Coupling of nuclear and electron modes in relativistic stellar matter

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The conditions under which nuclear collective modes couple to plasmon modes in asymmetric nuclear matter (ANM) neutralized by electrons, which is of interest for the study of neutron stars and supernovae, are investigated. The calculations were performed within a relativistic mean-field approach to nuclear matter, and the Coulomb field was included. We show that the coupling may be so strong that it affects the onset of the nuclear mode at low densities and may also affect its isovector/isoscalar character.

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I. INTRODUCTION

The understanding of compact stars, supernova cores, and neutron stars requires a multidisciplinary theoretical effort, including astrophysics, nuclear and particle physics, and thermodynamics. From low density to few times the nuclear saturation, these stellar objects are essentially composed of neutrons, protons, electrons, and possibly, when their mean free path is short enough, neutrinos. The electrons neutralize the proton charge and thus suppress the diverging Coulomb contribution to the energy. Not only does the equation of state of stellar matter have to be understood, but also the neutrino mean free path in the medium has to be well described. It has been shown that the neutrino opacity is affected by nucleon-nucleon interactions due to coherent scattering off density fluctuations [1]. Both single-particle and collective contributions have to be taken into account. It is, therefore, important to have a through out understanding of the collective modes in asymmetric nuclear matter to predict the behavior of neutrinos.

Moreover, the liquid-gas phase transition also plays an important role in the collapse of supernova into neutron stars or black holes because stellar matter at low densities consists of neutron-rich nuclei immersed in a gas of neutrons [2]. Hence, isospin asymmetry is a very important quantity when phase transition takes place. Moreover, at very low densities, a competition between the long-range Coulomb repulsion and the short-range nuclear attraction can lead to the formation of a nonhomogeneous matter known as nuclear pasta [3], which can appear in different structures and its properties have important consequences in the crust of neutron stars and in the core-collapse of supernova [4–7]. The existence of a nonhomogeneous phase is intrinsically related to the appearance of collective unstable modes.

In a previous work [8] we studied the longitudinal nuclear and mesonic collective stable and unstable modes arising from small oscillations around a stationary state in nuclear matter. This investigation was performed in the framework of a relativistic mean-field hadronic model within the Vlasov formalism. We also investigated the influence of the electromagnetic interaction and of the presence of electrons on the unstable modes [9] and compared the dynamical instability region with the thermodynamical one. In the same work we studied the role of isospin and the modification of the distillation phenomenon due to the contribution of the Coulomb field and electrons.

In the present work we investigate the role of isospin and the presence of the Coulomb field and electrons on the collective nuclear modes. It has been shown [8,10] that at lower densities an isovector-like collective mode exists. The onset density of the mode depends on the isospin asymmetry. At higher densities, two to three times the saturation density, this mode changes to an isoscalar-like mode and the authors of Ref. [10] have even suggested that the experimental observation of the *neutron wave* would identify the transition density. We expect the presence of electrons to affect the properties of these modes, namely the excitations with large wavelengths when protons and electrons must move together. We restrict ourselves to the zero-temperature case.

Collective electron modes known as plasmon modes have already been extensively studied within nonrelativistic [11] and relativistic [12] electron gas models. A comparison between these two models has been investigated in a recent work [13], where the one-photon pair creation (PC) appears as a possible dissipation mechanism in the relativistic case. As already stated, neutrino emission from the core of compact stars depends on the dispersive properties of the matter considered. In Ref. [14] dissipation and dispersion in plasmas was investigated and it was shown that dissipation by PC is not completely suppressed above the PC threshold. If PC is allowed, photons decay much faster into pairs than into neutrinos.

Different relativistic phenomenological models have been used in the description of nuclear and stellar properties. For densities up to nuclear saturation density they predict similar results for the equation of state of nuclear matter and ground-state properties of nuclei. However, differences occur at large densities and/or finite temperatures. They should, therefore, be tested on a larger interval of densities and temperatures to have predictive power.

In what follows we call neutral matter composed of protons, neutrons, and electrons *npe* matter and charged matter composed only of protons and neutrons *np* matter. In Sec. II we present a brief review of the Vlasov equation formalism for nuclear matter, including electrons and the electromagnetic field already discussed in Refs. [9,15]. A brief discussion of the plasmon modes predicted within the present formalism is also included. In Sec. III the numerical results are shown and discussed. Finally, in the last section the most important conclusions are drawn.

II. THE VLASOV EQUATION FORMALISM

We consider a system of baryons, with mass M interacting with and through an isoscalar-scalar field ϕ with mass m_s, an isoscalar-vector field V^μ with mass m_v, and an isovector-vector field $\mathbf{b}^\mu$ with mass m_ρ. We also include a system of electrons with mass m_e. Protons and electrons interact through the electromagnetic field A^μ. The Lagrangian density reads:

$$\begin{aligned}\mathcal{L} = \bar{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_v V^\mu - \frac{g_\rho}{2}\tau\cdot\mathbf{b}^\mu - eA^\mu\frac{1+\tau_3}{2}\right)\right. \\ \left. -(M-g_s\phi)\right]\psi + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) - \frac{1}{3!}\kappa\phi^3 \\ -\frac{1}{4!}\lambda\phi^4 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} \\ +\frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ +\bar{\psi}_e[\gamma_\mu(i\partial^\mu + eA^\mu) - m_e]\psi_e, \qquad (1)\end{aligned}$$

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - g_\rho(\mathbf{b}_\mu\times\mathbf{b}_\nu)$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The model comprises the following parameters: three coupling constants g_s, g_v, and g_ρ of the mesons to the nucleons; the nucleon mass M; the electron mass m_e; the masses of the mesons m_s, m_v, m_ρ; the electromagnetic coupling constant $e = \sqrt{4\pi/137}$; and the self-interacting coupling constants κ and λ. We have used the set of constants identified as NL3 taken from Ref. [16]. For this case, the saturation density that we refer as ρ_0 is 0.148 fm^{-3}.

A. Vlasov equation

The time evolution of the distribution function is described by the Vlasov equation

$$\frac{\partial f_i}{\partial t} + \{f_i, h_i\} = 0, \quad i = p,\, n,\, e, \qquad (2)$$

where $\{,\}$ denotes the Poisson brackets. Equation (2) expresses the conservation of the number of particles in phase space and is, therefore, covariant.

We denote by $f(\mathbf{r},\mathbf{p},t) = \mathrm{diag}(f_p, f_n, f_e)$ the one-body phase-space distribution function in isospin space and by $h_i = \sqrt{(\mathbf{p}-\boldsymbol{\mathcal{V}}_i)^2 + (M-g_s\phi)^2} + \mathcal{V}_{0i}$, with $i = p, n$, $h_e = \sqrt{(\mathbf{p}+e\mathbf{A})^2 + m_e^2} - eA_0$ the one-body Hamiltonian, where $\mathcal{V}_{0i} = g_v V_0 + \frac{g_\rho}{2}\tau_i b_0 + eA_0\frac{1+\tau_i}{2}$, $\boldsymbol{\mathcal{V}}_i = g_v\mathbf{V} + \frac{g_\rho}{2}\tau_i\mathbf{b} + e\mathbf{A}\frac{1+\tau_i}{2}$, with $i = p, n$, $\tau_i = 1$ (protons) or -1 (neutrons).

At zero temperature and for particles obeying Fermi-Dirac statistics, the value of the distribution function is either 1 or 0, because the single-particle state is either occupied by one particle or empty. The state that minimizes the energy of asymmetric nuclear matter is characterized by the Fermi momenta $P_{Fi}, i = p, n$, $P_{Fe} = P_{Fp}$ and is described by the distribution function $f_0(\mathbf{r},\mathbf{p}) = \mathrm{diag}[\Theta(P_{Fp}^2 - p^2),\ \Theta(P_{Fn}^2 - p^2),\ \Theta(P_{Fe}^2 - p^2)]$ and by the constant mesonic fields [defined with a (0) superscript] that obey the following equations $m_s^2\phi^{(0)} + \frac{\kappa}{2}\phi^{(0)^2} + \frac{\lambda}{6}\phi^{(0)^3} = g_s\rho_s^{(0)}$, $m_v^2 V_0^{(0)} = g_v j_0^{(0)}$, $V_i^{(0)} = 0$, $m_\rho^2 b_0^{(0)} = \frac{g_\rho}{2} j_{3,0}^{(0)}$, $b_i^{(0)} = 0$, $A_0^{(0)} = 0$, and $A_i^{(0)} = 0$.

A detailed description of the formalism with all the definitions for the meson fields, densities, etc., is given in Refs. [8,9,15], for instance.

B. Linearized Vlasov equation

Collective modes in the present approach correspond to small oscillations around the equilibrium state, and they are described by the linearized equations of motion [17]. We take for the distribution function $f = f_0 + \delta f$. As in Ref. [17] we introduce a generating function $S(\mathbf{r},\mathbf{p},t) = \mathrm{diag}\left(S_p, S_n, S_e\right)$, defined in isospin space such that the variation of the distribution function is

$$\delta f_i = \{S_i, f_{0i}\} = -\{S_i, p^2\}\delta\left(P_{Fi}^2 - p^2\right). \qquad (3)$$

In terms of this generating function, the linearized Vlasov equations for δf_i are equivalent to the following time evolution equations

$$\frac{\partial S_e}{\partial t} + \{S_e, h_{0e}\} = \delta h_e = -e\left(\delta A_0 - \frac{\mathbf{p}\cdot\delta\mathbf{A}}{\epsilon_{0e}}\right), \qquad (4)$$

$$\begin{aligned}\frac{\partial S_i}{\partial t} + \{S_i, h_{0i}\} = \delta h_i = -g_s\delta\phi\frac{M^*}{\epsilon_0} + \delta\mathcal{V}_{0i} \\ -\frac{\mathbf{p}\cdot\delta\boldsymbol{\mathcal{V}}_i}{\epsilon_0}, \quad i = p, n, \qquad (5)\end{aligned}$$

where $\delta\mathcal{V}_{0i} = g_v\delta V_0 + \tau_i\frac{g_\rho}{2}\delta b_0 + e\frac{1+\tau_i}{2}\delta A_0$ and $\delta\boldsymbol{\mathcal{V}}_i = g_v\delta\mathbf{V} + \tau_i\frac{g_\rho}{2}\delta\mathbf{b} + e\frac{1+\tau_i}{2}\delta\mathbf{A}$, which has to be satisfied only for $p = P_{Fi}$. In Eq. (4) $\epsilon_{0e} = \sqrt{p^2 + m_e^2}$ and in Eq. (5) $h_{0i} = \sqrt{p^2 + M^{*2}} + \mathcal{V}_{0i} = \epsilon_0 + \mathcal{V}_{0i}$. The linearized equations of the fields are obtained using the procedure already presented in Ref. [8]. The longitudinal modes, with momentum $\mathbf{k}$ and frequency ω, are well described by the ansatz $\mathcal{F}_i = \mathcal{F}_{i,\omega}\exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$ for the fields and for the generating functions $S_j(\mathbf{r},\mathbf{p},t) = \mathcal{S}_\omega^j(\cos\theta)\exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$, $j = e, p, n$, where θ is the angle between $\mathbf{p}$ and $\mathbf{k}$. A different choice of the generating function would allow us to study the transverse modes [18]. This, however, will not be carried out in the present work. For the longitudinal modes $\delta V_\omega^x = \delta V_\omega^y = 0$, $\delta b_\omega^x = \delta b_\omega^y = 0$, and $\delta A_\omega^x = \delta A_\omega^y = 0$.

C. Dispersion relation

Equations (4) and (5) are written in terms of the amplitudes $A_{\omega i}$ related to the transition densities by $\delta\rho_i = \frac{3}{2}\frac{k}{P_{Fi}}\rho_{0i}A_{\omega i}$, and they read

$$\begin{pmatrix} 1+F^{pp}L_p & F^{pn}L_p & C_A^{pe}L_p \\ F^{np}L_n & 1+F^{nn}L_n & 0 \\ C_A^{ep}L_e & 0 & 1-C_A^{ee}L_e \end{pmatrix}\begin{pmatrix} A_{\omega p} \\ A_{\omega n} \\ A_{\omega e}\end{pmatrix} = 0, \quad (6)$$

with $A_{\omega i} = \int_{-1}^{1} xS_{\omega i}(x)dx$, $L_i = L(s_i) = 2 - s_i \ln(s_i+1/s_i-1)$, where $s_i = \omega/\omega_{oi} = \omega/(kV_{Fi})$, $V_{F_i} = P_{F_i}/\epsilon_{F_i}$ being the Fermi velocity of particle i, $\epsilon_{Fi} = \sqrt{P_{Fi}^2 + M^{*2}}$, $i = p, n$, $\epsilon_{Fe} = \sqrt{P_{Fe}^2 + m_e{}^2}$, and with $F^{ij} = C_s^{ij} - C_v^{ij} - \tau_i\tau_j C_\rho^{ij} - C_A^{ij}\delta_{ip}\delta_{jp}$. We also have

$$C_s^{ij} = \frac{1}{2\pi^2}\frac{M^{*2}g_s^2}{\omega^2-\omega_s^2}\frac{1}{P_{Fi}}P_{Fj}V_{Fj},$$

$$C_v^{ij} = \frac{1}{2\pi^2}\frac{g_v^2}{\omega^2-\omega_v^2}\left(1-\frac{\omega^2}{k^2}\right)\frac{P_{Fj}^2}{V_{Fi}},$$

$$C_\rho^{ij} = \frac{1}{2\pi^2}\frac{g_\rho^2}{4(\omega^2-\omega_\rho^2)}\left(1-\frac{\omega^2}{k^2}\right)\frac{P_{Fj}^2}{V_{Fi}},$$

$$C_A^{ij} = -\frac{e^2}{2\pi^2}\frac{1}{k^2}\frac{P_{Fj}^2}{V_{Fi}},$$

where $\omega_s^2 = k^2 + m_{s,\mathrm{eff}}^2$, $\omega_v^2 = k^2 + m_v^2$, $\omega_\rho^2 = k^2 + m_\rho^2$, with $m_{s,\mathrm{eff}}^2 = m_s^2 + \kappa\phi_0 + \frac{\lambda}{2}\phi_0^2 + g_s^2 d\rho_s^0$.

From Eq. (6) we get the following dispersion relation

$$\begin{aligned}&[1-C_A^{ee}L_e][1+L_pF^{pp}+L_nF^{nn}\\ &\quad +L_pL_n(F^{pp}F^{nn}-F^{pn}F^{np})]\\ &\quad -C_A^{ep}C_A^{pe}L_eL_p(1+L_nF^{nn}) = 0. \end{aligned} \quad (7)$$

D. Plasmon modes

As stated in the Introduction, the longitudinal response of a relativistic degenerate gas of electrons was first studied by Jancovici [12]. If we discard the nucleon degrees of freedom the dispersion relation reduces to

$$1 - C_A^{ee}L(s_e) = 0. \quad (8)$$

The left-hand side term of Eq. (8) gives the dielectric constant of the electron gas in the limit $k \ll k_{Fe}$ and $\omega \ll \epsilon_{Fe}$, [12], which corresponds to the range of validity of the Vlasov equation. This is the semiclassical approximation that is obtained neglecting the quantum recoil terms [13]. To understand the coupling of the nuclear modes to the plasmon modes given by Eq. (8), we represent the response of the free electron gas in the figures of next section, whenever adequate, with thin lines. There are two modes: a soundlike mode and a zero-sound mode with a frequency

$$\omega_0 = \sqrt{\frac{e^2\rho_e}{\epsilon_{Fe}}}, \quad (9)$$

at zero-momentum transfer.

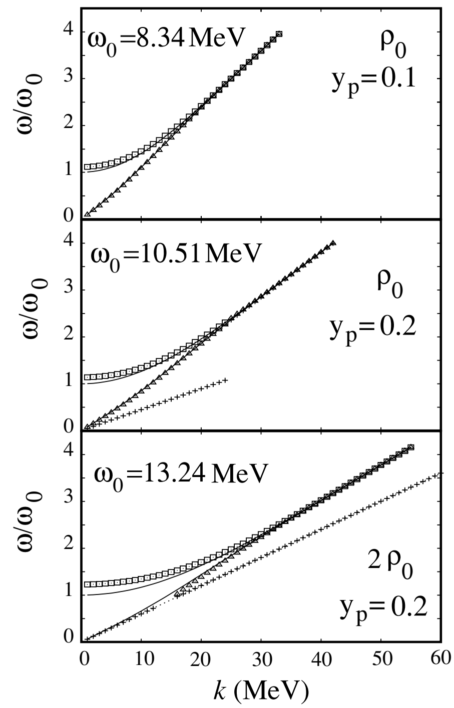

FIG. 1. Collective modes as a function of the momentum transfer k for two different densities ρ_0 and $2\rho_0$ and two proton fractions $y_p = 0.1$ and 0.2. The plasmon frequency at zero momentum ω_0 is defined in Eq. (9).

III. RESULTS AND DISCUSSION

In the present section we discuss the results obtained from Eq. (7). We first discuss the behavior of the energy of the collective modes in terms of the momentum transfer and the proton fraction.

Figure 1 shows, for high isospin asymmetry, the dependence of the energy of the collective modes on the momentum transfer. The plasmonlike modes are represented by squares and triangles and the nuclear mode by crosses. We also include the results for *np* matter (thin dotted line) and for a relativistic gas of free electrons (full thin lines). Above a given $k_{\max}$ the plasmonlike modes do not propagate, which means that Eq. (8) has no solution. The existence of low-lying nuclear modes depends on the density and on the proton fraction: for $\rho = \rho_0$ and $y_p = 0.1$ they do not exist for any value of k; for $\rho = \rho_0$ and $y_p = 0.2$ it propagates until a $k_{\max} = 25$ MeV; for $\rho = 2\rho_0$ and $y_p = 0.2$ the nuclear and plasmon modes couple and they only exist as independent modes above $k = 14$ MeV. Under the last conditions, the plasmonlike mode propagates up to a value of $k_{\max} = 55$ MeV. For large values of k, the nuclear mode propagates in *npe* matter just as in *np* matter.

We have just seen that the appearance of the collective nuclear mode and its possible coupling to the plasmon mode depend both on the density and the proton fraction. We therefore study the collective modes as a function of the proton fraction. In Fig. 2 we plot the energy of the collective modes for $k = 10$ MeV and two different densities: ρ_0 and $2\rho_0$. We identify three modes: the two modes with larger energies

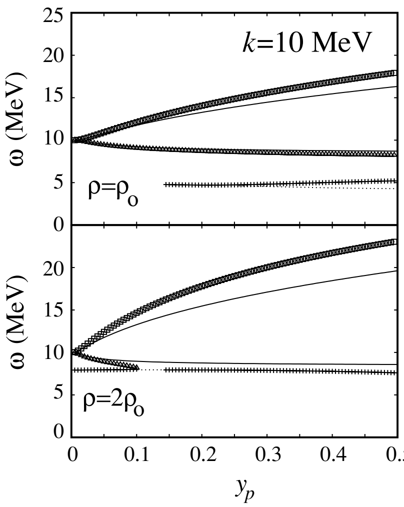

FIG. 2. Collective modes as a function of the proton fraction. The thin full lines are the plasmon energies of a gas of free relativistic electrons and the thin dotted line the nuclear mode of *np* matter.

are plasmonlike modes and the low-lying mode is a nuclear mode that also exists in *np* matter. Figure 2 represents two different situations: for $\rho = \rho_0$ the plasmon modes do not couple to the nuclear mode for any proton fraction, whereas the opposite occurs for $\rho = 2\rho_0$. A first conclusion is that the plasmonlike modes depend on the proton fraction that is equal to the electron fraction, whereas the collective mode is almost independent of this quantity. The energy of the highest mode, corresponding to the zero-sound mode of a free electron gas, increases with the electron fraction. In fact, in *npe* matter this mode increases more with the electron fraction than the prediction from the free electron gas. The energy of the lower plasmon like mode decreases with the electron fraction and, if the energy of the nuclear mode is high enough, the two modes couple.

Having identified the main features of the plasmonlike and nuclear modes, we next investigate the way the nuclear mode

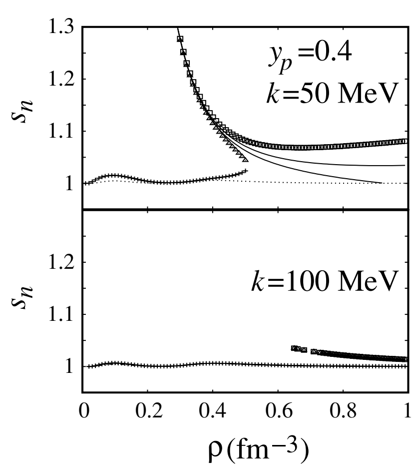

FIG. 3. Sound velocity of the collective modes for $k = 50$ and 100 MeV and proton fraction $y_p = 0.4$. The thin dotted line stands for *np* nuclear matter and the thin full lines are the plasmon modes of a free relativistic electron gas.

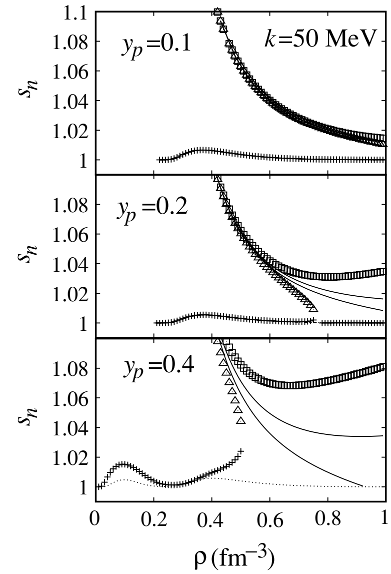

FIG. 4. Sound velocity as a function of density for $k = 50$ MeV and $y_p = 0.1, 0.2, 0.4$. Thin lines as described in the legend to Fig. 3.

is affected by the presence of electrons, in particular we study under which conditions the coupling between the nuclear mode and the plasmon mode is stronger. We discuss both the behavior of the sound velocity of the nuclear mode and of the density fluctuations for protons, neutrons, and electrons in terms of the density and the proton fraction.

In Fig. 3 we represent the sound velocity in units of the neutron Fermi velocity corresponding to the collective modes of *npe* matter as a function of density. We include the results for *np* matter (thin dotted line) and for a relativistic gas of free electrons (full thin lines). As discussed in Ref. [9], the influence of the electrons on the collective modes depends strongly on the momentum transfer. For $k = 100$ MeV the presence

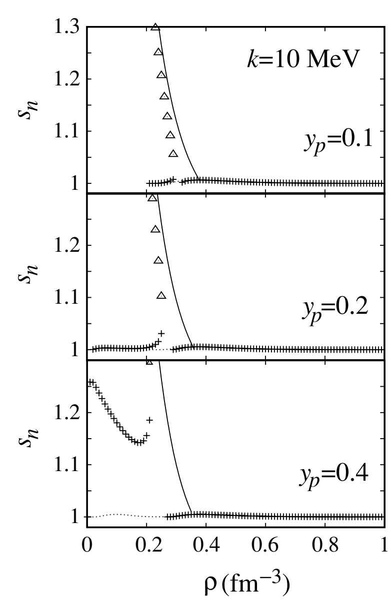

FIG. 5. Sound velocity as a function of density for $k = 10$ MeV and $y_p = 0.1, 0.2, 0.4$. Thin lines as described in the legend to Fig. 3.

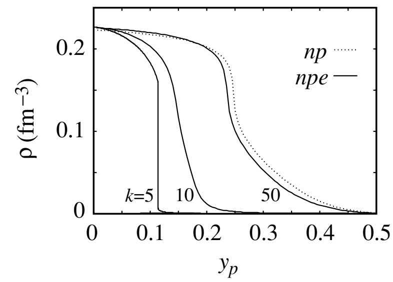

FIG. 6. Density values at which the nuclear *np* (thin dotted line) and the low-lying *npe* (full line) mode start propagating for different values of k given in MeV versus the proton fraction.

of electrons has almost no effect on the nuclear modes, but taking $k = 50\,\text{MeV}$ we have a different situation. The soundlike plasmon mode couples with the nuclear mode at $\rho \sim 0.5\ \text{fm}^{-3}$. Above this density the nuclear modes disappears. At low densities the presence of electrons affects the nuclear mode giving rise to higher sound velocities.

In Fig. 4 we compare the behavior of the nuclear collective modes for $k = 50$ MeV and different proton fractions y_p as a function of the density. We have already discussed the main features for $y_p = 0.4$ in Fig. 3. For the lower values of proton fraction the coupling of the nuclear to the plasmon mode occurs at much larger densities ($0.7\ \text{fm}^{-3}$ for $y_p = 0.2$ and above $1\ \text{fm}^{-3}$ for $y_p = 0.1$). Also the energy of the mode is not affected below the density at which the coupling occurs. Decreasing the proton fraction is equivalent to decreasing the electron fraction and therefore the effect of electrons on the nuclear modes is weaker for lower y_p. However, if a smaller momentum transfer is considered the results also change for $y_p = 0.1$, as seen in Fig. 5. For $y_p = 0.4$ the isovector mode below $0.2\ \text{fm}^{-3}$ has a much higher sound velocity due to the coupling to the plasmon mode. A similar effect occurs for $y_p = 0.2$: the onset of the mode occurs at lower densities than would occur in *np* matter [8]. This effect is not so strong for $y_p = 0.1$ but for still lower momentum transfers it would also be strong for this proton fraction, when the onset of the isovector mode occurs at much lower densities and the mode is

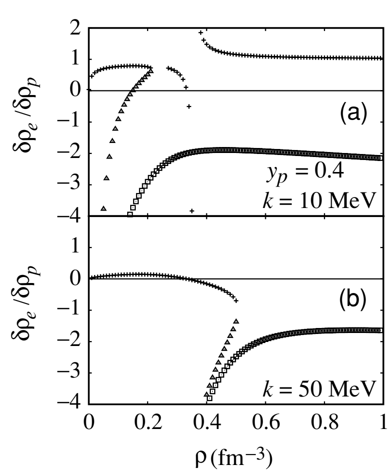

FIG. 7. The ratio of electron to proton density fluctuations as a function of density for $y_p = 0.4$ and (a) $k = 10\,\text{MeV}$; (b) $k = 50\,\text{MeV}$.

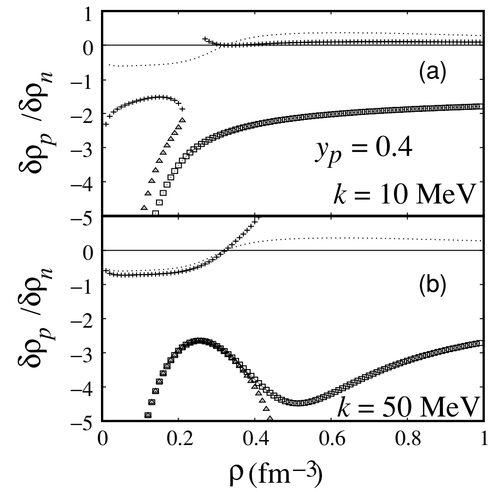

FIG. 8. The ratio of proton to neutron density fluctuations as a function of density for $y_p = 0.4$ and (a) $k = 10\,\text{MeV}$; (b) $k = 50\,\text{MeV}$. The thin dotted line stands for the nuclear mode of *np* matter.

well separated from the boundary $s_n = 1$ below which Landau damping does not allow the mode to propagate.

In Fig. 6 we compare the values of the onset density of the nuclear mode in *np* and *npe* matter, for $k = 5$, 10, and 50 MeV, respectively. In *np* matter, and independently of the momentum transfer, at $y_p \sim 0.25$ there is a sudden drop on the density, from $\sim 0.2\ \text{fm}^{-3}$ to a small value close to zero. In *npe* matter the behavior is similar for $k = 50$ MeV, with a drop on the density for $y_p \sim 0.24$. However, for $k = 5$ and 10 MeV the drop occurs at much lower values of y_p, respectively, $y_p \sim 0.12$ and 0.15, and is much steeper. The smaller the value of k the smaller the value of y_p at which the jump occurs. This is clearly due to the effect of electrons, namely the attractive proton-electron contributions that give rise to the last term in the dispersion relation (7).

In Figs. 7 and 8 we plot, respectively, the ratios of the electron-to-proton and proton-to-neutron density fluctuations in *npe* matter. In Fig. 8 we also include the result obtained for *np* matter. We take the proton fraction 0.4 and two values of k, 10 and 50 MeV. For the larger k value the nuclear mode in *npe* matter at densities below the crossing density, $0.3\ \text{fm}^{-3}$, has properties similar to the ones already found in *np* matter [10,19]: the mode is isovector-like with a slightly larger proton-to-neutron density fluctuation. In Fig. 7 we see that, for this mode, protons and electrons move in phase, but the ratio $\delta\rho_e/\delta\rho_p$ is smaller for larger k values. From Fig. 7 we also confirm the plasmonlike behavior of the higher energy modes (squares and triangles) because both modes present large negative values for the quantity $\delta\rho_e/\delta\rho_p$, i.e., electrons and protons move out of phase. This behavior is disturbed only by the coupling of the plasmon to the nuclear mode. For a small k value, e.g., Figs. 7(a) and 8(a) obtained with $k = 10\,\text{MeV}$, as expected, we observe at smaller densities a stronger coupling of the nuclear mode to the plasmon mode. This gives rise to a strong isovector-like character to the nuclear mode: $|\delta\rho_p/\delta\rho_n|$ is much larger for *npe* matter than for *np* matter. At higher densities the nuclear mode gets a plasmonlike character that can be seen from the large negative values the quantity $\delta\rho_e/\delta\rho_p$ gets. However, the ratio $|\delta\rho_p/\delta\rho_n|$ takes very small positive values: it seems that the proton fluctuations are frozen.

IV. CONCLUSIONS

In the present work, we studied the collective modes of nuclear neutral matter that is of interest for the study of neutron stars and supernovae. In particular, we studied the effect on the collective modes of including the Coulomb interaction and the possibility of the occurrence of a coupling between the nuclear and the plasmon modes. The longitudinal response of a degenerate relativistic electron gas presents two modes: a soundlike mode at lower energies and a zero-sound branch at higher energies. These modes only exist below a cutoff frequency. We have shown that the soundlike branch may couple to the nuclear mode at densities for which this mode has an isovector character if the wave number of the perturbation is low enough. For larger wave numbers/small proton fractions it will couple to the nuclear mode at densities above the isovector-isoscalar crossing density, $\sim 0.3\ \mathrm{fm}^{-3}$. However, for sufficiently small wave numbers and large enough proton fractions the coupling occurs even at small densities. We have also shown that due to this coupling, the onset density of the nuclear isovector mode is particularly sensitive to both the proton fraction and the wave number of the perturbation in *npe* matter. For *np* matter this onset depends only on the proton fraction: it occurs at $\sim 0.2\ \mathrm{fm}^{-3}$ for $y_p < 0.25$ and for larger asymmetries it drops to a small density close to zero. In *npe* matter the drop in density occurs at smaller y_p values for $k < 50$ MeV. This is due to proton-electron interaction. The calculations were performed within a relativistic mean-field approach to nuclear matter, namely the NL3 parametrization of the NLWM but we believe that the main conclusions do not depend on the model. Density-dependent models [20,21], however, may show a slightly different dependence on the momentum transfer.

The present results have implications mainly in astrophysical objects and the related transport properties because we are dealing with neutral matter. In particular, it is known that neutrino interactions are crucial in the dynamics of the core-collapse supernovae because they carry most of the energy away. The properties of the plasmon modes are of importance for the neutrino emission from dense compact matter. In particular, one of the possible mechanisms of the neutrino emissions involves the decay of a plasmon into a neutrino-anti neutrino pair. All the calculations were done at $T = 0$ MeV. The effect of including the Coulomb field at finite temperature is under investigation.

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[1] R. F. Sawyer, Phys. Rev. D **11**, 2740 (1975); N. Iwamoto and C. J. Pethick, Phys. Rev. D **25**, 313 (1982).
[2] C. Providência, D. P. Menezes, and L. Brito, Nucl. Phys. **A703**, 188 (2002); D. P. Menezes and C. Providência, Phys. Rev. C **64**, 044306 (2001); D. P. Menezes and C. Providência, Nucl. Phys. **A650**, 283 (1999); D. P. Menezes and C. Providência, Phys. Rev. C **60**, 024313 (1999).
[3] D. G. Ravenhall, C. J. Pethick, and J. R. Wilson, Phys. Rev. Lett. **50**, 2066 (1983); M. Hashimoto, H. Seki, and M. Yamada, Prog. Theor. Phys. **71**, 320 (1984).
[4] C. J. Horowitz, M. A. Perez-Garcia, and J. Piekarewicz, Phys. Rev. C **69**, 045804 (2004).
[5] G. Watanabe, K. Sato, K. Yasuoka, and T. Ebisuzaki, Phys. Rev. C **69**, 055805 (2004); G. Watanabe, T. Maruyama, K. Sato, K. Yasuoka, and T. Ebisuzaki, Phys. Rev. Lett. **94**, 031101 (2005).
[6] T. Maruyama, T. Tatsumi, D. N. Voskresensky, T. Tanigawa, and S. Chiba, Phys. Rev. C **72**, 015802 (2005).
[7] C. J. Horowitz, M. A. Perez-Garcia, D. K. Berry, and J. Piekarewicz, Phys. Rev. C **72**, 035801 (2005); C. J. Horowitz, M. A. Perez-Garcia, J. Carriere, D. K. Berry, and J. Piekarewicz, Phys. Rev. C **70**, 065806 (2004).
[8] S. Avancini, L. Brito, D. P. Menezes, and C. Providência, Phys. Rev. C **71**, 044323 (2005).
[9] C. Providência, L. Brito, S. S. Avancini, D. P. Menezes, and Ph. Chomaz, Phys. Rev. C **73**, 025805 (2006).
[10] V. Greco, M. Colonna, M. Di Toro, and F. Matera, Phys. Rev. C **67**, 015203 (2003).
[11] D. J. Lindhard, Mat. Fys. Medd. Dan. Vid. Selsk. **28**, 1 (1954).
[12] B. Jancovici, Nuovo Cimento **25**, 428 (1962).
[13] J. McOrist, D. B. Melrose, and J. I. Weise, J. Plasma Phys., arXiv:physics/0603227.
[14] D. B. Melrose, J. I. Weise, and J. McOrist, J. Phys. A **39**, 8727 (2006).
[15] L. Brito, C. Providência, A. M. Santos, S. S. Avancini, D. P. Menezes, and Ph. Chomaz, Phys. Rev. C **74**, 045801 (2006).
[16] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997).
[17] M. Nielsen, C. Providência, and J. da Providência, Phys. Rev. C **44**, 209 (1991); M. Nielsen, C. Providência, and J. da Providência, *ibid*. **47**, 200 (1993).
[18] M. Nielsen, C. da Providência, J. da Providência, and Wang-Ru Lin, Mod. Phys. Lett. A **10**, 919 (1994).
[19] S. S. Avancini, L. Brito, D. P. Menezes, and C. Providência, Phys. Rev. C **70**, 015203 (2004).
[20] S. Typel and H. H. Wolter, Nucl. Phys. **A656**, 331 (1999).
[21] T. Gaitanos, M. Di Toro, S. Typel, V. Baran, C. Fuchs, V. Greco, and H. H. Wolter, Nucl. Phys. **A732**, 24 (2004); G. Hua, L. Bo, and M. Di Toro, Phys. Rev. C **62**, 035203 (2000).