Maximum freeze-out baryon density in nuclear collisions

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Using simple parametrizations of the thermodynamic freeze-out parameters extracted from the data over a wide beam-energy range, we reexpress the hadronic freeze-out line in terms of the underlying dynamic quantities, the net baryon density ρ_B and the energy density ε, which are subject to local conservation laws. This analysis reveals that ρ_B exhibits a maximum as the collision energy is decreased. This maximum freeze-out density has $\mu = 400$–500 MeV, which is above the critical value, and it is reached for a fixed-target bombarding energy of 20–30 GeV/nucleon.

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In recent years, it has become abundantly clear that the collision energy plays an important role in determining the properties of the final state in relativistic heavy ion collisions. While the extracted freeze-out temperature generally increases monotonically with the collision energy, the corresponding net baryon density exhibits a more complicated behavior: It initially increases steadily as the collision energy is raised, but ultimately, at sufficiently high energies, it decreases as a result of nuclear transparency. Therefore, there is a certain beam energy for which the resulting baryon density at freeze-out acquires a maximum value. In this paper, we discuss this optimal beam energy on the basis of the most up-to-date results on the properties of hadronic freeze-out.

For this purpose, we employ statistical considerations which have been quite successful in accounting for the observed yields of most hadronic species (see Refs. [1–5], for example). In fact, the evidence has become truly overwhelming in recent years that chemical equilibrium is established in relativistic heavy ion collisions throughout the entire range of collision energies. The resulting picture shows that the freeze-out temperature T increases steadily with the collision energy as the corresponding baryon chemical potential μ_B decreases steadily toward zero.

However, it is not always convenient to work with the thermodynamic variables T and μ_B. This is particularly true within a dynamic context, since they are not subject to any conservation laws, in contrast with the more basic dynamic variables, namely, the energy density ε and net baryon density ρ_B. Furthermore, when a first-order phase transition is present, T and μ_B become nonmonotonic functions of ε and ρ_B throughout the associated phase-coexistence region of the phase diagram (as does the pressure p). This feature in turn causes the inverse functions to be multivalued, a clearly inconvenient situation. It is therefore of interest to reexpress the thermodynamic freeze-out information in terms of the dynamic quantities. The resulting representation also serves to more clearly demonstrate that the freeze-out density exhibits a maximum value, a feature that may be important in the planning of experiments that seek to explore compressed baryonic matter.

In the grand-canonical ensemble of hadron species i, the partition function factorizes into separate contributions for each specie, $\mathcal{Z} = \Pi_i \mathcal{Z}_i$, with

$$\ln \mathcal{Z}_i(T, V, \{\mu\}) = \pm \frac{V T g_i m_i^2}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\pm\lambda_i)^n}{n^2} K_2\left(\frac{n m_i}{T}\right), \quad (1)$$

where plus is for bosons and minus is for fermions. The hadron mass is m_i and $g_i = 2J_i + 1$ is the spin degeneracy. The fugacity associated with the hadron specie i is

$$\lambda_i(T, \{\mu\}) = \exp\left[(\mu_B B_i + \mu_Q Q_i + \mu_S S_i)/T\right], \quad (2)$$

where B_i, Q_i, and S_i denote its baryon number, electric charge, and strangeness, respectively. The ensemble is characterized by the temperature T and the three chemical potentials $\{\mu\} = \{\mu_B, \mu_Q, \mu_S\}$. Finally, V denotes the enclosing volume of the system (which plays no role in the analysis since only yield ratios are considered).

The corresponding spatial number density of a given specie is then readily obtained as

$$n_i(T, \{\mu\}) = \pm \frac{g_i T^3}{2\pi^2} \sum_{n=1}^{\infty} \frac{(\pm\lambda_i)^n}{n^3} \left(\frac{n m_i}{T}\right)^2 K_2\left(\frac{n m_i}{T}\right) \quad (3)$$

$$= \frac{g_i \lambda_i}{2\pi^2} m_i^2 T K_2\left(\frac{m_i}{T}\right) \pm \cdots. \quad (4)$$

In the first line, the factors are arranged such that the terms in the sum are regular in the small-mass limit, $m_i \to 0$, where K_2 diverges, and the second line exhibits the leading term representing the classical result. Since the sign of the second term in (3) depends on the quantum-statistical nature of the specie, the leading term overestimates fermion densities while it underestimates boson densities. For all the parameter values employed in the yield estimates, the first term in the expansion (4) is an excellent approximation for all baryon species, being accurate to within a few per thousand. For kaons it is off by a few percent, and even the pion densities are underestimated by at most 10%. The first term in (4) would thus be a quite reasonable approximation for rough estimates of the yields.

Once we know the number density of each specie, it is straightforward to calculate the corresponding baryon, charge, and strangeness densities. In particular, the net baryon density is

$$\rho_B(T,\{\mu\}) = T\frac{\partial \ln \mathcal{Z}}{\partial \mu_B} = \sum_i B_i n_i(T,\{\mu\}). \tag{5}$$

Furthermore, the total energy density is $\varepsilon = \sum_i \varepsilon_i$, where the contribution by a particular specie i is

$$\varepsilon_i(T,\{\mu\}) = -\frac{\partial \ln \mathcal{Z}_i}{\partial \beta} = \pm\frac{g_i T^4}{2\pi^2}\sum_{n=1}^{\infty}\frac{(\pm\lambda_i)^n}{n^4} \tag{6}$$

$$\times\left[\left(\frac{nm_i}{T}\right)^2 K_2\left(\frac{nm_i}{T}\right) + \left(\frac{nm_i}{T}\right)^3 K_1\left(\frac{nm_i}{T}\right)\right]$$

$$\approx \frac{g_i\lambda_i}{2\pi^2}m_i^2T^2\left[K_2\left(\frac{m_i}{T}\right) + \frac{m_i}{T}K_1\left(\frac{m_i}{T}\right)\right], \tag{7}$$

where the last line [Eq. (7)] is the classical result.

As it turns out, it is possible to fit the observed yield ratios reasonably well with the statistical model by adjusting the Lagrange multipliers at each particular collision energy [1–5]. In this analysis, T and μ_B are treated as free parameters, while the values of μ_S and μ_Q are fixed by requiring overall strangenss neutrality, $\langle S\rangle = 0$, and that the ratio of the net charge to the net baryon number be equal to that of the collision system, e.g., $\langle Q\rangle = 0.4\langle B\rangle$ for Au+Au. As discussed in Ref. [6], the effect of the small change in this ratio in going to Pb+Pb is negligible. The extracted values of the freeze-out temperature can be approximately represented as [6]

$$T(\mu_B) \approx 166 - 139\mu_B^2 - 53\mu_B^4, \tag{8}$$

where T and μ_B are expressed in MeV.

Using the relationship (8), we have calculated the corresponding freeze-out values of the net baryon density ρ_B [Eq. (5)] and the energy density ε [Eq. (6)]. The result is displayed in Fig. 1. Before turning to the calculated results, we note that the lower-right section of the ρ_B-ε plane is not physically accessible, since a given net baryon density ρ_B gives rise to a certain minimum energy density. [The corresponding curve, $\varepsilon_{\min}(\mu_B)$, is simply the pressure at zero temperature and is therefore occasionally, and somewhat misleadingly, referred to as the equation of state.] In the ideal-gas scenario, where binding and compression effects are absent, this lower bound is given by $\varepsilon_{\min} = m_N\rho_B$, which is indicated on the figure.

In order to illustrate the importance of adjusting μ_Q and μ_S to ensure average conservation of charge and strangeness (see above), we have also calculated those values of ρ_B and ε that would result if μ_Q and μ_S were taken to vanish. As can be seen from Fig. 1, the failure to adjust μ_Q and μ_S increases the values of both densities by amounts that are especially significant in the region of maximal density [7]. These results were calculated both classically (first terms only) and quantally (all terms) to illustrate how unimportant this distinction is for the present analysis.

At the highest energies, freeze-out occurs for a negligible value of the chemical potential μ_B (and hence the net baryon density ρ_B is practically zero), and the energy density ε is nearly one-half GeV/fm^3. As the collision energy is lowered, ρ increases rapidly while T initially remains fairly constant but gradually begins to drop. Then, in the range of

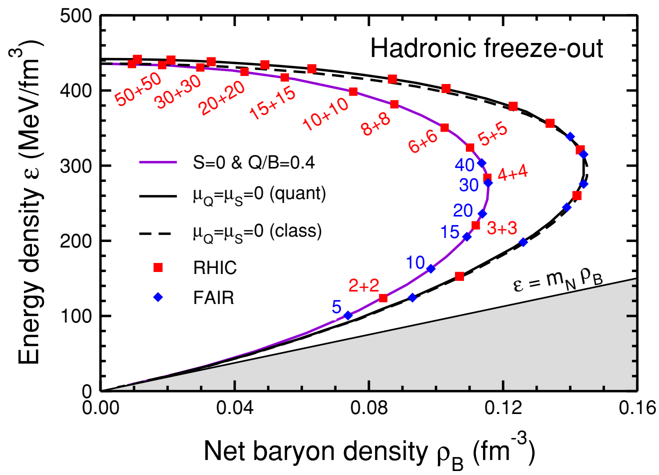

FIG. 1. (Color online) The hadronic freeze-out line in the $\rho_B - \varepsilon$ phase plane as obtained in the statistical model with the values of μ_B and T that have been extracted from the experimental data in Ref. [6]. The curves on the right have been calculated for $\mu_Q = \mu_S = 0$ using either quantal (solid) or classical (dashed) statistics, while the curve on the left employs values of μ_Q and μ_S that have been adjusted to ensure $\langle S\rangle = 0$ and $\langle Q\rangle = 0.4\langle B\rangle$ for each value of μ_B. Also indicated are the beam energies (in GeV/nucleon) for which the particular freeze-out conditions are expected at either RHIC (solid squares, total energy in each beam), starting at 100+100 and going down to 2+2, or FAIR (solid dots, kinetic energy of the beam for a stationary target), starting at 5 and going up to 40. as based on fits to existing data [6]. The triangular area (grey) corresponds to energy densities below the minimum required at the given net baryon density, $\varepsilon = m_N\rho_B$ (ignoring binding and compression), and is thus inaccessible.

T = 140–130 MeV, the freeze-out line (ρ_B, ε) exhibits a backbend and approaches the origin. The resulting maximum value of the net baryon density at freeze-out is about three-quarters of the normal nuclear saturation density of $\rho_0 \approx 0.15\,\mathrm{fm}^{-3}$.

The value of the baryon chemical potential μ_B extracted from the data decreases monotonically with the collision energy and can be parametrized on a simple form [6]

$$\mu_B(\sqrt{s}) \approx \frac{1308}{1000 + 0.273\sqrt{s}}, \tag{9}$$

where μ_B as well as the NN c.m. energy $\sqrt{s}$ are expressed in MeV. By using this result, it is possible to attach beam energies along the freeze-out curves shown in Fig. 1. This has been done for both collider experiments, such as those being carried out at the BNL Relativistic Heavy Ion Collider (RHIC), or for a fixed target, as has been done at the CERN Super Proton Synchrotron (SPS) and the BNL Alternating Gradient Synchrotron (AGS) and is being planned at Facility for Antiproton and Ion Research (FAIR).

We note that if this region of maximum freeze-out density were to be explored with RHIC, the appropriate beam kinetic energies would be about 2–4 GeV/nucleon, corresponding to $s = (6\text{–}10\,\mathrm{GeV/nucleon})^2$, which would be a rather challenging proposition. By contrast, it appears that the region would be well within reach of FAIR which is planned to bombard fixed targets with heavy ion beams having kinetic energies of 5–40 GeV/nucleon.

Finally, according to the most refined lattice QCD studies [8–12], it appears that the region of highest freeze-out density lies well beyond the critical region where the transformation between the quark-gluon plasma and the hadronic resonance gas changes from being merely a crossover to being a genuine first-order phase transition. This fact is helpful to the prospect of probing this phase transition by means of heavy ion collisions.

Let us briefly summarize: Analyses of experimentally obtained hadronic yield ratios at a variety of collision energies have shown that the data can be well reproduced within the conceptually simple statistical model that describes an ideal hadron resonance gas in statistical equilibrium. Furthermore, the extracted freeze-out values of the temperature and the baryon chemical potential exhibit a smooth and monotonic dependence on the collision energy and can be simply parametrized. We have used these results to examine how freeze-out appears when represented in terms of the basic baryon and energy densities, rather than chemical potential and temperature. These quantities are more basic, and they are of more direct relevance to the collision dynamics, because they are subject to corresponding conservation laws.

We have found that the freeze-out value of the net baryon density exhibits a maximum as the collision energy is being scanned, with a value of about three-quarters of the saturation density. In a fixed-target configuration, this maximum freeze-out density is reached for a beam kinetic energy of 20–30 GeV/nucleon. This is well within the range of the planned FAIR at GSI, and it may also be accessible in the low-energy campaign now under preparation at RHIC.

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