Multiplicity associated to high p_T events and multiplicity fluctuations

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It is shown that the difference between the multiplicity associated to high p_T events and the unbiased multiplicity is given by the normalized variance of the multiplicity distribution, as a consequence of high p_T events being self-shadowed. We discuss the possibility of checking the nonmonotonic behavior with centrality of the normalized variance by measuring the difference between multiplicities.

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In the last few years very interesting phenomena related to high p_T physics have been observed at RHIC experiments [1,4], namely, a strong supression of inclusive high p_T hadron production in Au-Au central collisions compared to the scaling with the number of binary nucleon-nucleon collisions. The data [6] also show the disappearence of back-to-back jet-like hadron correlations in Au-Au collisions, contrary to what is observed in d-Au and p-p collisions, and a nonmonotonic behavior of the fluctuations in transverse momentum and multiplicity [6–9] with a maximum around low centralities. In order to explore further the physical phenomena involved [10], different correlations related to high p_T events are being studied.

High p_T events are self-shadowed [11]. In this paper we show that due to this property the difference between the multiplicity associated to high p_T events and the total multiplicity is directly related to the variance of the multiplicity distribution. In this way, the observed dependence on centrality of the normalized variance can be translated into the difference between multiplicities. In particular, the supression of the normalized variance at large centrality will correspond to a decrease in the difference between multiplicities with increasing centrality.

The behavior of the normalized variance has been explained in the string percolation approach as a consequence of the dependence on centrality of the number of clusters with different number of strings. At low centrality there is no overlapping of strings, all the clusters have only one string and the fluctuations arise only from the multiplicity ditribution of one string. As the degree of centrality increases, clusters of different number of strings are formed giving rise to different multiplicity distributions due to the different color of the clusters and therefore different cluster tensions. Now, there are additional fluctuations coming from the different distributions. Above the percolation threshold, essentially only one large cluster is formed and again the fluctuations are suppressed. In this approach, the formation of such a large cluster implies that both multiplicities will be equal, independently of the hardness of the event.

We start our discussion by describing hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions, which we label HH, as a superposition of independent elementary interactions. We are specifically interested in such collisions involving an elementary interaction or process of type C. We shall say that a particular HH collision or event is of type C if at least one of the elementary collisions involved in that event is also of type C. It is easy to show that C events are self-shadowed. Indeed, in hadron-nucleus collisions, the inelastic cross section can be written as

$$\sigma^{hA}(b) = \sum_{n=1}^{A} \binom{A}{n} (\sigma T(b))^n (1 - \sigma T(b))^{A-n}, \qquad (1)$$

where we can write

$$(\sigma T(b))^n = \sum_{i=0}^{n} \binom{n}{i} (\sigma_C)^i (\sigma_{NC})^{n-i} T(b)^n, \qquad (2)$$

σ_C and σ_{NC} being the elementary nucleon-nucleon cross sections for events of type C and for the rest of events, respectively. The final cross section for events of type C must contain at least one elementary σ_C in the sum, therefore [11]

$$\sigma_C^{hA}(b) = \sum_{n=1}^{A} \binom{A}{n} \sum_{i=1}^{n} \binom{n}{i} \sigma_C^i \sigma_{NC}^{n-i} T(b)^n \times (1 - (\sigma_C + \sigma_{NC}) T(b))^{A-n} = 1 - (1 - \sigma_C T(b))^A. \qquad (3)$$

Equation (3) shows that C-events are self-shadowed, in the sense that their cross section in HH depends only on their cross section in nucleon-nucleon collision. Similar considerations can be done for hadron-hadron and nucleus-nucleus collisions [12]. There are many different self-shadowed events, for instance nondiffractive, annihilation, nonisolated fast baryons or high p_T events. In all of them, depending on wether σ_C is small or large, σ_C^{hA} behaves like A or $A^{\frac{2}{3}}$, respectively.

If we denote by α_C the probability for an elementary collision to be of type C, one can trivially write

$$N(\nu) = \sum_{i=0}^{\nu} \binom{\nu}{i} (1 - \alpha_C)^{\nu - i} \alpha_C^i N(\nu), \qquad (4)$$

and

$$N_C(\nu) = \sum_{i=1}^{\nu} \binom{\nu}{i} (1-\alpha_C)^{\nu-i} \alpha_C^i N(\nu) \simeq \nu\alpha_C N(\nu), \quad (5)$$

$$N_{NC}(\nu) = (1-\alpha_C)^{\nu} N(\nu) \simeq (1-\nu\alpha_C) N(\nu), \quad (6)$$

where $N(\nu)$, $N_C(\nu)$, and $N_{NC}(\nu)$ stand for the total number of events, the total number of events of type C, and the total number of non-C events produced with ν collisions, respectively. The last equalities of Eqs. (5) and (6) hold in the limit of small α_C. Since

$$\sum_{\nu} N(\nu) = N, \quad (7)$$

$$\sum_{\nu} \nu N(\nu) = \langle\nu\rangle N \quad (8)$$

and

$$\sum_{\nu} N_C(\nu) = \sum_{\nu} \alpha_C \nu N(\nu) = N_C, \quad (9)$$

with N the total number of events and N_C the total number of events of type C, we then have that the probability distribution for C events with ν collisions is

$$P_C(\nu) = \frac{\alpha_C \nu N(\nu)}{\sum_{\nu} N_C(\nu)} = \frac{\nu N(\nu)}{\langle\nu\rangle \sum_{\nu} N(\nu)} = \frac{\nu P(\nu)}{\langle\nu\rangle}. \quad (10)$$

In Eqs. (7)–(9), the sums are over the index ν, the number of collisions, whose range is from 1 to 1, A and AB for hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions, respectively. However, instead of the number of collisions, another elementary blocks, such as number of Pomerons, number of strings or number of parton-parton interactions could be used as far as the total amplitude of the whole process can be expressed as a superposition of the elementary amplitudes of the mentioned blocks. For instance, the hadron-hadron cross section is usually expressed in terms of a sum over elementary cross sections corresponding to Pomeron exchanges. In this case, the hadron-hadron cross section corresponding to events of type C is related to the elementary cross section of type C obtained from cutting a Pomeron.

The total multiplicity $P(n)$ is obtained by the convolution of the elementary multiplicity distributions $p(n)$,

$$P(n) = \sum_{\nu} \sum_{n_1+\ldots+n_\nu=n} P(\nu) p(n_1)\ldots p(n_\nu). \quad (11)$$

If we denote by $G(z)$ and $g(z)$ the generating functions of $P(n)$ and $p(n)$, respectively,

$$G(z) = \sum_{n} z^n P(n), \qquad g(z) = \sum_{n} z^n p(n), \quad (12)$$

we have

$$G(z) = \sum_{\nu} P(\nu) g(z)^{\nu}. \quad (13)$$

Doing the first two derivatives of Eq. (13) we relate the total dispersion, D, $D^2 = \langle n^2\rangle - \langle n\rangle^2$, to the dispersion d and multiplicity $\overline{n}$, of the distribution of the elementary interaction,

$$\frac{D^2}{\langle n\rangle^2} = \frac{\langle\nu^2\rangle - \langle\nu\rangle^2}{\langle\nu\rangle^2} + \frac{d^2}{\langle\nu\rangle\overline{n}^2}, \quad \langle n\rangle = \langle\nu\rangle\overline{n}. \quad (14)$$

Since ν is very high in nucleus-nucleus collisions, the second term of Eq. (14) can be neglected, and the normalized dispersion of the total multiplicity is approximated by the normalized dispersion of the number of elementary interactions. This provides an argument to extend Eq. (10) to the multiplicity distribution [13,14]:

$$P_C(n) \simeq \frac{nP(n)}{\langle n\rangle}. \quad (15)$$

Notice that the right hand side of Eq. (15) is independent of C. Equation (15) has been checked in high energy pp collisions for the multiplicity associated to $W^{\pm}$ and Z^0 production, and also for the multiplicity distribution associated to jet production and annihilation [14]. In nucleus-nucleus collisions, data of ISR experiments on events with $p_T \geqslant 3\,\mathrm{GeV}/c$ produced in α-α collisions also satisfy Eq. (15) [15].

From Eq. (15) we have

$$\langle n\rangle_C - \langle n\rangle = \frac{D^2}{\langle n\rangle}. \quad (16)$$

Equation (16) is our main result: the difference between the average multiplicity associated to high p_T events and the unbiased average multiplicity is given by the normalized variance of the unbiased distribution.

Equations (10), (14), (15), and (16) have been obtained assuming independent superposition of elementary interactions. This assumption is not justified at RHIC energies where collective interactions are at work. However, we think that Eqs. (10), (14), (15), and (16) are valid even in this case. In fact, the experimental data on the multiplicity distributions on nucleus-nucleus are well described by the negative binomial distribution

$$P(n,k) = \frac{\gamma^k}{\Gamma(k) n!} \frac{\Gamma(n+k)}{(1+\gamma)^{n+k}}, \quad (17)$$

where k is given by

$$\frac{1}{k} \equiv \frac{D^2}{\langle n\rangle^2} - \frac{1}{\langle n\rangle} \quad (18)$$

and

$$\gamma = \frac{k}{\langle n\rangle}. \quad (19)$$

$P(n,k)$ can be rewritten in the form

$$P(n,k) = \int_0^{\infty} dN\, W(N) P(N,n), \quad (20)$$

where $P(N,n)$ is the Poisson distribution

$$P(N,n) = \exp(-N)\frac{N^n}{n!} \quad (21)$$

and $W(N)$ is the Gamma function

$$W(N) = \frac{\gamma}{\Gamma(k)} (\gamma N)^{k-1} \exp(-\gamma N), \quad (22)$$

with $\frac{1}{k}$ given by

$$\frac{1}{k} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2}. \tag{23}$$

In the framework of percolation of strings [16], Eq. (20) represents the superposition of clusters of different number of strings which decay according to a Poisson multiplicity distribution of mean value N, and this average value of the multiplicity of each cluster is related to the number of strings of each cluster [17,18]. In this approach $W(N)$ plays the same role as $P(\nu)$, the probability of ν elementary collisions, and therefore we expect that for events of type C

$$W_C(N) = \frac{N}{\langle N \rangle} W(N) \tag{24}$$

and

$$\begin{aligned} P_C(n,k) &= \int_0^\infty dN W_C(N) P(N,n) \\ &= \int_0^\infty dN \frac{N}{\langle N \rangle} W(N) P(N,n) = \frac{n+k}{\langle n \rangle + k} P(n,k). \end{aligned} \tag{25}$$

In the limit of high $\langle n \rangle$ we recover Eq. (15). We emphasize that Eq. (25) is obtained assuming that Eq. (10), valid for the distribution of the number of collisions $P(\nu)$, can be extended to $W(N)$, the cluster size distribution. RHIC experimental data can test directly Eq. (15), checking whether any deviation occurs. Equation (16), taking into account that $D \simeq \langle n \rangle$ for the minimum bias distribution [19], predicts $\langle n \rangle_C \simeq 2\langle n \rangle$ which can be easily experimentally checked.

We can go further exploring the centrality dependence of Eq. (16). NA49 Collaboration [7] has measured $\frac{D^2}{\langle n \rangle}$ as a function of the number of participants for Pb-Pb collisions in the rapidity interval $4 < y < 5.5$ and in the p_T range $0.0005 < p_T < 1.5\,\text{GeV}/c$. The data show that $\frac{D^2}{\langle n \rangle}$ decreases from a value of 1.8 at $N_{\text{part}} \simeq 50$ down to 1 for the highest number of participants. The values for $\overline{p}p$ are around one. However differences between multiplicities of order 1 or 2 are impossible to measure. We must require at least differences of the order of 20–30 charged particles. Thus, instead of $\frac{D^2}{\langle n \rangle}$ at a fixed number of participants we must look at broader bins of the number of participants in such a way that the multiplicity distribution becomes wider and thus D becomes more similar to $\langle n \rangle$, what would make the difference between multiplicities larger.

For instance we can look at eight equal spacing intervals of N_{part}, such as 0–50, 50–100, 100–150, 150–200, 200–250, 250–300, 300–350 and 350–400, measuring $\frac{D^2}{\langle n \rangle}$ for each interval. Equation (16) can then be checked by measuring in every interval the difference between the average multiplicity of those selected events with at least one particle with transverse momentum larger than a fixed value, say $p_T = 2\,\text{GeV}/c$, and the average multiplicity of all events.

To be more explicit, for the above mentioned intervals, at SPS energies and in the rapidity range $-0.75 < y < 0.75$, the mean multiplicity of charged particles is approximately [5]: 20, 65, 112, 164, 215, 268, 323, 380. On the other hand, the corresponding values of $\frac{D^2}{\langle n \rangle}$ for charged particles at the same energy are approximately: 101, 160, 142, 110, 88, 76, 71, 70 [7]. (We have used a rapidity gap of 1.5 to coincide with the rapidity interval where $\frac{D^2}{\langle n \rangle}$ was measured.) Therefore we predict that the multiplicity of events with at least one particle with $p_t > 2\,\text{GeV}/c$ for the corresponding centrality bins would be: 121, 225, 254, 274, 303, 344, 394, 450.

In the framework of percolation of strings, as we said before, we expect a nonmonotonic behavior of $\frac{D^2}{\langle n \rangle}$ with centrality, increasing at low centrality up to $N_{\text{part}} \simeq 50$–100 and decreasing at high centrality. The quantitative prediction will depend on the range of rapidity, transverse momentum and centrality bins used.

Equation (16) will test two aspects of the collisions: the self-shadowing nature of high p_T events and the dependence of the multiplicity on the clustering of particle sources.

In conclusion, we have shown that the difference between the multiplicity associated to high p_T events and the unbiased multiplicity is given by the normalized variance of the multiplicity distribution. We predict a nonmonotonic behavior with centrality of this difference which can be checked experimentally.

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