usss\bar{s}\rangle$ | (−1, 1/2, +1/2) | $-(\sqrt{\frac{4}{15}}\|ussu\bar{u}\rangle + \sqrt{\frac{3}{15}}\|ussd\bar{d}\rangle + \sqrt{\frac{8}{15}}\|usss\bar{s}\rangle)$ |
| (−1, 1/2, −1/2) | $\sqrt{\frac{1}{2}}\|dssu\bar{u}\rangle + \sqrt{\frac{1}{3}}\|dssd\bar{d}\rangle + \sqrt{\frac{1}{6}}\|dsss\bar{s}\rangle$ | (−1, 1/2, −1/2) | $\sqrt{\frac{3}{15}}\|dssu\bar{u}\rangle - \sqrt{\frac{4}{15}}\|dssd\bar{d}\rangle - \sqrt{\frac{8}{15}}\|dsss\bar{s}\rangle$ |
| (−2, 0, 0) | $\sqrt{\frac{1}{2}}(\|sssu\bar{u}\rangle + \|sssd\bar{d}\rangle)$ | (0, 0, 0) | $-(\sqrt{\frac{3}{10}}\|udsu\bar{u}\rangle + \sqrt{\frac{3}{10}}\|udsd\bar{d}\rangle + \sqrt{\frac{4}{10}}\|udss\bar{s}\rangle)$ |

## III. THE BARYON OCTET MAGNETIC MOMENTS

### A. Wave functions

The baryon wave function is formed as combinations of the color, space, flavor, and spin wave functions with appropriate Clebsch-Gordan coefficients. Here the states, in which the antiquark is in its ground state, so that the flavor-spin state of the $qqqq$ system is completely symmetric ($[4]_{FS}$) are considered. The flavor-spin configuration of the $qqqq$ system, which is expected to have the lowest energy for an octet baryon, is for the reasons mentioned above, the mixed symmetry configuration $[4]_{FS}[22]_F[22]_S$. The flavor-spin decomposition of this wave function is

$$|[4]_{FS}[22]_F[22]_S\rangle = \frac{1}{\sqrt{2}}\{[22]_{F_1}[22]_{S_1} + [22]_{F_2}[22]_{S_2}\}. \quad (7)$$

This expression may be rewritten more pictorially in terms of Young tableaux as

$$\boxed{\phantom{1}}\boxed{\phantom{1}}\boxed{\phantom{1}}\boxed{\phantom{1}}_{FS}\begin{array}{|c|c|}\hline \phantom{1} & \phantom{1} \\ \hline \phantom{1} & \phantom{1} \\ \hline\end{array}_F\begin{array}{|c|c|}\hline \phantom{1} & \phantom{1} \\ \hline \phantom{1} & \phantom{1} \\ \hline\end{array}_S = \frac{1}{\sqrt{2}}\left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}_F\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}_S + \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}_F\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}_S\right). \quad (8)$$

The explicit forms of the $qqqq\bar{q}$ flavor wave functions, in which the $qqqq$ subsystem has flavor symmetry $[22]_F$ are listed in Table III. The corresponding spin wave functions are readily derived from these flavor wave functions by the substitutions: $u \to \uparrow$, $d \to \downarrow$ and $s \to \downarrow$.

The corresponding flavor-spin configuration of the decuplet baryons is $[4]_{FS}[31]_F[31]_S$. These wave functions take the

TABLE III. The $qqqq\bar{q}$ components, in which the $qqqq$ subsystem has the flavor symmetry $[22]_F$ and $[211]_F$.

| $(\frac{1}{2}^+, Y, I, I_3)$ | $qqqq\bar{q}$ component ($[22]_F$) | $qqqq\bar{q}$ component ($[211]_F$) |
|---|---|---|
| (1, 1/2, +1/2) | $\sqrt{\frac{2}{3}}\|uudd\bar{d}\rangle + \sqrt{\frac{1}{3}}\|uuds\bar{s}\rangle$ | $\|uuds\bar{s}\rangle$ |
| (1, 1/2, −1/2) | $\sqrt{\frac{2}{3}}\|uddu\bar{u}\rangle - \sqrt{\frac{1}{3}}\|udds\bar{s}\rangle$ | $\|udds\bar{s}\rangle$ |
| (0, 1, +1) | $\sqrt{\frac{1}{3}}\|uusd\bar{d}\rangle + \sqrt{\frac{2}{3}}\|uuss\bar{s}\rangle$ | $-\|uusd\bar{d}\rangle$ |
| (0, 1, 0) | $-(\sqrt{\frac{1}{6}}\|udsu\bar{u}\rangle - \sqrt{\frac{1}{6}}\|udsd\bar{d}\rangle) + \sqrt{\frac{2}{3}}\|udss\bar{s}\rangle$ | $\sqrt{\frac{1}{2}}(\|udsu\bar{u}\rangle - \|udsd\bar{d}\rangle)$ |
| (0, 1, −1) | $-(\sqrt{\frac{1}{3}}\|ddsu\bar{u}\rangle - \sqrt{\frac{2}{3}}\|ddsd\bar{d}\rangle)$ | $\|ddsu\bar{u}\rangle$ |
| (−1, 1/2, +1/2) | $-(\sqrt{\frac{2}{3}}\|ussu\bar{u}\rangle - \sqrt{\frac{1}{3}}\|ussd\bar{d}\rangle)$ | $-\|ussd\bar{d}\rangle$ |
| (−1, 1/2, −1/2) | $-(\sqrt{\frac{1}{3}}\|dssu\bar{u}\rangle + \sqrt{\frac{2}{3}}\|dssd\bar{d}\rangle)$ | $\|ddsu\bar{u}\rangle$ |
| (0, 0, 0) | $-\sqrt{\frac{1}{2}}(\|udsu\bar{u}\rangle + \|udsd\bar{d}\rangle)$ | $\sqrt{\frac{1}{6}}\|udsu\bar{u}\rangle - \sqrt{\frac{1}{6}}\|udsd\bar{d}\rangle + \sqrt{\frac{2}{3}}\|udss\bar{s}\rangle)$ |

general form:

$$|[4]_{FS}[31]_F[31]_S\rangle = \frac{1}{\sqrt{3}}\{[31]_{F_1}[31]_{S_1} + [31]_{F_2}[31]_{S_2} + [31]_{F_3}[31]_{S_3}\}. \quad (9)$$

The explicit forms of the $qqqq\bar{q}$ flavor wave functions, in which the $qqqq$ subsystem have flavor symmetry $[31]_F$ are listed in Table II. The corresponding spin wave functions are obtained from these flavor wave functions by the same substitutions as above.

The remaining 19 different symmetric combinations of mixed symmetry flavor and spin wave functions for the $qqqq$ system are listed in Table 3 of Ref. [26]. The appropriate $S_4$ Clebsch-Gordan coefficients for the completely symmetric combinations of these flavor and spin wave functions are listed in Table 4.13 of Ref. [25].

In the nonrelativistic quark model, the magnetic moment of a baryon is defined as the expectation value of the magnetic moment operator:

$$\hat{\mu} = \sum_i \frac{Q_i}{2m_i}(\hat{l}_{iz} + \hat{\sigma}_{iz}). \quad (10)$$

Here the sum over $i$ runs over the quark content of the baryon, and $Q_i$ denotes the corresponding electric charge of the quark and $m_i$ are the constituent quark masses.

With the combination of $qqq$ and $qqqq\bar{q}$ state wave functions (6) the magnetic moment will have contributions from the diagonal matrix elements of the operator (10) between the $qqq$ component and the $qqqq\bar{q}$ components, respectively, and from the off-diagonal matrix elements between the $qqq$ and the $qqqq\bar{q}$ components. These transition matrix elements between the $qqq$ and $qqqq\bar{q}$ components typically give rise to larger contributions to the baryon magnetic moments than the diagonal contributions from the $qqqq\bar{q}$ components. The contributions to the magnetic moment operator from the nondiagonal terms, which involve $q\bar{q}$ pair annihilation and creation, are obtained as matrix elements of the operator:

$$\hat{\mu} = \sum_i \frac{Q_i}{2}(\vec{r}_i \times \hat{\sigma}_i)_z. \quad (11)$$

The calculation of these nondiagonal contributions to the magnetic moments calls for a specific orbital wave function model. Here, for simplicity, harmonic oscillator constituent quark model wave functions are employed:

$$\phi_{00}(\vec{p};\omega) = \frac{1}{(\omega^2\pi)^{3/4}}\exp\left\{-\frac{p^2}{2\omega^2}\right\}, \quad (12)$$

$$\phi_{1m}(\vec{p};\omega) = \sqrt{2}\,\frac{p_m}{\omega}\,\phi_{00}(\vec{p};\omega). \quad (13)$$

Here $\phi_{00}(\vec{p};\omega)$ and $\phi_{1,m}(\vec{p};\omega)$ are the s-wave and p-wave orbital wave functions of the constituent quarks, respectively. The oscillator parameters of the $qqq$ and $qqqq\bar{q}$ components, $\omega_3$ and $\omega_5$, will in general be different.

The relation between the oscillator parameters $\omega_3$ and $\omega_5$ depends on the color dependence of the effective confining interaction. If the confining interaction between two quarks is pairwise with the color factor $\tilde{\lambda}_i \cdot \tilde{\lambda}_j$, where $\lambda_i^a$ is a color SU(3) generator, the strength of the pairwise confining interaction between the quarks in the $qqqq$ subsystem is half of that between the quarks in a color singlet $qqq$ triplet [27]. This would imply the relation:

$$\omega_5 = \sqrt{5/6}\,\omega_3. \quad (14)$$

The parameter $\omega_3$ may be determined by the nucleon radius as $\omega_3 = 1/\sqrt{\langle r^2\rangle}$ or as half the splitting between the nucleon and its lowest positive parity resonance. Both methods yield the same value ∼246 MeV. The parameter $\omega_5$ may be set by the relation (14) or be treated as a free phenomenological parameter. In Ref. [21] it was noted that the best description of the extant empirical strangeness form factors is obtained with $\omega_5 \sim 1$ GeV, which would imply that the $qqqq\bar{q}$ component is very compact.

### B. Magnetic moment expressions

The magnetic moments of the octet baryons are formed of diagonal matrix elements in the $qqq$ and $qqqq\bar{q}$ subspaces, respectively, and off-diagonal transition matrix elements of the form $qqq \to qqqq\bar{q}$ and $qqq\bar{q} \to qqq$. The former only depend on the group theoretical factors, while the latter also depend on the spatial wave function model. The diagonal contributions to the octet magnetic moments may be expressed in the form:

$$\mu_p = P_{3q}\frac{M_N}{m} + P_{(p)s\bar{s}}\left(\frac{M_N}{6m} - \frac{M_N}{6m_s}\right), \quad (15)$$

$$\mu_n = -P_{3q}\frac{2M_N}{3m} + P_{(n)u\bar{u}}\frac{M_N}{3m} - P_{(n)s\bar{s}}\frac{M_N}{6m_s}, \quad (16)$$

$$\mu_{\Sigma^+} = P_{3q}\left(\frac{8M_N}{9m} + \frac{M_N}{9m_s}\right) + P_{(\Sigma^+)s\bar{s}}\left(\frac{2M_N}{9m} - \frac{2M_N}{9m_s}\right) + P_{(\Sigma^+)d\bar{d}}\left(\frac{M_N}{18m} - \frac{M_N}{18m_s}\right), \quad (17)$$

$$\mu_{\Sigma^0} = P_{3q}\left(\frac{2M_N}{9m} + \frac{M_N}{9m_s}\right) + P_{(\Sigma^0)s\bar{s}}\left(\frac{M_N}{18m} - 2\frac{M_N}{9m_s}\right) - P_{(\Sigma^0)d\bar{d}}\left(\frac{M_N}{9m} + \frac{M_N}{18m_s}\right) + P_{(\Sigma^0)u\bar{u}}\left(7\frac{M_N}{18m} - \frac{M_N}{18m_s}\right), \quad (18)$$

$$\mu_{\Sigma^-} = -P_{3q}\left(\frac{4M_N}{9m} - \frac{M_N}{9m_s}\right) - P_{(\Sigma^-)s\bar{s}}\left(\frac{M_N}{9m} + \frac{2M_N}{9m_s}\right) - P_{(\Sigma^-)u\bar{u}}\left(\frac{2M_N}{9m} + \frac{M_N}{18m_s}\right), \quad (19)$$

$$\mu_{\Sigma^0\to\Lambda} = -P_{3q}\left(\frac{M_N}{\sqrt{3}m}\right) + \frac{1}{4\sqrt{3}}P_{5q}\frac{M_N}{m}, \quad (20)$$

$$\mu_{\Xi^0} = P_{3q}\left(-\frac{2M_N}{9m} - \frac{4M_N}{9m_s}\right) + P_{(\Xi^0)u\bar{u}}\left(\frac{4M_N}{9m} - \frac{M_N}{9m_s}\right) - P_{(\Xi^0)d\bar{d}}\left(\frac{M_N}{18m} + \frac{M_N}{9m_s}\right), \quad (21)$$

$$\mu_{\Xi^-} = P_{3q}\left(\frac{M_N}{9m} - \frac{4M_N}{9m_s}\right) + P_{(\Xi^-)u\bar{u}}\left(\frac{5M_N}{18m} - \frac{M_N}{9m_s}\right) - P_{(\Xi^-)d\bar{d}}\left(\frac{2M_N}{9m} + \frac{M_N}{9m_s}\right), \quad (22)$$

$$\mu_\Lambda = -P_{3q}\frac{M_N}{3m_s} + P_{(\Lambda)u\bar{u}}\left(7\frac{M_N}{18m} - \frac{M_N}{18m_s}\right) - P_{(\Lambda)d\bar{d}}\left(\frac{M_N}{9m} + \frac{M_N}{18m_s}\right). \quad (23)$$

The factors $P_{(B)q_i\bar{q}_i}$ are the probabilities of the $qqqq_i\bar{q}_i$ components in the baryon $B$. These are related to the corresponding amplitudes $A_i$ and the probability of the $qqqq\bar{q}$ components, Eq. (6), as

$$P_{(B)q_i\bar{q}_i} = P_{5q}A_i^2. \quad (24)$$

The contributions to the octet magnetic moments from the off-diagonal matrix elements take the following forms in the harmonic oscillator model:

$$\mu_p = -\frac{2\sqrt{3}}{9}F_{35}(P_{(p)s\bar{s}}) - \frac{2\sqrt{6}}{9}F_{35}(P_{(p)d\bar{d}}), \quad (25)$$

$$\mu_n = -\frac{4\sqrt{6}}{9}F_{35}(P_{(n)u\bar{u}}) - \frac{2\sqrt{3}}{9}F_{35}(P_{(n)s\bar{s}}), \quad (26)$$

$$\mu_{\Sigma^+} = -\frac{2\sqrt{3}}{9}F_{35}(P_{(\Sigma^+)d\bar{d}}) - \frac{2\sqrt{6}}{9}F_{35}(P_{(\Sigma^+)s\bar{s}}), \quad (27)$$

$$\mu_{\Sigma^0} = +\frac{2\sqrt{6}}{18}F_{35}(P_{(\Sigma^0)s\bar{s}}) - \frac{2\sqrt{6}}{9}F_{35}(P_{(\Sigma^0)u\bar{u}}) + \frac{\sqrt{6}}{9}F_{35}(P_{(\Sigma^0)d\bar{d}}), \quad (28)$$

$$\mu_{\Sigma^-} = -\frac{2\sqrt{6}}{9}F_{35}(P_{(\Sigma^-)s\bar{s}}) + \frac{4\sqrt{3}}{9}F_{35}(P_{(\Sigma^-)u\bar{u}}), \quad (29)$$

$$\mu_{\Sigma^0\to\Lambda} = -\frac{2\sqrt{3}}{3}F_{35}(P_{5q}) \quad (30)$$

$$\mu_{\Xi^0} = -\frac{2\sqrt{3}}{9}F_{35}(P_{(\Xi^0)d\bar{d}}, 0) + \frac{4\sqrt{6}}{9}F_{35}(P_{(\Xi^0)u\bar{u}}), \quad (31)$$

$$\mu_{\Xi^-} = \frac{4\sqrt{3}}{9}F_{35}(P_{(\Xi^-)u\bar{u}}) - \frac{2\sqrt{6}}{9}F_{35}(P_{(\Xi^-)d\bar{d}}), \quad (32)$$

$$\mu_\Lambda = -\tfrac{1}{3}F_{35}(P_{5q}). \quad (33)$$

Here the phase factors for the off-diagonal matrix element between the $qqq$ and $qqqq\bar{q}$ components have been taken to be +1. The functions $F_{35}(P_{(B)q\bar{q}})$ above are defined as

$$F_{35}\big(P_{(B)q\bar{q}}\big) = C_{35}\frac{M_N}{\omega_5}\sqrt{P_{3q}P_{(B)q\bar{q}}}, \quad (34)$$

where the factor $C_{35}$,

$$C_{35} = \left(\frac{2\omega_3\omega_5}{\omega_3^2 + \omega_5^2}\right)^{9/2}, \quad (35)$$

is the overlap integral of the s-wave wave functions of the quarks in the $qqq$ and $qqqq\bar{q}$ configurations.

Note that the [22] symmetry of the flavor wave function rules out any contribution to the magnetic moments of the $\Xi$ hyperons from $s\bar{s}$ components. For the same reason there is no contribution to the proton magnetic moment from $u\bar{u}$ nor to the neutron magnetic moment from $d\bar{d}$ components.

### C. Numerical results

Here the factor $C_{35}$, Eq. (35), is treated as a free phenomenological parameter. It is found that addition of $qqqq\bar{q}$ admixtures reduces the deviation of the calculated values from the experimental magnetic moment values only if the $qqqq\bar{q}$ component is much more compact than the $qqq$ component. This will be shown by comparing the calculated values for $C_{35} \sim 0.24$ and $C_{35} = 1$. The latter value implies equal oscillator parameters in the $qqq$ and $qqqq\bar{q}$ systems, while the former value corresponds to $\omega_5 \sim 2.3\,\omega_3$. The former value would imply that the radius of the $qqqq\bar{q}$ component is only ~0.3 fm.

The other model parameters are the probabilities of the $qqqq\bar{q}$ components $P_{5q}$ and the constituent quark masses. To reproduce the experimentally measured values $\bar{d} - \bar{u} = 0.12$ in the proton [28] the $qqqq\bar{q}$ probability is set to $P_{5q} = 1 - P_{3q} = 0.18$. The constituent masses of the up and down quarks are set to be $m_u = m_d = m = 274$ MeV and that of the strange quark to be $m_s = 419$ MeV. The oscillator parameter $\omega_5$ is taken to have the values 0.57 GeV, which corresponds to a compact $qqqq\bar{q}$ extension in the baryon octets. The corresponding numerical results of the baryon octet magnetic moments are shown in Table IV.

The results show that the average deviation from the empirical magnetic moment values drops from ~9% to ~4%, when the $qqqq\bar{q}$ admixture is included in the quark model. The improvement is most notable in the case of the $\Sigma^-$ and the $\Xi^o$ hyperons, which deviate most notably from the empirical values in the conventional $qqq$ model [14]. For these the $qqq$ model results differ by 14% and 21%, respectively, from the experimental values. Inclusion of the contributions of the $qqqq\bar{q}$ components reduces these differences to ~4% and 3%, respectively.

TABLE IV. Magnetic moments of the baryon octet. The column $qqq$ contains the results of the conventional quark model from Ref. [29] and the column Exp the experimental data from Ref. [30]. The present results are listed in column $P_1$. Columns D ($qqq$) and D ($P_1$) contain the deviations of the calculated results from the data, respectively.

| Baryon | Exp | $qqq$ | $P_1$ | D ($qqq$) | D ($P_1$) |
|---|---|---|---|---|---|
| $p$ | 2.79 | 2.76 | 2.72 | 1.1% | 2.5% |
| $n$ | −1.91 | −1.84 | −1.93 | 3.6% | 0.8% |
| $\Lambda$ | −0.61 | −0.67 | −0.61 | 9.8% | 0.0% |
| $\Sigma^+$ | 2.46 | 2.68 | 2.63 | 8.9% | 6.9% |
| $\Sigma^0$ | ? | 0.84 | 0.87 | ? | ? |
| $\Sigma^-$ | −1.16 | −1.00 | −1.11 | 13.7% | 4.3% |
| $\Xi^0$ | −1.25 | −1.51 | −1.21 | 20.8% | 3.0% |
| $\Xi^-$ | −0.65 | −0.59 | −0.58 | 9.2% | 10.3% |
| $\Sigma^0 \to \Lambda$ | \|1.61\| | −1.59 | −1.67 | 1.2% | 3.7% |

TABLE V. Magnetic moments of the baryon octet. The column Exp contains the experimental data from Ref. [30]. The present results are listed in columns $P_1$, with model parameters $C_{35} = 0.24$, $P_{3q} = 0.82$; $P_2$, with $C_{35} = 0.24$, $P_{3q} = 0.90$; $P_3$, with $C_{35} = 1$, $P_{3q} = 0.82$; $P_4$, $C_{35} = 1$, $P_{3q} = 0.90$, respectively.

| Baryon | Exp | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|
| $p$ | 2.79 | 2.72 | 3.01 | 2.40 | 2.76 |
| $n$ | −1.91 | −1.93 | −2.12 | −2.46 | −2.54 |
| $\Lambda$ | −0.61 | −0.61 | −0.68 | −0.71 | −0.80 |
| $\Sigma^+$ | 2.46 | 2.63 | 2.90 | 2.31 | 2.65 |
| $\Sigma^0$ | ? | 0.87 | 0.94 | 1.03 | 1.07 |
| $\Sigma^-$ | −1.16 | −1.11 | −1.18 | −1.11 | −1.18 |
| $\Xi^0$ | −1.25 | −1.21 | −1.43 | −0.89 | −1.18 |
| $\Xi^-$ | −0.65 | −0.58 | −0.60 | −0.58 | −0.60 |
| $\Sigma^0 \to \Lambda$ | \|1.61\| | −1.67 | −1.84 | −2.12 | −2.19 |

In Table V the calculated magnetic moments are shown for both $P_{5q} = 0.1$ and $P_{5q} = 0.18$ and for both $C_{35} = 0.24$ and $C_{35} = 1.0$. The results indicate a clear preference for the smaller value of $C_{35}$ and the larger value of $P_{5q}$.

## IV. DECUPLET MAGNETIC MOMENTS

In the case of the decuplet baryons the lowest energy $qqqq\bar{q}$ configuration is expected to be that, for which the $qqqq$ subsystem is assumed to have the $[4]_{FS}[31]_F[31]_S$ mixed flavor-spin symmetry. The corresponding flavor wave functions are listed in Table II, and the combined flavor-spin wave function in Eq. (9). This wave function, and the corresponding conventional wave function in the $qqq$ quark model lead to the following diagonal matrix element contribution to the magnetic moment values for the $\Delta^{++}$ and the $\Omega^-$ members of the baryon decuplet for which experimental data exist:

$$\mu_{\Delta^{++}} = P_{3q}\frac{2M_N}{m} + P_{(\Delta^{++})d\bar{d}}\frac{35M_N}{24m} + P_{(\Delta^{++})s\bar{s}}\left(\frac{13M_N}{12m} + \frac{3M_N}{8m_s}\right), \quad (36)$$

$$\mu_{\Omega^-} = -P_{3q}\frac{M_N}{m_s} + P_{(\Omega^-)u\bar{u}}\left(-\frac{13M_N}{24m_s} - \frac{3M_N}{4m_s}\right) + P_{(\Omega^-)d\bar{d}}\left(\frac{3M_N}{8m} - \frac{13M_N}{24m_s}\right). \quad (37)$$

The off-diagonal $qqq \to qqqq\bar{q}$ matrix element contributions are the following:

$$\mu_{\Delta^{++}} = -\frac{\sqrt{3}}{12}F_{35}(P_{(\Delta^{++})d\bar{d}}, 0) - \frac{\sqrt{3}}{12}F_{35}(P_{(\Delta^{++})s\bar{s}}, 0), \quad (38)$$

$$\mu_{\Omega^-} = \frac{\sqrt{3}}{6}F_{35}(P_{(\Omega^-)u\bar{u}}, 0) - \frac{\sqrt{3}}{12}F_{35}(P_{(\Omega^-)d\bar{d}}, 0). \quad (39)$$

Note that the $qqqq$ subsystem of the $qqqq\bar{q}$ component in baryon decuplet can have both total angular momentum $J = 1$ and $J = 2$. Here only the contribution from $J = 1$, which, with 40% and 20% proportion of the $qqqq\bar{q}$ component in baryon deculplet, leads to the values in Table VI.

TABLE VI. The magnetic moments of the baryon decuplet. The column $qqq$ contains $qqq$ model results [29] and the column Exp the experimental data from Ref. [30]. The present results are listed in columns $P_1$ with model parameters $C_{35} = 0.24$, $P_{3q} = 0.60$; $P_2$ with $C_{35} = 1$, $P_{3q} = 0.60$; $P_3$ with $C_{35} = 0.24$, $P_{3q} = 0.80$; $P_4$ with $C_{35} = 1$, $P_{3q} = 0.80$.

| Baryon | Exp | CQ | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|---|---|---|
| $\Delta^{++}$ | 3.7–7.5 | 5.52 | 4.98 | 4.94 | 5.87 | 5.84 |
| $\Omega^-$ | −2.02 | −2.01 | −2.02 | −2.00 | −2.11 | −2.09 |

Note that the [31] symmetry of the flavor wave function rules out any contribution to the magnetic moments of the $\Omega^-$ hyperon from $s\bar{s}$ components. For the same reason there is no contribution to the $\Delta^{++}$ magnetic moment from $u\bar{u}$ components.

The values in Table VI shows that the inclusion of the $qqqq\bar{q}$ components leads to improved agreement with the empirical value of the the empirically well determined magnetic moment of $\Omega^-$ only if the $qqqq\bar{q}$ system is again very compact. In the case of the magnetic moment of the $\Delta^{++}$ the uncertainty range of the empirical values is too wide to allow any definite conclusion.

## V. STRANGENESS MAGNETIC MOMENTS

The contribution to the strangeness magnetic moment of the proton can be inferred directly from Eqs. (15) and (25). With $P_{5q} = 0.18$ SU(3) symmetry suggests $P_{s\bar{s}} \sim 0.06$. With this value the strangeness magnetic moment of the proton is $\mu_s = 0.17$, which falls well within the ranges of the empirical data $(0.37 \pm 0.2 \pm 0.26 \pm 0.07)$ given by the SAMPLE experiment [7] and of the combined value from all the presently completed experiments $(0.28 \pm 0.20)$ [31].

The contributions from $s\bar{s}$ seaquark configurations to any of the octet and decuplet magnetic moments can be directly calculated from the terms in Eqs. (15)–(23) and (34), (36) that are proportional to $P_{(B)(s\bar{s})}$ and inversely proportional to the constituent mass $m_s$ of the strange quark. From these it emerges that the strangeness magnetic moments of the proton $\mu_s(P)$ and the neutron $\mu_s(N)$ are equal as required by SU(3) flavor symmetry. For the same reason $\mu_s(\Sigma^+) = \mu_s(\Sigma^-)$.

As noted above there are no $s\bar{s}$ contributions to the magnetic moments of the $\Xi$ hyperons because of the restriction to $[22]_F$, nor to that of the $\Omega^-$ hyperon because of the restriction to $[31]_F$ symmetry. Such contributions are nevertheless possible in the case of the $\Xi$ hyperons if the flavor symmetry is $[31]_F$ and in the case of the $\Omega^-$ hyperon if the flavor symmetry is $[4]_F$ instead. Those configurations are however expected to be energetically unfavorable.

## VI. LIGHT FLAVOR SEAQUARK MAGNETIC MOMENTS

The contributions to the magnetic moments from one $u\bar{u}$ seaquark component can be derived from those terms in the expressions (15)–(23) and (37), (39), which are proportional to $P_{(B)(u\bar{u})}$ and inversely proportional to the light flavor constituent mass $m$. In the case of the octet baryons the resulting expressions naturally only apply to the symmetry $qqqq$ configuration $[4]_{FS}[22]_F[22]_S$. In the case of the decuplet baryons the corresponding expressions (37), (39) are restricted to the symmetry configuration $[4]_{FS}[31]_F[31]_S$.

These two symmetries imply that in the case of the proton only $d\bar{d}$ contributions, but no $u\bar{u}$ contributions appear. This is in line with empirical observation that the $\bar{d}$ contributions are larger than the $\bar{u}$ contribution [32]. Similarly there are no $d\bar{d}$ contributions to the magnetic moment of the neutron.

## VII. CONCLUSIONS

Here the magnetic moment contributions from $qqqq\bar{q}$ admixtures in the baryon octet and decuplet have been derived within the framework of the flavor SU(3) symmetry. In addition explicit expressions for all the possible flavor wave functions for $qqqq\bar{q}$ systems with completely symmetric combinations of flavor and spin wave functions are given. The calculated magnetic moments include the contributions from the $qqq$ components and the $qqqq\bar{q}$ components with both light and strange $q\bar{q}$ pairs. The magnetic moment expressions readily allow separation of the strangeness components from $s\bar{s}$ pairs as well as individual components from $u\bar{u}$ and $d\bar{d}$ pairs.

If the $qqqq\bar{q}$ components are more compact than the $qqq$ components of the wave functions an improved description of the experimental magnetic moments is possible, with appropriately chosen model parameter values. With a $qqqq\bar{q}$ probability in the range ∼10–20% the $qqqq\bar{q}$ components lead to small corrections to the magnetic moment values given by the conventional $qqq$ model apart from the $\Sigma^-$ and $\Xi^0$ hyperons, where these corrections are large and notably improve the agreement with the empirical values.

The restriction in the calculation of the baryon octet magnetic moments to the configuration with flavor-spin symmetry $[4]_{FS}[22]_F[22]_S$ given in Table III, is motivated on the one hand by its expected low energy and on the other hand by the indications of the experimentally observed positive sign of the strangeness magnetic moment of the proton, which is best described by this configuration [19–21]. The configuration with $[4]_{FS}[31]_F[31]_S$ symmetry given in Table II is expected to have the next lowest energy and to give an at most very insignificant contribution to the baryon octet magnetic moments. In the case of the decuplet this configuration is, however, expected to have the lowest energy as the configuration $[4]_{FS}[22]_F[22]_S$ cannot contribute. The contribution of admixtures of $qqqq\bar{q}$ components with the flavor-spin symmetry $[4]_{FS}[31]_F[31]_S$ were found to be very small in the case of the $\Omega^-$.

It is of course not obvious that those $qqqq\bar{q}$ configurations that have the lowest energy for a given hyperfine interaction model should have the largest probability in the nucleons. The main terms should be expected to be those with the strongest coupling to the $qqq$ component. This coupling depends both on the confining interaction in the transition amplitude and (inversely) on the difference in energy from the rest energy of the nucleon.

It should be noted that the $qqqq\bar{q}$ components here, for which the antiquark is in its ground state, do not correspond to the pion contributions considered e.g. in Refs. [13,14]. In the quark model those pion contributions correspond to $qqqq\bar{q}$ configurations, in which the antiquark is in the $P-$state, and the $qqqq$ system is in the ground state. The present approach is motivated by the empirical observation that the strangeness magnetic moment of the proton is positive, which only can be described by $qqqq\bar{q}$ configurations with the antiquark in the ground state [19–21].

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