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Baryonic strangeness and related susceptibilities in QCD

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The ratios of off-diagonal to diagonal conserved charge susceptibilities, e.g., $\chi_{\rm BS}/\chi_S$, $\chi_{\rm QS}/\chi_S$, related to the quark flavor susceptibilities, have proven to be discerning probes of the flavor carrying degrees of freedom in hot strongly interacting matter. Various constraining relations between the different susceptibilities are derived based on the Gell-Mann-Nishijima formula and the assumption of isospin symmetry. Using generic models of deconfined matter and results from lattice quantum chromodynamics, it is demonstrated that the flavor-carrying degrees of freedom at a temperature above $1.5T_c$ are quarklike quasiparticles. A new observable related by isospin symmetry to $C_{\rm BS} = -3\chi_{\rm BS}/\chi_S$ and equal to it in the baryon free regime is identified. This new observable, which is blind to neutral and nonstrange particles, carries the potential of being measured in relativistic heavy-ion collisions.

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I. INTRODUCTION

The goal of of the heavy-ion program at the Relativistic Heavy Ion Collider (RHIC) is the creation and study of heated strongly interacting matter at nearly vanishing baryon density [1,2]. Detailed models of nuclear reactions predicted that the energy deposition in the center-of-mass frame should be sufficient to cause temperatures at midrapidity in central collisions of gold nuclei to reach upwards of 300 MeV [3]. These predictions have been confirmed by the experimental results of the four detector collaborations at the BNL RHIC, which set a lower bound of about 5 GeV/fm^3 on the energy density at a time $\tau = 1$ fm/c in central Au+Au collisions [4]. This estimate for the attained energy density should place the produced matter well into the region of the quantum chromodynamics (QCD) phase diagram that cannot be described as a dilute hadronic resonance gas. Until recent results from the RHIC experiments had raised doubts about this interpretation [2], the matter in this domain of the QCD phase diagram was expected to be a colored plasma composed of quasiparticle excitations with the quantum numbers of quarks and gluons [1,5].

Signals of the excited matter produced in the early stages of such a collision, buried in the pattern of detected particles, may be divided into two categories: hard probes, such as the modification of partonic jets by the medium [6], bound states of heavy quarks [7] and electromagnetically produced particles [8]; and bulk observables, dealing with low-momentum particles that make up a large fraction of the produced matter. Bulk observables include the single inclusive spectra of identified hadrons [9] and the event-by-event fluctuations of conserved charges [10]. The latter are the subject of our present study.

The theoretical analysis of the observed suppression of energetic hadron emission in Au+Au collisions at RHIC ("jet quenching" [6]) confirms that matter with a very high energy density is produced. This matter clearly exhibits collective behavior as evidenced by its radial and elliptic flow [11]. As a large elliptic flow requires large pressure gradients that can be present only during the earliest stage of the collision, the matter produced at such times must exhibit the properties of a fluid. A quantitative analysis of the observed magnitude of the flow indicates that this fluid must be nearly ideal, i.e., endowed with a very low viscosity [12]. This result suggests that there must exist a strong interaction between the constituents of the medium.

At vanishing baryon density, the entire range of thermal conditions expected to be attained at RHIC may be simulated by the numerical methods of lattice QCD (LQCD) at finite temperature [13]. Here one computes the grand canonical partition function of a system whose states are thermally weighted by the QCD action at a temperature T, with baryon (B), electric charge (Q) and strangeness (S) chemical potentials set to zero ($\mu_B = \mu_Q = \mu_S = 0$). Explorations on the lattice consist of a threefold approach [13]: studies of the behavior of the components of the stress energy tensor, i.e., the energy density ϵ and the pressure P; spatial and temporal correlation functions; and the recently measured derivatives of the free energy such as the various conserved flavor susceptibilities.

Investigations of the first kind, exploring the QCD equation of state at vanishing chemical potentials, provide the most solid evidence for the expectation that, when hadronic matter is heated beyond a critical temperature $T_c \sim 170$ MeV, a transition to a new state of matter, the quark-gluon plasma (QGP), will occur. The transition is signaled by a steep rise in the energy density and the pressure as a function of the temperature. The slow rise of both quantities prior to the sudden transition has come to be understood in a picture of a hadronic resonance gas [14]. However, attempts to describe the excited phase as a weakly interacting plasma of quasiparticles [15,16] have not met with success in the region $T_c \leqslant T \leqslant 3T_c$.

This finding might indicate that matter in this region may not be a weakly coupled plasma where quarks and gluons are deconfined over large distances as it was originally proposed [17]. It has been established that such a weakly coupled state, indeed, occurs at much higher temperatures (see Ref. [18] and references therein). Assuming that temperatures at RHIC do not exceed $3T_c$, a strongly coupled state is not inconsistent with the strong collective behavior observed in experiments. It is clear that a microscopic understanding of the emergent degrees of freedom in this regime is essential for an explanation of the "perfect liquid" character of the matter created in nuclear collisions at RHIC.

There already exists a candidate model for such matter, proposed by Shuryak and Zahed [19,20], consisting of a tower of colored bound states of heavy quasiparticulate quarks and gluons. In spite of the presence of such heavy quasiparticles, this model was able to account for the large pressure required by the RHIC data because of the proliferation of excited bound states. The large cross sections resulting from resonance scattering were put forward as the cause for the short mean free path that is a requisite for the appearance of hydrodynamic behavior. The model has, however, fared poorly in a comparison with lattice susceptibilities. Koch *et al.* [21] recently proposed the susceptibilities of conserved charges B, Q, S and their off-diagonal analogs as diagnostics of the conserved charge or flavor carrying degrees of freedom in a strongly interacting system. The primary quantity of interest is the ratio of the covariance between baryon number and strangeness $\sigma^2_{\rm BS} \propto \chi_{\rm BS}$ to the variance in strangeness $\sigma^2_S \propto \chi_S$, renormalized to be unity in a quasiparticle gas of quarks and gluons,

$$C_{\rm BS} = -3\frac{\langle \delta B \delta S\rangle}{\langle \delta S^2\rangle} = -3\frac{\chi_{\rm BS}}{\chi_S}. \quad (1)$$

This quantity may be calculated on the lattice and estimated in the dynamical model of Ref. [19]. The calculated values on the lattice were found to be 50% higher than those computed in the model [21]. Such comparisons demonstrate the capacity of ratios such as $C_{\rm BS}$ to serve as tests for models of the QGP. The origin of the difference between the results from lattice QCD and those from the bound state model are further discussed in the upcoming sections.

The objective of the present work is to continue the study of diagonal and off-diagonal flavor susceptibilities. In the next section, we point out that there exists an entire gamut of such diagonal and off-diagonal susceptibilities for conserved quantum numbers and they are related by a simple transformation to the equivalent basis of quark flavor susceptibilities. This presentation builds on the work of Gavai and Gupta [22]. In the next section, a complete set of such susceptibilities is introduced and related via the Gell-Mann-Nishijima formula. The alternative set of quark flavor susceptibilities, which appear more often in the literature, is also compared. In Sec. III, operator relations between the off-diagonal susceptibilities in different bases are derived and the generic behavior of such operators, depending on the prevalent degrees of freedom, is outlined. In Sec. IV, several models for the flavor-carrying sector of the QGP are analyzed and arguments favoring a picture of quasiparticle quarks are presented. In Sec. V, we formulate observables based on the ratios of susceptibilities that may be estimated from experimental measurements. Concluding discussions are presented in Sec. VI.

II. DIAGONAL AND OFF-DIAGONAL SUSCEPTIBILITIES

In the lattice formulation of QCD, the fundamental degrees of freedom are local quark and gluon fields. Under conditions where deconfinement has been achieved, the elementary set of conserved charges is given by the quark flavors: the net "upness" ($\Delta u = u - \bar{u}$), "downness" ($\Delta d = d - \bar{d}$), and "strange-quarkness" ($\Delta s = s - \bar{s}$). An alternate basis is provided by the hadronically defined conserved charges of B, Q, and S. The two bases are related by

$$\begin{aligned} B &= \tfrac{1}{3}(\Delta u + \Delta d + \Delta s),\\ Q &= \tfrac{2}{3}\Delta u - \tfrac{1}{3}\Delta d - \tfrac{1}{3}\Delta s,\\ S &= -\Delta s. \end{aligned} \quad (2)$$

In what follows, the Δ is omitted and the variables u, d, s are understood to denote the net flavor contents.

The mean values of any conserved charge may be measured in a thermal ensemble of interacting quarks and gluons on the lattice. The grand canonical partition function may be defined using either basis, i.e.,

$$\begin{aligned} &\mathcal{Z}(T, \mu_B, \mu_Q, \mu_S) = \mathcal{Z}(T, \mu_u, \mu_d, \mu_s),\\ &\text{if } \mu_u = \frac{\mu_B}{3} + \frac{2\mu_Q}{3} \;\&\; \mu_d = \frac{\mu_B - \mu_Q}{3} \;\&\; \mu_s = -\mu_S. \end{aligned} \quad (3)$$

The mean values and variances of any combination of conserved charges may be obtained from appropriate differentiation of either partition function:

$$\langle x\rangle = T\frac{\partial}{\partial \mu_x}\log\mathcal{Z}(T, \mu_x, \mu_y), \quad (4)$$

$$\sigma^2_{xy} = T^2\frac{\partial^2}{\partial \mu_x \partial \mu_y}\log\mathcal{Z}(T, \mu_x, \mu_y). \quad (5)$$

Although the mean values in a given basis are related to the other via Eq. (2), the variances exhibit a more complex structure. A similar and larger matrix relates the 6 fluctuation measures, viz. the variances $\sigma^2_B, \sigma^2_Q, \sigma^2_S$ and the covariances $\sigma^2_{\rm BS}, \sigma^2_{\rm BQ}, \sigma^2_{\rm QS}$, to the 6 diagonal and off-diagonal quantities constructed from the quark flavors. The 6×6 matrix relating these two sets of (co-)variances is given by

$$\begin{pmatrix} B^2\\ Q^2\\ \rm BQ\\ \rm BS\\ \rm QS\\ S^2 \end{pmatrix} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 2 & 2 & 2 & 1\\ 4 & 1 & -4 & -4 & 2 & 1\\ 2 & -1 & 1 & 1 & -2 & -1\\ 0 & 0 & 0 & -3 & -3 & -3\\ 0 & 0 & 0 & -6 & 3 & 3\\ 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}\begin{pmatrix} u^2\\ d^2\\ ud\\ us\\ ds\\ s^2 \end{pmatrix}, \quad (6)$$

where the corresponding subscripts of σ^2_{xy} are used to indicate the corresponding variance. The above matrix immediately demonstrates the utility of using ratios of $\sigma^2_{\rm BS}, \sigma^2_{\rm QS}$, and σ^2_S as opposed to the other three variances, as these form a smaller subgroup with the quark flavor covariances $\sigma^2_{us}, \sigma^2_{ds}$ and the strangeness variance σ^2_s. The (co-)variances are extensive quantities:

$$\sigma^2_{xy} = VT\chi_{xy}, \quad (7)$$

where χ_{xy} is the intensive diagonal or off-diagonal susceptibility. These susceptibilities can be measured on the lattice. In heavy-ion experiments, the variances and covariances are measured by means of an event-by-event analysis of the corresponding conserved quantities, i.e.,

$$\sigma^2_{xy} = \frac{1}{N_E}\sum_{i\in E} X_i Y_i - \left(\frac{\sum_{i\in E} X_i}{N_E}\right)\left(\frac{\sum_{i\in E} Y_i}{N_E}\right), \quad (8)$$

where E represents the set of events, N_E is the number of events considered, and X_i, Y_i are the net values of the conserved charge in a given event i. The volume-independent ratios of variances, measured event-by-event in heavy-ion collisions, may then be directly compared with the lattice estimate for the ratio of susceptibilities.

In addition to relating the susceptibilities or variances from one basis to another, simplifying relations may be obtained between the variances in a given basis using the Gell-Mann-Nishijima formula,

$$Q = I_3 + \frac{B+S}{2}, \quad (9)$$

where I_3 denotes the third component of the isospin. Because the mass difference between the u, d quarks is small compared with the typical scale of hadron masses, the masses of all hadrons in a given isospin multiplet are degenerate. One may also assume that any quasiparticle excitations in highly excited chromodynamic matter display a large degree of isospin symmetry. It should be pointed out that in actual experiments, in addition to a small mass split between the u and the d quark, there is an overall isospin asymmetry in the entire system as all heavy nuclei have a large isospin due to their proton-neutron imbalance. Such an isospin asymmetry is, however, dominantly correlated with the net baryon number of the nuclei. As the matter formed in the central rapidity region has nearly vanishing net baryon density, one may assume that the produced matter has almost no net I_3 component, on the average. In the following, we also ignore the isospin-violating effects of weak decays and electromagnetic interactions.

To obtain relations between quadratic variables involving strangeness, we obtain the variance of the left- and right-hand side of the equation

$$\mathrm{QS} = I_3 S + \frac{\mathrm{BS} + S^2}{2}. \quad (10)$$

Taking the ensemble or event average of the above quantity, we obtain

$$\begin{aligned}\sigma^2_{\mathrm{QS}} &= \frac{1}{N_E}\left[\sum_{i\in E} Q_i S_i - \frac{1}{N_E}\sum_{i\in E} Q_i \sum_{j\in E} S_j\right] \\ &= \left[\frac{1}{N_E}\sum_{i\in E}\sum_f n_i^f Q_f \sum_g n_i^g S_g \right. \\ &\quad \left. - \frac{1}{N_E}\sum_{i\in E}\sum_f n_i^f Q_f \frac{1}{N_E}\sum_{j\in E}\sum_g n_j^g S_g\right]. \quad (11)\end{aligned}$$

In the above, the total charge and strangeness measured in an event i is denoted as Q_i, S_i. In the case that the degrees of freedom or active flavors f are eigenstates of charge or strangeness, this expression may be decomposed as $Q_i = \sum_f n_i^f Q_f$, where n_i^f are the number of states of flavor f in event i. For independent flavors, where $\langle n^f n^g\rangle = \sum_i^{\mathrm{Ens.}} n_i^f n_i^g = \langle n^f\rangle\langle n^g\rangle$, one may easily demonstrate that

$$\sigma^2_{\mathrm{QS}} = \sum_f \sigma_f^2 Q_f S_f \overset{\mathrm{P.S.}}{=} \sum_f \langle n^f\rangle Q_f S_f. \quad (12)$$

The last equality in the above equation holds solely in the case that Poisson statistics is applicable to the independent flavors (i.e., the variance σ^2 is equal to the mean $\langle n\rangle$), e.g., in the case where the masses of the various flavors exceeds the temperature.

If isospin symmetry is maintained by the system, then $\sigma_f^2 = \sigma_g^2$ when f, g belong to the same isospin multiplet. The associated physical picture is that fluctuations of isospin, or of a product quantum number involving isospin, are brought about by fluctuations in the populations of flavors that carry isospin. If all the members of an isospin multiplet have the same mass and there exist no chemical potentials that favor one species over another, the fluctuations of carriers with opposing values of I_3 compensate for each other. As a result, one obtains the equalities,

$$\sum_{i=-I}^{I} I_{3i}\sigma_i^2 \simeq \sum_{i=-I}^{I} I_{3i} S_i \sigma_i^2 \simeq \sum_{i=-I}^{I} I_{3i} B_i \sigma_i^2 \simeq 0, \quad (13)$$

where the sum is restricted to lie in a given isospin multiplet. One uses the notation of an approximate equality ($\simeq$) as opposed to an exact equality (=) to highlight the fact that such relations hold only in the case of exact isospin invariance. However, even in the case with a small mass split between the u and d quarks, the equalities are still approximately true and are applied in this spirit. In actual calculations involving quarks and hadrons, in the upcoming sections, the physical masses of all known species are used. Based on the above, the following simplifying relations between the various covariances may be easily derived,

$$\sigma^2_{\mathrm{QS}} = \sigma^2_{I_3 S} + \frac{\sigma^2_{\mathrm{BS}} + \sigma^2_S}{2} \simeq \frac{\sigma^2_{\mathrm{BS}} + \sigma^2_S}{2} \quad (14)$$

$$\sigma^2_{\mathrm{QB}} = \sigma^2_{I_3 B} + \frac{\sigma^2_B + \sigma^2_{\mathrm{BS}}}{2} \simeq \frac{\sigma^2_B + \sigma^2_{\mathrm{BS}}}{2}. \quad (15)$$

In the definitions introduced in Refs. [21,22], for the two coefficients: $C_{\mathrm{BS}} = -3\sigma^2_{\mathrm{BS}}/\sigma^2_S$ and $C_{\mathrm{QS}} = 3\sigma^2_{\mathrm{QS}}/\sigma^2_S$, one may use Eq. (14) to obtain the following simplifying relation:

$$C_{\mathrm{QS}} \simeq \frac{3 - C_{\mathrm{BS}}}{2}. \quad (16)$$

The validity of the above equation is amply demonstrated by Fig. 1. The solid red and blue data points are taken from the lattice calculations of Ref. [22], where both these quantities were computed independently of each other. The hazed cyan points represent an estimation of C_{QS} from the C_{BS} points using Eq. (16). In the lattice computation, the masses of the u and d quarks are set equal to each other, i.e., the lattice calculation displays exact isospin symmetry, clearly demonstrated by the exact coincidence of the hazed cyan and solid blue circles. It should be reiterated that C_{QS} may be calculated given a C_{BS} using Eq. (16).

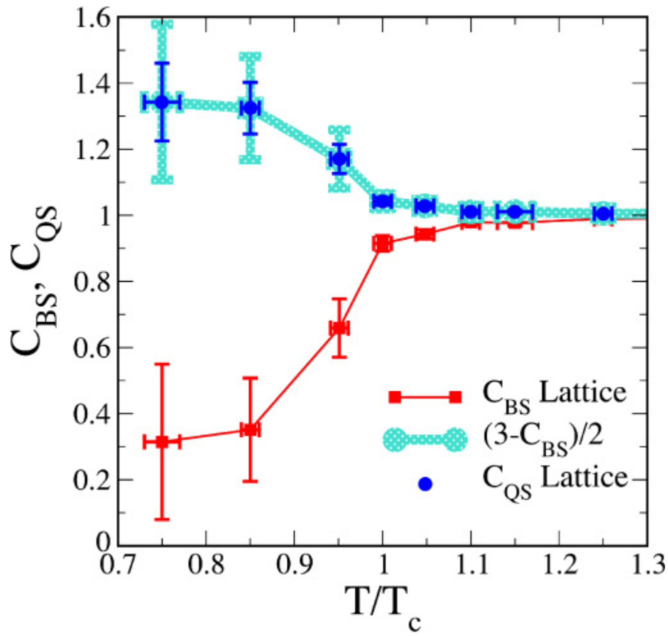

FIG. 1. (Color online) A test of the formula $C_{\mathrm{QS}} = 3 - C_{\mathrm{BS}}/2$. Both ratios, C_{BS} and C_{QS}, computed on the lattice in Ref. [22], are presented as the solid red and blue points. The C_{BS} points are then used to calculate C_{QS} resulting in the hazed cyan points.

Similarly, one may also define another set of related observables,

$$C_{\mathrm{QB}} = \frac{\sigma^2_{\mathrm{QB}}}{\sigma^2_B}, \quad C_{\mathrm{SB}} = \frac{\sigma^2_{\mathrm{SB}}}{\sigma^2_B}. \tag{17}$$

We refrain from ascribing any overall normalization factor, as in the cases of $C_{\mathrm{BS}}, C_{\mathrm{QS}}$: none is self-evident. Both these quantities may be estimated on the lattice, in models, as well as in an experiment. In all such cases, independent of the phase of matter involved, one will recover the equality imposed by isospin symmetry [Eq. (15)],

$$C_{\mathrm{QB}} = \frac{1 + C_{\mathrm{SB}}}{2}. \tag{18}$$

An interesting situation is afforded for flavor SU(2), i.e., in a theory without strangeness. Although $C_{\mathrm{BS}}, C_{\mathrm{SB}}$, and C_{QS} are undefined, $C_{\mathrm{QB}} = \frac{1}{2}$ (a similar situation is that of the canonical ensemble where S is held fixed). This last value serves as an important test of any model devised to reproduce the properties of any phase of strongly interacting matter and is used in the upcoming sections where we compare model predictions to results from lattice simulations.

III. OPERATOR RELATIONS

In the previous section, we outlined various properties and relations between the diagonal and off-diagonal susceptibilities of heated strongly interacting matter. In this section, we extend the discussion to the operator structure of these susceptibilities and derive general expectations for the value of the off-diagonal flavor susceptibilities in different quasiparticle bases. The reader not interested in such a study can skip to the next section, in which comparisons of lattice results with phenomenological models is made.

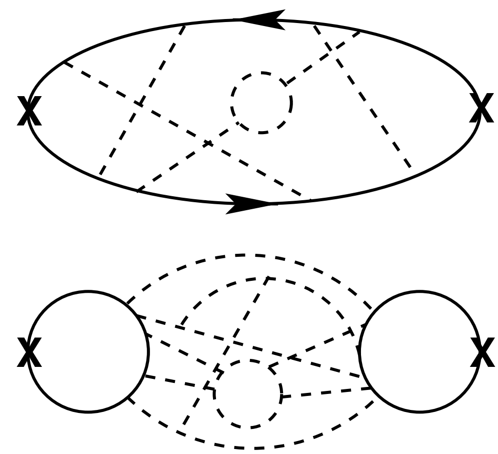

FIG. 2. Connected and disconnected quark number operators.

In numerical simulations of lattice QCD, one evaluates thermal expectation values of operators weighted with the SU(3) gauge action S_g and the fermionic determinant det$[M]$,

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \mathcal{O} (\det[M])^{n_f} e^{-S_g}}{\int \mathcal{D}U (\det[M])^{n_f} e^{-S_g}}. \tag{19}$$

In the case of the quark number susceptibility, the operator in question is

$$N_{q_i} N_{q_j} = \int d^3x d^3y n_{q_i}(x) n_{q_j}(y), \tag{20}$$

where $n_{q_i}(x) =: \bar{\Psi}_i(x)\gamma^0\Psi_i(x):$ and q_i, q_j represent quarks of flavor i and j. The normal ordering removes the leading short distance piece, which is proportional to the four-volume. The operator may be decomposed into connected and disconnected diagrams as shown in Fig. 2. The locations x, y of the two operator insertions are indicated by the crosses in the figures. The solid lines represent the valence quarks, whereas the dashed lines represent virtual gluons or quarks as allowed by the Lagrangian. If the flavors of the two quarks are the same $(i = j)$, corresponding to a diagonal susceptibility, contributions emerge from both types of diagrams. Whereas if the two flavors are different, contributions arise solely from the second diagram.

In the interaction picture, the operator of Eq. (20), for the off-diagonal susceptibility may be expressed in the simplified form,

$$N_{q_i} N_{q_j} = \int d^3q_i \int d^3q_j \sum_{r,s} \left[a_i^{r\dagger} a_i^r - b_i^{r\dagger} b_i^r\right] \times \left[a_j^{s\dagger} a_j^s - b_j^{s\dagger} b_j^s\right], \tag{21}$$

where r, s denote the spin orientations. This expression may then be evaluated in a basis of weakly interacting partons, with the effect of the interaction introduced perturbatively in the basis of states. Such a starting point for the evaluation of the off-diagonal susceptibility is most appropriate for the case of a weakly interacting plasma of quarks and antiquarks, e.g., in the high temperature limit. In this weak coupling limit, the expectation of the susceptibility operator may be obtained order by order in the strong coupling constant g. The leading order contribution is vanishing:

$$\langle \chi_{ij} \rangle = \frac{\langle N_{q_i} N_{q_j} \rangle - \langle N_{q_i} \rangle \langle N_{q_j} \rangle}{VT} \simeq 0, \tag{22}$$

if $i \neq j$ as these create and annihilate different flavors. The first nonzero correction to the off-diagonal susceptibility at $\mu = 0$ occurs at order g^6 (see Ref. [23]).

The opposite case is that of the strong coupling limit, where the system is composed of bound states of quarks (or antiquarks) of flavor i with antiquarks (or quarks) of flavor j. The operator of Eq. (20) may no longer be meaningfully evaluated perturbatively starting from free particle states. A convenient starting point is afforded by the basis of bound states: in the interest of simplicity we focus on the specific example of the low-temperature phase of two-flavor QCD, i.e., a system with only u, d quarks (antiquarks) and gluons. At vanishing baryon and charge chemical potentials, the system may be effectively described in the basis of pions π^+, π^-, π^0. As derived in the appendix, in such a system, the quark number operators assume the simplified forms,

$$\begin{aligned} \hat{N}_u &= \hat{N}_{\pi^+} - \hat{N}_{\pi^-}, \\ \hat{N}_d &= \hat{N}_{\pi^-} - \hat{N}_{\pi^+}. \end{aligned} \tag{23}$$

In the above, $\hat{N}_{\pi^+}, \hat{N}_{\pi^-}$ is the number operator for π^+, π^-. There exists a subtle difference in the meaning of $\hat{N}_q$ (where q stands for either quark) and $\hat{N}_\pi$: the quark number operators are meant to indicate the net amount of a certain quark flavor, i.e., $\hat{N}_u$ is the difference between the number of u quarks and $\bar{u}$ antiquarks, whereas the pion number operators $\hat{N}_\pi$ merely indicate the number of pions of a certain flavor and not the difference between the π^+ and π^- populations.

Within the effective form of the quark number operators, given by Eq. (23), the off-diagonal quark number covariance in two-flavor QCD may be constructed as

$$\begin{aligned} \sigma_{ud}^2 &= \langle \hat{N}_u \hat{N}_d \rangle - \langle \hat{N}_u \rangle \langle \hat{N}_d \rangle \\ &= \langle (\hat{N}_{\pi^+} - \hat{N}_{\pi^-})(\hat{N}_{\pi^-} - \hat{N}_{\pi^+}) \rangle \\ &\quad - \langle \hat{N}_{\pi^+} - \hat{N}_{\pi^-} \rangle \langle \hat{N}_{\pi^-} - \hat{N}_{\pi^+} \rangle. \end{aligned} \tag{24}$$

In the case of a dilute pion gas in the grand canonical ensemble, the various flavors may be considered to be uncorrelated. As a result,

$$\langle \hat{N}_{\pi^+} \hat{N}_{\pi^-} \rangle \simeq \langle \hat{N}_{\pi^+} \rangle \langle \hat{N}_{\pi^-} \rangle. \tag{25}$$

Incorporating the above approximation in the expression for σ_{ud}^2 leads to the simplified form for the off-diagonal covariance in a dilute pion gas at low temperature and vanishing chemical potentials [we also denote such contributions as $\sigma_{ud}^2(M)$, where M denotes contributions solely from the mesonic sector; at a higher temperature, this will also include contributions from more massive mesons, e.g., ρ, ω, etc.],

$$\begin{aligned} \sigma_{ud}^2 &= -\langle \hat{N}_{\pi^+}^2 \rangle + \langle \hat{N}_{\pi^+} \rangle^2 - \langle \hat{N}_{\pi^-}^2 \rangle + \langle \hat{N}_{\pi^-} \rangle^2 \\ &= -\left(\sigma_{\pi^+}^2 + \sigma_{\pi^+}^2\right) \simeq \sigma_{ud}^2(M). \end{aligned} \tag{26}$$

As the variance of either pion species is always positive, we obtain the general result that at low temperature and vanishing chemical potentials, the off-diagonal susceptibility $\chi_{ud} = \sigma_{ud}^2 / V$ is negative. This prediction has been verified in lattice calculations of χ_{ud} in Ref. [24].

As the temperature of the system is raised, the pion populations (and populations of heavier mesons) as well as the fluctuations in the populations will increase, leading to a drop in χ_{ud}. This trend will continue until substantial baryon populations appear. The expression for σ_{ud}^2 in the baryon sector is quite different from that in the meson sector, owing to the fact that there are no valence antiquarks in a baryon, nor any valence quarks in an antibaryon. Hence, one obtains σ_{ud}^2 in the baryon sector as

$$\begin{aligned} \sigma_{ud}^2(B) &= \langle (3\hat{N}_{uuu} + 2\hat{N}_{uud} + \hat{N}_{udd} \\ &\quad - 3\hat{N}_{\bar{u}\bar{u}\bar{u}} - 2\hat{N}_{\bar{u}\bar{u}\bar{d}} - \hat{N}_{\bar{u}\bar{d}\bar{d}}) \\ &\quad \times (3\hat{N}_{ddd} + 2\hat{N}_{udd} + \hat{N}_{uud} \\ &\quad - 3\hat{N}_{\bar{d}\bar{d}\bar{d}} - 2\hat{N}_{\bar{u}\bar{d}\bar{d}} - \hat{N}_{\bar{u}\bar{u}\bar{d}}) \rangle \\ &\quad - \langle (3\hat{N}_{uuu} + 2\hat{N}_{uud} + \hat{N}_{udd} \\ &\quad - 3\hat{N}_{\bar{u}\bar{u}\bar{u}} - 2\hat{N}_{\bar{u}\bar{u}\bar{d}} - \hat{N}_{\bar{u}\bar{d}\bar{d}}) \rangle \\ &\quad \times \langle (3\hat{N}_{ddd} + 2\hat{N}_{udd} + \hat{N}_{uud} \\ &\quad - 3\hat{N}_{\bar{d}\bar{d}\bar{d}} - 2\hat{N}_{\bar{u}\bar{d}\bar{d}} - \hat{N}_{\bar{u}\bar{u}\bar{d}}) \rangle \\ &= 2\sigma_{uud}^2 + 2\sigma_{udd}^2 + 2\sigma_{\bar{u}\bar{u}\bar{d}}^2 + 2\sigma_{\bar{u}\bar{d}\bar{d}}^2. \end{aligned} \tag{27}$$

As a result, the baryon contribution to the off-diagonal susceptibility is always positive. As the temperature of the system is raised, the contributions from mesons and baryons begin to compensate each other. In a weakly interacting hadron gas the two contributions are additive: $\sigma_{ud}^2 = \sigma_{ud}^2(M) + \sigma_{ud}^2(B)$. The increasing density of states in the baryon sector relative to the meson sector at higher energies, as well as the larger prefactors involved in $\sigma_{ud}^2(B)$, leads to an increasing cancellation between the two contributions as the temperature is raised.

In lattice computations of the temperature dependence of χ_{ud} one notes an initial drop followed by a rise to zero at $T \to T_c$. If the picture of a weakly interacting hadron gas remained valid past $T = T_c$, σ_{ud}^2 would continue to rise to larger positive values. The absence of such behavior is an indication of the breakdown of the picture of a weakly interacting hadron gas near and beyond $T = T_c$. Further comparisons between the behavior of the off-diagonal susceptibility, as well as its derivatives as computed on the lattice, with expectations within the picture of a weakly interacting hadron gas are carried out in the upcoming section.

IV. LATTICE VERSUS MODELS

As pointed out in the previous sections, the various susceptibilities and their ratios may be measured on the lattice [25] by evaluating the average of certain operators over a set of configurations. The appropriate choice of observables and their sensitivity to composite structures was discussed in the previous section. Presently we focus on the results obtained from such a calculation on the lattice and use it to isolate the subset of models that describe the emergent degrees of freedom at various temperatures in strongly interacting matter. The models used are rather empirical and require very little beyond arguments based on general symmetry principles. In all comparisons, the focus lies expressly on two regions: the region below T_c, where one expects a hadronic resonance gas to be the correct set of degrees of freedom and above $1.5T_c$, where one would expect to be firmly in the deconfined phase.

A. Hadron gas to quasiparticle plasma

In the region below T_c, the monotonic rise of the pressure and energy density with temperature has come to be understood in the picture of a weakly interacting hadronic resonance gas [14], as originally introduced by Hagedorn. This picture remains true for the case of the diagonal and off-diagonal susceptibilities. The susceptibilities are computed, assuming the condition of Eqs. (11) and (12), i.e.,

$$\chi_{ud} = \frac{\sum_f \langle n_f \rangle u_f d_f}{VT}, \tag{28}$$

where $\langle n_f \rangle, u_f, d_f$ are the thermal average, upness, and downness of a given hadron species f. The sum is, in principle, over all hadrons but is usually truncated at an appropriately chosen upper mass limit. As a comparison, we plot in Fig. 3 the coefficients $C_{\rm BS}$ and $C_{\rm QS}$ as obtained from a hadronic resonance gas spectrum, truncated at the mass of the Ω^-. As in the case of the computation on the lattice, the plots correspond to all chemical potentials vanishing. One notes that the hadron resonance gas provides a good description of the behavior of the ratio of susceptibilities up to the point of the phase transition. Here the behavior of the truncated spectrum fails to reproduce the sharp rise in $C_{\rm BS}$ and the corresponding sharp drop in $C_{\rm QS}$. It should be pointed out that although in the lattice simulations exact isospin symmetry has been imposed, no such condition has been required of the hadron spectrum: the masses are taken directly from the particle data book.

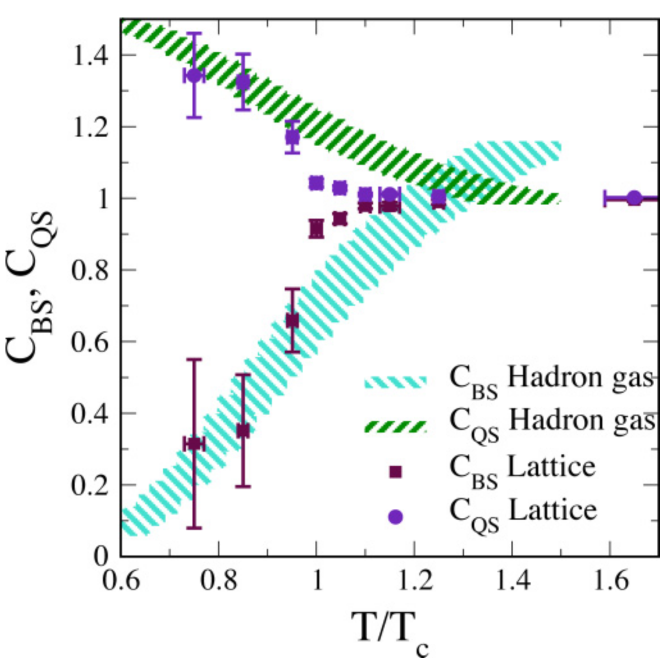

FIG. 3. (Color online) A comparison of the $C_{\rm BS}$ and $C_{\rm QS}$ calculated in a truncated hadron resonance gas at $\mu_B = \mu_S = \mu_Q = 0$ MeV compared to lattice calculations at $\mu = 0$ from Ref. [22]. The two hazed bands for $C_{\rm BS}$ and $C_{\rm QS}$ for the hadron gas plots reflect the uncertainty in the actual value of the phase transition temperature T_c, which is assumed to lie in the range $T_c = 170 \pm 10$ MeV.

The results from the lattice simplify in the high temperature phase, where a general statement regarding susceptibilities may be made: off-diagonal flavor susceptibilities are vanishing compared to the diagonal susceptibilities [24,25]; susceptibilities inclusive of strangeness are smaller compared to those which involve lighter flavors. This may be stated as,

$$\chi_{us} = \chi_{ds} \leqslant \chi_{ud} \ll \chi_s \leqslant \chi_d = \chi_u, \tag{29}$$

where the last and the first equality applies in the case of isospin symmetry. For simulations at vanishing chemical potentials, the mean values of the conserved flavor charges are vanishing, as a result $\langle B \rangle = \langle S \rangle = \langle Q \rangle = 0$. Using the above equalities derived from the lattice, we may formulate the correlation ratio $C_{\rm BS}$ as

$$-3\frac{\langle \mathrm{BS} \rangle}{\langle S^2 \rangle} = \frac{\langle (u+d+s)s \rangle}{\langle s^2 \rangle} \approx \frac{\chi_{us} + \chi_{ds} + \chi_s}{\chi_s} \approx 1. \tag{30}$$

Models of the deconfined phase must obey the above constraint. The simplest model of deconfined matter is that of noninteracting quark, antiquark, and gluon quasiparticles. As has been demonstrated in Ref. [21], in such a situation, off-diagonal susceptibilities are identically zero as the degrees of freedom may carry only a single flavor. To wit, using the notation $\bar{n}_f = \langle n_f \rangle + \langle n_{\bar{f}} \rangle$, for the total number of quarks and antiquarks of flavor f,

$$\frac{\chi_{us} + \chi_{ds} + \chi_s}{\chi_s} = \frac{\bar{n}_u(1 \times 0) + \bar{n}_d(1 \times 0) + \bar{n}_s(1^2)}{\bar{n}_s(1^2)} = 1. \tag{31}$$

The zero entries in the above equation indicate that the up and down flavors carry no strangeness. We also work in the limit described in Sec. II, where the masses of the quasiparticles are large enough for classical Poisson statistics to apply. Thus a model of quark quasiparticles presents a $C_{\rm BS}$ that is in agreement with that derived from the lattice. It should be pointed out that the nature of the gluon sector is irrelevant in this test. The gluons carry no conserved flavor and are thus oblivious to any such constraints. As a result, such comparisons yield no clues to the structure of the gluon sector.

The next independent set of ratios of susceptibilities is that involving the covariances of Eq. (15). Expressions for $C_{\rm SB}$ may be expressed as above,

$$\begin{aligned} C_{\rm SB} &= -3\frac{\chi_{us} + \chi_{ds} + \chi_s}{\chi_u + \chi_d + \chi_s + 2\chi_{us} + 2\chi_{ds} + 2\chi_{ud}} \\ &= -3\frac{\chi_s}{\chi_u + \chi_d + \chi_s} \geqslant -1, \end{aligned} \tag{32}$$

where the last inequality holds in the general case. $C_{\rm SB} = -1$ in the case of exact $SU(3)_f$ symmetry, i.e., when mass of the s quark equals the mass of its lighter counterparts. Once again, it may be demonstrated that the simplified model of quark quasiparticles satisfies this requirement for the ratio of susceptibilities. Thus, from this standpoint, it is a viable candidate for the degrees of freedom of hot strongly interacting matter. Both these conclusions may be reduced to the single observation that the degrees of freedom of excited matter have to be such, as to display minimal strength in the covariance between flavors, i.e., off-diagonal flavor susceptibilities have to be tiny compared to the diagonal ones.

B. A plasma with colored meson and diquark bound states?

The picture of quasiparticle quarks is a rather simple solution to the constraint of Eq. (30). Indeed, such a picture of a quasiparticle plasma has been the subject of numerous studies in weak coupling expansions [16,18,26]. Such approximations do indeed obtain a similar behavior as a function of temperature as compared to the lattice for both the off-diagonal and the diagonal susceptibility. Recently, an alternate picture of the QGP has been proposed: one where a tower of bound states of quarks and gluons are present in addition to the quasiparticles themselves [19,20]. The larger number of particles and larger scattering cross sections that result in such a mixture is shown to account for the pressure observed on the lattice as a function of the temperature. The large cross sections imply very short mean free paths and lend consistency to the macroscopic hydrodynamic picture henceforth used to describe the dynamical evolution of the matter produced at RHIC.

However, in a plasma containing bound states of quarks, in the form of colored mesons, diquarks, and quark-gluon bound states, the correlation between flavors is no longer negligible compared to the diagonal susceptibility. Consider a gas consisting solely of quark-antiquark bound states, built from three flavors of quarks. The possible states are $u\bar{d}, d\bar{u}, u\bar{s}, s\bar{u}, d\bar{s}, s\bar{d}$. The flavor singlets $q\bar{q}$ are ignored as they carry no conserved flavor and thus do not contribute to any susceptibility or covariance. In such a system, the ratio $C_{\rm BS}$ must vanish, because all states have vanishing baryon number. Expressed via the flavor (co-)variances, the numerator of $C_{\rm BS}$ is given as

$$\begin{aligned}\langle (u+d+s)s\rangle = & -n_{u\bar{s}} - n_{s\bar{u}} - n_{d\bar{s}} - n_{s\bar{d}} \\ & + n_{u\bar{s}} + n_{s\bar{u}} + n_{d\bar{s}} + n_{s\bar{d}} \\ = & \ 0. \end{aligned} \quad (33)$$

Thus the inclusion of mesonic states changes ratios such as $C_{\rm BS}$ as they cause the off-diagonal susceptibilities to become nonvanishing (in this case $\chi_{us} = -\chi_s/2$).

Quark-gluon bound states contribute similarly as quark-antiquark quasiparticles, whereas gluonic bound states make no contribution. States such as diquarks have a somewhat opposite effect, diquarks belonging to the flavor antitriplet (ud, us, ds) contribute $us = ds = s^2 = +1$, thus leading to (we here assume $\mu_B = 0$):

$$C_{\rm BS} = \frac{2n_{us} + 2n_{ds} + 2n_{us} + 2n_{ds}}{2n_{us} + 2n_{ds}} = 2, \quad (34)$$

whereas the states belonging to the flavor hexaplet produce a coefficient,

$$C_{\rm BS} = \frac{2n_{us} + 2n_{ds} + 2n_{us} + 2n_{ds} + 8n_{ss}}{2n_{us} + 2n_{ds} + 8n_{ss}} > 1.0. \quad (35)$$

Reference [19] provides masses and degeneracies for the various bound states. The masses of all the bound states exceed the temperature in this model, allowing us to use a Maxwell-Boltzmann (MB) distribution to calculate the populations of such states. Such a computation was carried out in Ref. [21] at a temperature of $T = 1.5T_c$ and yielded a value $C_{\rm BS} = 0.62$, quite different from the value of unity found on the lattice.

C. Introduction of baryonic bound states

These results have motivated the inclusion of a variety of baryonic states into the model outlined above [27]. We illustrate the effect of such additions on the correlation between flavors in the following simple model. In the interest of simplicity, the flavor group will be restricted to $SU(2)_f$. Lattice results for susceptibilities and their derivatives with respect to baryon chemical potential using dynamical quarks exist in this case [24]. The model will consist of quark and antiquark quasiparticles u, d and $\bar{u}, \bar{d}$; mesonlike bound states $u\bar{u}, d\bar{d}, u\bar{d}, d\bar{u}$; diquark states uu, dd, ud and their antiparticles; as well as baryons uuu, uud, udd, ddd and their corresponding antibaryons. We assume that there is no significant covariance between these quasiparticles, which are assumed to be massive enough for MB statistics to apply. We will compute general contributions to the off-diagonal susceptibility and its various derivatives.

In such a situation, the off-diagonal susceptibility at vanishing chemical potential may be decomposed as

$$\chi_{ud} = \frac{1}{VT}\left[-2n^0_{u\bar{d}} + 2n^0_{ud} + 4n^0_{uud} + 4n^0_{udd}\right], \quad (36)$$

where, as before, $2n^0_x$ includes similar contributions from both particles and antiparticles at vanishing chemical potentials. It is also assumed that populations of higher excited states, e.g., the hexaplet of diquarks, as well as states lying in the baryon decuplet are included in the respective populations. In the remaining, we deal with densities as opposed to the absolute numbers:

$$\rho^0_x = \frac{n^0_x}{V}. \quad (37)$$

Given the off-diagonal susceptibility in a range of temperatures, one obtains a temperature-dependent relation between the baryonic and mesonic densities, i.e.,

$$\begin{aligned}2\rho^0_{u\bar{d}}(T) = & \ 2\rho^0_{ud}(T) + 4\rho^0_{uud}(T) + 4\rho^0_{udd}(T) \\ & - T\chi_{ud}(T, \mu = 0), \end{aligned} \quad (38)$$

where $\rho^0_x(T)$ represent the densities of various quasiparticle species at temperature T and vanishing chemical potential. Unlike the conventional use of the term baryonic density, ρ^0_{uud} and ρ^0_{udd} denote the density of a certain type of baryon and not the difference between the baryon and antibaryon densities (i.e., $\rho^0_{uud}, \rho^0_{udd}$ do *not* denote the net baryon density). Adjusting the baryonic densities compared to the mesonic densities, one may obtain the requisite off-diagonal susceptibility. Introducing a large-enough baryon density, one may engineer a vanishing χ_{ud} and as an extension a vanishing χ_{us}. With such densities, a $C_{\rm BS} = C_{\rm QS} = 1$ may also be achieved by a plasma of bound states.

To differentiate a plasma of quasiparticle quarks and antiquarks that naturally produces a $\chi_{ud} \to 0$ from a plasma of colored bound states with a similar property, one needs to consider the derivatives of the off-diagonal susceptibility χ_{us}.

At finite baryon chemical potential, the susceptibility will vary, as the populations of the diquarks and the baryons change under the influence of the baryon chemical potential (quark antiquark bound states carry no baryon number and hence remain unaffected). One obtains:

$$T\chi_{ud}(T,\mu) \simeq -2\rho^0_{u\bar{d}}(T) + \{2\rho^0_{ud}(T) + 4\rho^0_{uud}(T) + 4\rho^0_{udd}(T)\}\cosh(\mu\beta), \quad (39)$$

where β is the inverse temperature. The expression is valid in the regime where MB statistics may be used instead of the full quantum statistics. Differentiating Eq. (39), with respect to $\mu\beta$, one obtains the relation

$$T\left[\frac{\partial^2\chi_{ud}}{\partial(\mu\beta)^2}\right]_{\mu=0} = 2\rho^0_{ud}(T) + 4\rho^0_{uud}(T) + 4\rho^0_{udd}(T) = T\left[\frac{\partial^4\chi_{ud}}{\partial(\mu\beta)^4}\right]_{\mu=0}. \quad (40)$$

The model at this stage is applicable to any situation where there are baryonic and mesonic degrees of freedom that carry well-defined quantum numbers of upness or downness. The baryons and mesons may be colored or color singlets. The considerations outlined above are applicable to all such models, including the hadron resonance gas model used below T_c.

In this way, one may divide the contributions to the off-diagonal susceptibility and its derivatives in terms of mesonic and baryonic contributions. Using the measured susceptibility and its derivatives, these contributions may be estimated as a function of the temperature. In the lattice computations, results are expressed in units of T_c; we assume $T_c = 0.17$ GeV for definiteness. These are plotted as the thick solid line (mesons)

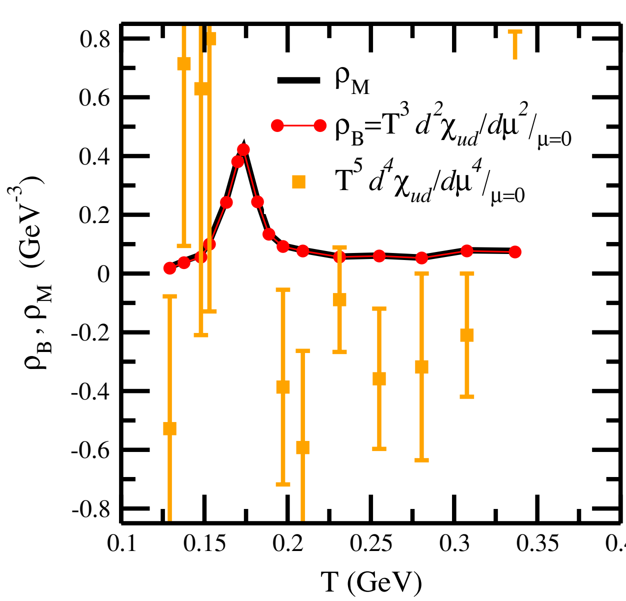

FIG. 4. (Color online) A plot of the mesonic and baryonic contributions to $\chi_{ud}(T,\mu=0)$ and a plot of the derivatives $T\partial^2\chi_{ud}/\partial(\mu\beta)^2|_{\mu=0}$ (which are equal to the baryonic contribution ρ_B) and $T\partial^4\chi_{ud}/\partial(\mu\beta)^4|_{\mu=0}$ (which is negative beyond $T_c =$ 170 MeV and thus is inconsistent with a bound state interpretation of the data). See text for details.

and the solid circles (baryons) in Fig. 4. These densities for $\rho_B = 2\rho^0_{ud}(T) + 4\rho^0_{uud}(T) + 4\rho^0_{udd}(T)$ and $\rho_M = 2\rho^0_{u\bar{d}}(T)$ satisfy Eq. (38) and the first equality of Eq. (40). The condition imposed by Eq. (40) has to be satisfied by the derivatives of the susceptibility in such a picture of bound states. The fourth derivative of the susceptibility has been plotted as the square symbols. Despite large error bars one notes that although the baryon density or the second derivative of the susceptibility is consistent with Eq. (40) below the phase transition temperature, it becomes inconsistent with such a condition above the phase transition temperature. Above T_c, $T\partial^4\chi_{ud}/\partial(\mu\beta)^4$ is actually negative; hence, no composite quasiparticle picture is compatible with such results. It has already been pointed out in Ref. [24] that the signs of the various derivatives of the susceptibility are consistent with the picture of a weakly interacting quasiparticle gas.

In the above simple model, numerous approximations were made. Maxwell-Boltzmann statistics was used throughout, variances were replaced with the mean populations of the various flavors, and masses of various flavors were assumed to be independent of chemical potential. Such approximations were made to clearly illustrate the central point of this article that the measured values of the off-diagonal susceptibilities and their derivatives are inconsistent with a picture of a composite quasiparticle plasma. However, the susceptibility in a weakly interacting model of quark quasiparticles, computed in the hard-thermal loop approximation, has been shown to be consistent with the diagonal and off-diagonal susceptibilities derived from lattice simulations [23]. It has also been pointed out that the signs of the derivatives of the susceptibilities are consistent with the quasiparticle picture [24]. A computation of the absolute values of the derivatives in such a picture is currently underway.

V. EXPERIMENTAL OBSERVABLES

In the previous sections, a theoretical study of various diagonal and off-diagonal susceptibilities has been carried out and their relations with the degrees of freedom in heated strongly interacting matter has been elucidated. In the present section, our focus lies on the possible measurement of such correlations in heavy-ion experiments. Our considerations are restricted to the measurement of the ratio $C_{\rm BS}$, which in the view of the authors is the most favorable from an experimental point of view.

It is believed that thermalized, strongly interacting, and deconfined matter is transiently produced in central heavy-ion collisions at RHIC. If one divides the whole system into small rapidity bins, then the fluctuations of conserved charges within a given rapidity bin are controlled by the degrees of freedom prevalent at the temperatures achieved. As the system expands and cools, it reconverts to a hadronic gas prior to freeze-out. If the transition to the confined phase is sudden, as in the case of a continuous transition and the longitudinal expansion is sufficiently large, then the net charge in the rapidity bin, set in the deconfined phase, is maintained through the hadronic phase up to freeze-out. Such fluctuations may then be measured event by event. The two major hurdles in the survival of such fluctuations through the hadronic phase are the contamination by hadronic fluctuations and measurability of the particles sensitive to the partonic fluctuations. The first issue is of lesser importance for an observable such as C_{BS}, as the lightest carriers of strangeness are the kaons that are much heavier than the temperatures reached in RHIC collisions in the hadronic phase. They are produced in far fewer number than pions and hence do not manage to diffuse through multiple rapidity bins in the short time available in the hadronic phase. The measurement of baryon number is the primary problem in the estimation of C_{BS}.

The detector most suitable to measurements of bulk fluctuations at RHIC is the STAR detector. In the measurement of baryon-strangeness correlations, the detector has to accurately assess the baryon number and strangeness in a given rapidity bin in each event. As the STAR detector is blind to stable uncharged particles it cannot measure the neutron and antineutron populations. As a result, a measurement of σ_{BS} may become rather difficult. Based on the discussion of Sec. II, we present the following recourse. A new quantum number is constructed,

$$M = B + 2I_3, \tag{41}$$

and fluctuations of M with respect to S are studied. In theoretical calculations of the the quantity σ_{MS} as outlined in Secs. II and IV, one notes that the assumption of isospin symmetry [see Eq. (13)] reduces the covariance σ_{MS} to simply σ_{BS}, i.e.,

$$\begin{aligned}\sigma_{MS} &= \langle (B + 2I_3)S\rangle - \langle B + 2I_3\rangle\langle S\rangle \\ &= \sigma_{BS} + 2\sigma_{I_3S} \simeq \sigma_{BS}.\end{aligned} \tag{42}$$

As a result, in all theoretical models with isospin symmetry $C_{MS} = C_{BS}$.

In the experimental determination, M has the advantage that it is vanishing for all particles that do not carry charge or strangeness, thus $M = 0$ for neutrons, antineutrons, neutral pions, and so on. The experimental measure is thus

$$C_{MS} = -3\frac{\sum_n M^{(n)}S^{(n)} - (\sum_n M^{(n)})(\sum_n S^{(n)})}{\sum_n (S^{(n)})^2 - (\sum_n S^{(n)})^2}. \tag{43}$$

In the above equation $M^{(n)}$ and $S^{(n)}$ are the total M and total strangeness within the given rapidity bin in event (n). One may not make the simplification of counting the product quantum number MS for individual flavors as in Eq. (11) as the fluctuations in M and S are set in the partonic phase and the final hadrons are the result of decay from the deconfined phase. Hence, the different flavors are no longer uncorrelated as assumed in the derivation of Eq. (11).

The presence of I_3 in the observable, introduces a new problem in the experimental measurement. The lightest carriers of M are the charged pions that are numerous in the hadronic phase and may lead to contamination of the conserved charge in the chosen rapidity bin from neighboring bins. However, as the central rapidity bins at RHIC are practically charge neutral, the possibility of contamination by charged pion fluctuations is greatly reduced. One may divide the measured correlation between I_3 and S into a genuine correlation and a contamination,

$$\sigma_{I_3S} = \sigma^{\text{act}}_{I_3S} + \sigma^{\text{cont}}_{I_3S}. \tag{44}$$

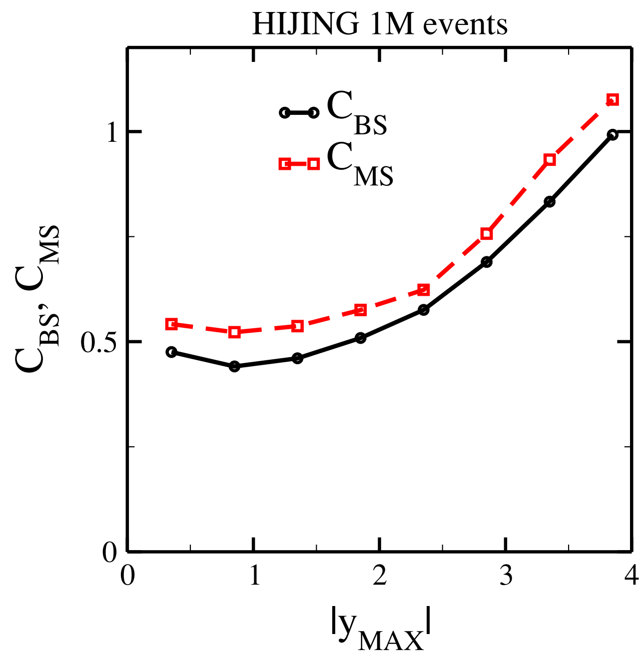

FIG. 5. (Color online) A comparison of the two related ratios of variances C_{BS}, C_{MS} as a function of the acceptance in rapidity from $-|y_{\max}|$ to $|y_{\max}|$.

As the fluctuations that result in $\sigma^{\text{cont}}_{I_3S}$ are driven by pions in the hadronic phase, a sum over a relatively large number of events will lead to this quantity becoming rather small compared to the actual correlation $\sigma^{\text{act}}_{I_3S}$ if violations of isospin symmetry are negligible. This condition should hold for the produced hadronic phase over a range of rapidities at RHIC. This effect is illustrated in Fig. 5 where both C_{BS} and C_{MS} are calculated from model simulations using the HIJING code [28].

In Fig. 5 the correlations C_{BS} and C_{MS} are estimated in a central $Au - Au$ event at $\sqrt{s} = 200$ AGeV. The acceptance in rapidity ranges from $-|y_{\max}|$ to $|y_{\max}|$, hence a larger $y_{\max}$ indicates a larger acceptance. In an effort to further mimic the experimental acceptance, K_L mesons are ignored, and K_S mesons are identified either as a K^0 or a $\bar{K}^0$ with 50% probability for either case. The results are presented as a function of $y_{\max}$. One notes that over the range of $y_{\max}$ the two correlations C_{BS} and C_{MS} are rather similar. The increased fluctuations in isospin are the cause of the slightly larger value of C_{MS} as compared to C_{BS}. This bodes well for the measurement of C_{BS} in RHIC experiments via a measurement of the quantity C_{MS} over a range of rapidity intervals.

VI. CONCLUSIONS

Both heavy-ion collisions and lattice simulations of QCD at finite temperature present components in the study of heated strongly interacting matter. In this article we demonstrated that off-diagonal susceptibilities and ratios of susceptibilities have the ability to discern the prevalent flavor-carrying degrees of freedom in heated strongly interacting matter. The latter quantity may be measured on the lattice as well as in heavy-ion collisions, under the assumption that event-by-event fluctuations in a heavy-ion collision are set in the deconfined phase and are maintained through the hadronic phase.

In this article, the behavior of a number of observables based on the ratio of susceptibilities $C_{\rm BS}, C_{QS}$, etc., was explored both in the confined as well as the deconfined phase. Under the assumption of isospin symmetry, simplifying relations between such observables were derived that reveal the interdependence of such ratios, e.g., Eqs. (16) and (18). Results of the computations of such quantities in a hadronic resonance gas as well as in a non-interacting plasma of quarks and gluons were compared with calculations on the lattice (Fig. 3). Such comparisons demonstrate that the flavor-carrying sector of QCD is consistent with a deconfined plasma of weakly interacting quarks and antiquarks above T_c. Below T_c the behavior of $C_{\rm BS}$ and $C_{\rm QS}$ is consistent with that of a hadron resonance gas.

The various relations between the ratios of susceptibilities relate the behavior of $C_{\rm QS}$ to that of $C_{\rm BS}$. The behavior of $C_{\rm BS}$ above and below T_c is caused primarily by the vanishing of the the off-diagonal susceptibility $\chi_{us} = \chi_{ds}$ at $T \geqslant T_c$ and the negative value of its expectation below T_c. A similar behavior is shown by other off-diagonal susceptibilities such as χ_{ud}. The remainder of our study focused on the behavior of the two flavor off-diagonal susceptibility χ_{ud}, as calculations of the temperature dependence of χ_{ud} as well as its various derivatives have been carried out in full unquenched lattice simulations.

In Secs. III and IV, it was demonstrated that the behavior of χ_{ud} as well as its various derivatives is consistent with that of a hadron gas below T_c and a weakly interacting plasma of quarks and antiquarks above T_c. The behavior of the various derivatives of χ_{ud} with baryon chemical potential was shown to be inconsistent with a bound-state picture above $T = T_c$. Such an inconsistency remained even if baryonic bound-state populations above T_c were artificially enhanced to be consistent with the $C_{\rm BS}$ and $C_{\rm QS}$ measured on the lattice.

Finally, we proposed a new experimental observable $C_{\rm MS}$ related by isospin symmetry to $C_{\rm BS}$. This observable is blind to uncharged and nonstrange particles. We showed that it is equivalent to $C_{\rm BS}$ and may be measurable in experiments at RHIC. Estimates of $C_{\rm MS}$ and $C_{\rm BS}$ in HIJING simulations demonstrate the similarity of the two quantities over a range of rapidities at RHIC. This bodes well for its use as an experimental proxy for $C_{\rm BS}$. Such measurements offer the possibility to directly probe the degrees of freedom in the deconfined matter produced in high-energy heavy-ion collisions.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix, we outline the derivation of Eq. (23). Imagine strongly interacting matter at low temperature, confined in a box of volume V. The temperature is assumed to low enough for the prevalent degrees of freedom to be a dilute gas of pions. The state vector representing a pion with momentum $p = 2n_p\pi/V^{1/3}$ may be expressed as

$$|\pi^+_{\vec{p}}\rangle = \sum_{\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\},\{\mathbf{l}\}} \Psi_1^{\vec{p}}(\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\},\{\mathbf{l}\}) \times |\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\}\rangle \times \delta\left(\sum_i n_i - \bar{n}_i - 1\right)\delta\left(\sum_i \bar{m}_i - m_i - 1\right) \otimes |\{\mathbf{l}\}\rangle. \tag{A1}$$

In the above equation, the vectors of integers $\{\mathbf{n}\}(\{\bar{\mathbf{n}}\}), \{\mathbf{m}\}(\{\bar{\mathbf{m}}\})$, represent the set of occupation numbers in different momentum states of u quarks ($\bar{u}$ antiquarks) and d quarks ($\bar{d}$ antiquarks), i.e.,

$$\{\mathbf{n}\} \equiv \{n_1, n_2, n_3, \cdots\}. \tag{A2}$$

Values for n_i may be 0 or 1. The vector $\{\mathbf{l}\}$ represents the occupation numbers of the gluon sector and is a vector of integers $l_i \geqslant 0$. In Eq. (A1), $|\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\}\rangle$ represents the general state vector of the quark (antiquark) sector. The function $\Psi_1^{\vec{p}}(\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\},\{\mathbf{l}\})$ represents the wave function of the one pion state with the constraint that the total momentum residing in this sector is $\vec{p}$. Hence,

$$\Psi_1^{\vec{p}}(\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\},\{\mathbf{l}\}) = \widetilde{\Psi}(\{\mathbf{n}\},\{\bar{\mathbf{n}}\},\{\mathbf{m}\},\{\bar{\mathbf{m}}\},\{\mathbf{l}\}) \times \delta\left(\vec{p} - \sum_i \vec{p}_{u,i}n_i + \vec{p}_{\bar{u},i}\bar{n}_i + \vec{p}_{d,i}m_i + \vec{p}_{\bar{d},i}\bar{m}_i + \vec{p}_{g,i}l_i\right), \tag{A3}$$

where, $\vec{p}_{u,i}$ is the momentum of the ith u-quark state with occupation n_i. Although not explicitly pointed out, the wave function Ψ_1^p also maintains over all color neutrality.

Given the form of the one-pion state, it is a trivial matter to formulate general expressions for multiple-pion states. In this way, an effective basis of states at low temperature is constructed: $|0\rangle, |\pi^+_{p_1}\rangle, |\pi^+_{p_1}\pi^+_{p_2}\rangle, |\pi^+_{p_1}\pi^-_{p_2}\rangle$, etc. Interactions between these various states, is assumed to be small enough to be estimated in a perturbative formalism. The reader will note that the various states outlined above are orthogonal, given the orthogonality of the various states in the quark-gluon occupation number basis used in Eq. (A1). Such n-pion states may also be expressed in an occupation number basis as above, e.g.,

$$|\pi^+_{p_1}\rangle \equiv |0_1, 0_2, \cdots, 0_{p_1-1}, 1_{p_1}, 0_{p_1+1}, \cdots\rangle. \tag{A4}$$

One may now express the quark number operators as a matrix in the occupation number basis of pion states, i.e.,

$$\hat{N}_u = \sum_{\{\mathbf{n}\},\{\mathbf{m}\}} |n_1, n_2, \cdots\rangle \times \langle n_1, n_2, \cdots|\hat{N}_u|m_1, m_2, \cdots\rangle\langle m_1, m_2, \cdots|. \tag{A5}$$

Using the expression for the one-pion state from Eq. (A1), we obtain the simple relation,

$$\hat{N}_u|0_1, 0_2, \cdots, 0_{p_1-1}, 1_{p_1}, 0_{p_1+1}, \cdots\rangle. = \hat{N}_u|\pi^+_{p_1}\rangle = |\pi^+_{p_1}\rangle. \quad \text{(A6)}$$

Similarly, the action of the upness operator on the one π^- state may be computed to be

$$\hat{N}_u|\pi^-_{p_1}\rangle = -|\pi^-_{p_1}\rangle. \quad \text{(A7)}$$

Generalizing to the n-pion states, one obtains the general relations for the quark number operators in the basis of pions (in the limit that the pion gas is dilute, i.e., the interactions between the different states are small),

$$\hat{N}_u = \hat{N}_{\pi^+} - \hat{N}_{\pi^-}, \quad \hat{N}_d = \hat{N}_{\pi^-} - \hat{N}_{\pi^+}. \quad \text{(A8)}$$

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