Nuclear matter saturation point and symmetry energy with modern nucleon-nucleon potentials

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We determine the saturation properties of nuclear matter within the Brueckner-Hartree-Fock approach based on a large set of modern nucleon-nucleon potentials and confirm the validity of the Coester band. The improvement of the saturation point when including nuclear three-body forces is pointed out and comparison with the Dirac-Brueckner-Hartree-Fock results is made.

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Since the early 1980s we have known that the classical Brueckner-Hartree-Fock (BHF) approach to nuclear matter [1] is unable to reproduce exactly the empirical saturation point of symmetric nuclear matter, i.e., the saturation density $\rho_0 \approx 0.17\ \text{fm}^{-3}$ and binding energy per nucleon $B/A \approx (-16 \pm 1)$ MeV [2]. Instead the BHF saturation points obtained with different realistic nucleon-nucleon (NN) potentials were found to lie within a narrow band, close to the so-called Coester line [3], yielding either a too large saturation density or a too small binding energy.

However, the concept of the Coester line was originally established based on BHF calculations using the simple but rather unrealistic gap choice prescription for the single-particle energies [1,3], whereas nowadays the continuous choice is routinely used. Moreover, the numerical accuracy of the BHF calculations has been greatly improved due to more advanced computational facilities. Nevertheless it has been confirmed that this deficiency persists up to the third order in the hole-line expansion [4], hinting at the necessity of including the effect of three-nucleon forces (TBF) [5–7] in the BHF approach. Indeed, such a procedure leads to substantial improvement of the saturation properties [6–8]. Furthermore, new accurate *NN* potentials based on novel theoretical concepts have become available, for example, the N3LO potential of Ref. [9] based on chiral symmetry or the strongly nonlocal IS potential of Ref. [10].

We therefore think it is useful to give an update of the situation employing the most recent NN potentials and also to demonstrate the improvement of saturation achieved by including three-body forces within the BHF scheme. This is the main purpose of the present report. In addition we present results for the symmetry energy obtained with the different potentials, which is of great importance in particular for astrophysical applications: It essentially determines the proton fraction of β-stable matter and thus influences physical properties like the equation of state (EOS) and cooling processes [8,11]. Furthermore, the density derivative of the symmetry energy appears to be strongly correlated with the difference between neutron and proton rms radii (neutron skin thickness) of heavy nuclei [12].

We begin with a short review of the theoretical framework: The microscopic Brueckner-Bethe-Goldstone description of nuclear matter is based on a linked cluster expansion of the energy per nucleon of nuclear matter [1]. The basic ingredient is the Brueckner reaction matrix G, which is the solution of the Bethe-Goldstone equation,

$$G[\omega;\rho] = V + \sum_{k_a,k_b>k_F} V \frac{|k_a k_b\rangle\langle k_a k_b|}{\omega - e(k_a) - e(k_b) + i\epsilon} G[\omega;\rho], \tag{1}$$

where V is the bare nucleon-nucleon interaction, ρ is the nucleon number density, and ω the starting energy. The single-particle energy is

$$e(k) = e(k;\rho) = \frac{k^2}{2m} + U(k;\rho) \tag{2}$$

and the propagation of intermediate nucleon pairs is constrained above the Fermi momentum k_F. The BHF approximation for the single-particle potential $U(k;\rho)$ using the continuous choice prescription is

$$U(k;\rho) = \text{Re} \sum_{k' \leqslant k_F} \langle kk'|G[e(k)+e(k');\rho]|kk'\rangle_a, \tag{3}$$

where the subscript a indicates antisymmetrization of the matrix elements. Because of the occurrence of $U(k;\rho)$ in Eq. (2), Eqs. (1)–(3) constitute a coupled system of equations that needs to be solved self-consistently. In the BHF approximation the energy per nucleon is given by

$$\frac{B}{A} = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho}\text{Re} \sum_{k,k' \leqslant k_F} \langle kk'|G[e(k)+e(k');\rho]|kk'\rangle_a. \tag{4}$$

The basic input quantity in the Bethe-Goldstone equation (1) is the NN interaction in free space, V. In this work we perform a survey adopting a large number of modern potentials, namely the Paris potential [13]; the Argonne V14 [14] and V18 [15] potentials; the Bonn A, B, C [16] and CD-Bonn [17] potentials; the Reid 93 and Nijmegen 93, I, and II potentials [18]; as well as the most recent N3LO [9] and IS [10] potentials.

Technically, we have tried to obtain accurate results by enforcing the self-consistency condition Eq. (3) up to $k = 9\ \text{fm}^{-1}$ in momentum space and by including a large number of partial waves ($J_{\max} = 9$) in the expansion of the G matrix. The remaining angle-average approximation for Pauli operator and single-particle energies in the Bethe-Goldstone equation has been shown to introduce errors well below 1 MeV for the binding energy at saturation [19].

Concerning the inclusion of three-body forces in the BHF approach, we use the formalism developed in Refs. [5–7], namely a microscopic model based on meson exchange with intermediate excitation of nucleon resonances (Delta, Roper, and nucleon-antinucleon). The meson parameters in this model are constrained to be compatible with the two-nucleon potential, where possible.

For the use in BHF calculations, this TBF is reduced to an effective, density-dependent, two-body force by averaging over the third nucleon in the medium, the average being weighted by the BHF defect function g, which takes account of the nucleon-nucleon in-medium correlations [6,8,20]:

$$\overline{V}_{ij}(\boldsymbol{r}) = \rho \int d^3 r_k \sum_{\sigma_k,\tau_k} [1-g(r_{ik})]^2 [1-g(r_{jk})]^2 V_{ijk}. \quad (5)$$

The resulting effective two-nucleon potential has the operator structure

$$\overline{V}_{ij}(\boldsymbol{r}) = (\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)V_C^{\tau\sigma}(r) + (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)V_C^{\sigma}(r) + V_C(r) + S_{ij}(\hat{\boldsymbol{r}})\Big[(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)V_T^{\tau}(r) + V_T(r)\Big] \quad (6)$$

and the components $V_C^{\tau\sigma}$, V_C^{σ}, V_C, V_T^{τ}, V_T are density dependent. They are added to the bare potential in the Bethe-Goldstone equation (1) and are recalculated together with the defect function in every iteration step until convergence is reached. This approach has so far been followed with the Paris [6], the V14, and the V18 [7] potentials and the results will be shown in the following presentation of our results. For complete details, the reader is refered to Refs. [5–7].

We begin in Fig. 1 with the saturation curves obtained with our set of NN potentials. On the standard BHF level (black curves) one obtains in general too strong binding, varying between the results with the Paris, V18, and Bonn C potentials (less binding), and those with the Bonn A, N3LO, and IS (very strong binding). Including TBF (with the Paris, V14, and V18 potentials; red curves) adds considerable repulsion and yields results slightly less repulsive than the DBHF ones with the Bonn potentials [16] (green curves). This is not surprising, because it is well known that the major effect of the DBHF approach amounts to including the TBF corresponding to nucleon-antinucleon excitation by 2σ exchange within the BHF calculation [6,7]. This is illustrated for the case of the V18 potential (open stars) by the dashed (red) curve in the figure, which includes only the 2σ-exchange "Z-diagram" TBF contribution. The remaining TBF components are overall attractive and produce the final solid (red) curve in the figure.

Figure 2 shows the saturation points of symmetric matter extracted from the previous results. Indeed there is a strong linear correlation between saturation density and energy, confirming the concept of the Coester line. One can roughly identify three groups of results: The DBHF results with the Bonn potentials as well as the BHF+TBF results with the Paris, V14, and V18 potentials lie in close vicinity of the empirical value. The BHF results with Paris, V14, V18, and Bonn C form a group with about 1–2 MeV too-large binding and saturation

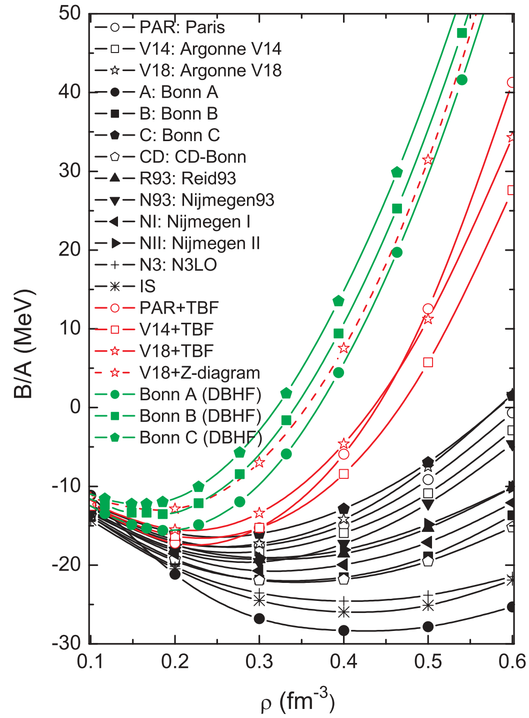

FIG. 1. (Color online) Energy per nucleon of symmetric nuclear matter obtained with different potentials and theoretical approaches. For details see text.

at about 0.27 fm^{-3}. The remaining potentials, in particular the most recent CD-Bonn, N3LO, and IS, yield strong overbinding at larger density, more than twice saturation density in the latter cases. From a practical point of view, it would therefore appear convenient to use the potentials of the former group for approximate many-body calculations, because the required corrections are smaller, at least for Brueckner-type approaches.

Historically, there is the observation that the position of a saturation point on the Coester line seems to be strongly

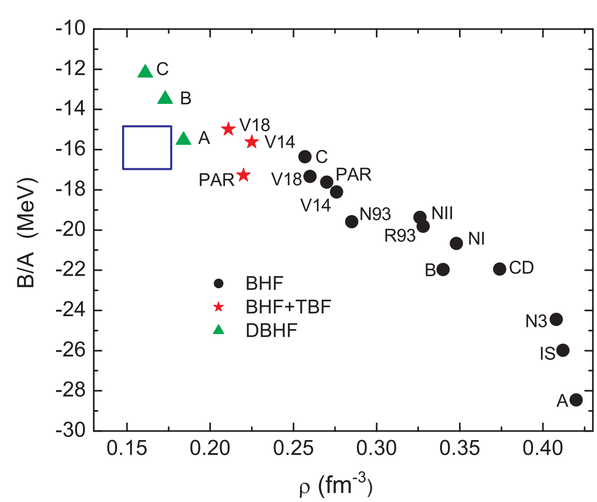

FIG. 2. (Color online) Saturation points obtained with different potentials and theoretical approaches. The (online blue) square indicates the empirical region.

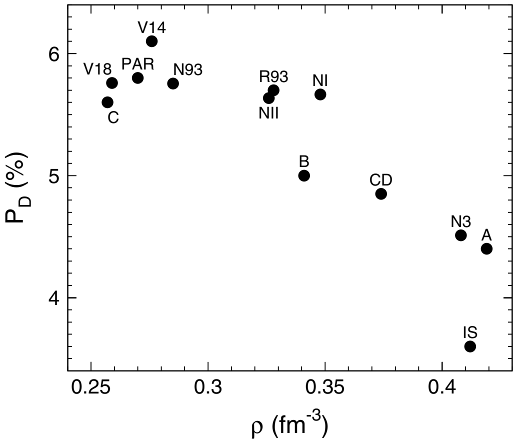

FIG. 3. Correlation between deuteron D-state probability and saturation density for the BHF results with different potentials.

correlated with the value of the deuteron D-state probability $P_D = \int d^3 r u_{L=2}^2(r)$, i.e., with the strength of the tensor force of the given potential [3]. We test this supposition in Fig. 3. In fact, a definite linear dependence is found only for the various Bonn and the N3LO potentials, whereas the remaining potentials do not exhibit any well-defined correlation between the two quantities.

Finally, Fig. 4 shows the symmetry energies, defined as difference between the binding energies of pure neutron matter and symmetric matter, obtained with the different models. One observes in all cases values that increase nearly linearly with increasing density. Generally, including TBF increases strongly the symmetry energy at high densities and leads therefore in astrophysical applications to larger proton fractions in β-stable matter, a stiffer EOS, and larger maximum neutron star masses than the (unrealistic) pure BHF results [8]. In particular, the threshold value of an 11% proton fraction allowing direct Urca cooling is easily traversed with the EOS including TBF. The symmetry energies obtained at saturation density $\rho = 0.17\ \text{fm}^{-3}$ range from 28.5 MeV (Bonn C) to 32.6 MeV (IS) and are shown in the inset of the figure.

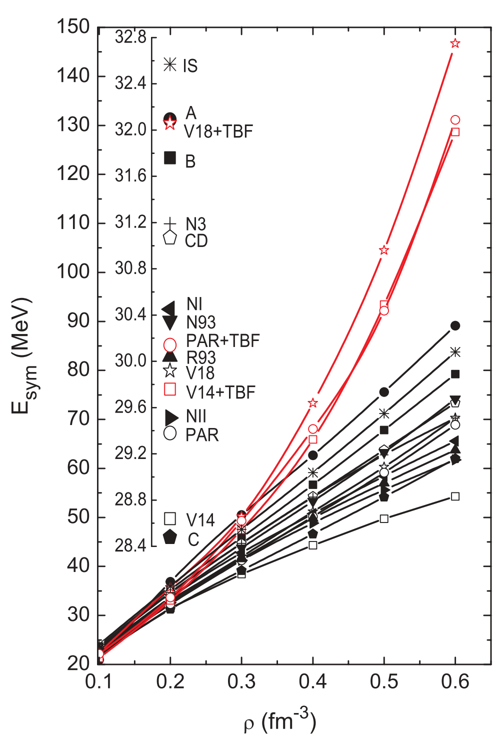

FIG. 4. (Color online) Symmetry energy obtained with different potentials within the BHF approach with (upper, red curves) and without (lower, black curves) TBF. The inset shows the values at normal density $\rho_0 = 0.17\ \text{fm}^{-3}$ on a magnified scale.

TABLE I. Properties of nuclear matter obtained with different potentials.

	ρ (fm^{-3})	$-B/A$ (MeV)	E_{sym} (MeV)	P_D (%)
Paris	0.270	17.6	29.4	5.8
Argonne V14	0.276	18.1	28.6	6.1
Argonne V18	0.259	17.3	29.9	5.76
Bonn A	0.419	28.4	32.1	4.4
Bonn B	0.341	22.0	31.8	5.0
Bonn C	0.257	16.4	28.5	5.6
CD-Bonn	0.374	21.9	31.1	4.85
Reid 93	0.328	19.8	30.0	5.70
Nijmegen 93	0.285	19.6	30.4	5.76
Nijmegen I	0.348	20.7	30.5	5.66
Nijmegen II	0.326	19.4	29.5	5.64
N3LO	0.408	24.5	31.2	4.51
IS	0.412	26.0	32.6	3.60
Paris + TBF	0.220	17.3	30.1	
V14 + TBF	0.225	15.6	29.8	
V18 + TBF	0.211	15.0	32.1	

We summarize our results for the saturation point, symmetry energy, and D-state probability obtained with the different potentials in Table I. In conclusion, we have reviewed the current status of the Coester line, i.e., the saturation points of nuclear matter obtained within the BHF approach using continuous single-particle energies and employing the most recent accurate nucleon-nucleon potentials. Our results confirm the concept of a "line" or "band," density and energy of the various saturation points being strongly linearly correlated. The BHF results including TBF (as well as the DBHF results), predict saturation in close proximity of the empirical point, whereas some of the most recent potentials yield strong overbinding of nuclear matter within the Brueckner scheme. The supposition of a strict correlation between the deuteron D-state probability and saturation is not generally confirmed by our extended data set. Furthermore, all calculations yield symmetry energies of about 30 MeV at normal density, which are increasing monotonously and roughly linearly with density.

For the future we recommend studying and refining in even more detail the microscopic TBF to narrow the margin of uncertainty associated with these forces. Also, DBHF calculations employing potentials other than the Bonn ones or including the effect of TBF are thus far unavailable and would be very useful for drawing more definite conclusions regarding this theoretical approach.

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[1] J. P. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Rep. **25C**, 83 (1976); M. Baldo, *Nuclear Methods and the Nuclear Equation of State* (World Scientific, Singapore, 1999).
[2] B. D. Day, Comm. Nucl. Part. Phys. **11**, 115 (1983); W. D. Myers and W. J. Swiatecki, Nucl. Phys. **A601**, 141 (1996).
[3] F. Coester, S. Cohen, B. Day, and C. M. Vincent, Phys. Rev. C **1**, 769 (1970); F. Coester, B. Day, and A. Goodman, *ibid.* **5**, 1135 (1972); B. D. Day, Phys. Rev. Lett. **47**, 226 (1981).
[4] B. D. Day, Phys. Rev. C **24**, 1203 (1981); H. Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, Phys. Rev. Lett. **81**, 1584 (1998); M. Baldo, G. Giansiracusa, U. Lombardo, and H. Q. Song, Phys. Lett. **B473**, 1 (2000); M. Baldo, A. Fiasconaro, H. Q. Song, G. Giansiracusa, and U. Lombardo, Phys. Rev. C **65**, 017303 (2002); R. Sartor, *ibid.* **73**, 034307 (2006).
[5] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt, and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979); R. G. Ellis, S. A. Coon, and B. H. J. McKellar, *ibid.* **A438**, 631 (1985).
[6] P. Grangé, A. Lejeune, M. Martzolff, and J.-F. Mathiot, Phys. Rev. C **40**, 1040 (1989).
[7] W. Zuo, A. Lejeune, U. Lombardo, and J.-F. Mathiot, Nucl. Phys. **A706**, 418 (2002); Eur. Phys. J. A **14**, 469 (2002).
[8] X. R. Zhou, G. F. Burgio, U. Lombardo, H.-J. Schulze, and W. Zuo, Phys. Rev. C **69**, 018801 (2004).
[9] D. R. Entem and R. Machleidt, Phys. Lett. **B524**, 93 (2002); Phys. Rev. C **68**, 041001(R) (2003).
[10] P. Doleschall, I. Borbely, Z. Papp, and W. Plessas, Phys. Rev. C **67**, 064005 (2003); M. Baldo and C. Maieron, *ibid.* **72**, 034005 (2005).
[11] I. Bombaci and U. Lombardo, Phys. Rev. C **44**, 1892 (1991); M. Baldo, I. Bombaci, and G. F. Burgio, Astron. Astrophys. **328**, 274 (1997); L. Engvik, M. Hjorth-Jensen, R. Machleidt, H. Müther, and A. Polls, Nucl. Phys. **A627**, 85 (1997); A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998).
[12] B. A. Brown, Phys. Rev. Lett. **85**, 5296 (2000); M. Baldo, C. Maieron, P. Schuck, and X. Viñas, Nucl. Phys. **A736**, 241 (2004); S. Yoshida and H. Sagawa, Phys. Rev. C **73**, 044320 (2006).
[13] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirés, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[14] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[15] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[16] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989); R. Brockmann and R. Machleidt, Phys. Rev. C **42**, 1965 (1990).
[17] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[18] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
[19] T. Cheon and E. F. Redish, Phys. Rev. C **39**, 331 (1989); R. Sartor, *ibid.* **54**, 809 (1996); K. Suzuki, R. Okamoto, M. Kohno, and S. Nagata, Nucl. Phys. **A665**, 92 (2000); F. Sammarruca, X. Meng, and E. J. Stephenson, Phys. Rev. C **62**, 014614 (2000).
[20] M. Baldo and L. S. Ferreira, Phys. Rev. C **59**, 682 (1999).