# Fluctuation analysis and the extraction of the collisional damping width of the giant dipole resonance in the system <sup>28</sup>Si+<sup>58</sup>Ni at E(<sup>28</sup>Si) = 100 and 125 MeV

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Simplistic shape and orientation fluctuation calculations are compared with the data for the system <sup>28</sup>Si+<sup>58</sup>Ni at  $E(^{28}Si)=100$  and 125 MeV. The collisional damping width ( $\Gamma_0$ ) is extracted under the shape fluctuation model analysis. A constant value of  $\Gamma_0$  cannot reproduce the measured  $\gamma$ -spectra. The extracted value of  $\Gamma_0$  is found to be independent of the average angular momentum  $\langle J \rangle$ , while showing a mild temperature dependence above an average temperature,  $\langle T \rangle \sim 1.3$  MeV. Below, a very small value of  $\Gamma_0 \sim 3.8$  MeV is found.

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### I. INTRODUCTION

The giant dipole resonance (GDR) is one of the fundamental modes of excitation. Its strength function is characterised by resonance energy  $(E_D)$  and width  $(\Gamma_D)$  [1,2]. One of the important issues in the GDR study is the separation of the contributions of widths from different damping mechanisms so that their dependence on temperature and angular momentum can be studied. The spreading of the width of the giant dipole resonance is mainly due to two mechanisms. The first is the collisional damping [3]. The mixing of the correlated one particle one hole states, which constitute the giant dipole resonance, with more complicated states lying at the same excitation energy is termed as the collisional damping. It arises due to coupling of giant vibrational modes to small amplitude quantal fluctuations of the nuclear surface. The collisional damping model [4,5] has been widely used in describing the GDR width at zero temperature (T). The other mechanism is the coupling of GDR vibration to large amplitude fluctuations (shape fluctuations) of nuclear surface that are induced by temperature. At finite temperature and angular momentum the nucleus can be viewed as an ensemble of shapes with a distribution governed by the Boltzmann factor [6-10]. An averaging of GDR vibrations over distribution of shapes is necessary to get the information of the GDR width.

Earlier we have reported angular momentum gated GDR measurements in the system  ${}^{28}\text{Si}+{}^{58}\text{Ni}$  at  $E({}^{28}\text{Si}) = 100$  and 125 MeV [11]. Experimental details and data analysis can be found in Refs. [11,12]. In the present paper, orientation and shape fluctuation analysis is presented. We also report the extraction of the collisional damping width ( $\Gamma_0$ ), under the shape fluctuation analysis.

## **II. ORIENTION FLUCTUATION ANALYSIS**

The GDR strength function is extracted from the measurement of the angle integrated  $\gamma$ -spectrum, which under

the shape fluctuation model, is related to the nuclear shape evolution and its fluctuations. The angular distribution of  $\gamma$ -rays emitted from compound nuclei, relative to the beam axis is given by

$$Y(\theta, E_{\gamma}) = Y_0(E_{\gamma})[1 + a_2(E_{\gamma})P_2(\cos\theta)],$$
(1)

where  $a_2$  is the anisotropy. The  $\gamma$ -ray energy dependence of the angular anisotropy should, in principle, differentiate between prolate and oblate shapes, rotating collectively or noncollectively. However, the angular anisotropy gets diluted due to the orientation fluctuation. For a given orientation ( $\theta$ ,  $\phi$ ) of the spin axis in the the body-fixed frame, the angular anisotropy coefficient  $a_2$  for the GDR  $\gamma$ -rays [13,14] is given by

$$a_{2} = \frac{1}{4} [F_{x}(3\sin^{2}\theta\cos^{2}\phi - 1) + F_{y}(3\sin^{2}\theta\sin^{2}\phi - 1) + F_{z}(3\cos^{2}\theta - 1)], \qquad (2)$$

where  $F_x$ ,  $F_y$ ,  $F_z$  are the relative probabilities for the  $\gamma$ -rays to originate from vibrations along the *x*, *y*, and *z* axis, respectively. The probability of a certain orientation is given by  $P(\theta, \phi) \sim \exp(-E_{\text{rot}}/kT)$ . The rotational energy  $(E_{\text{rot}})$  depends on the angular momentum *J* and the moment of inertia, which in turn depends on the angles describing the spin axis. Integrating over these probabilities, the average  $a_2$  and hence the ratio of  $\gamma$ -yields at various angles can be obtained.

The spectra at  $125^{\circ}$  are representatives of the angle integrated measurements since  $a_2 P_2(\cos \theta) = 0$  for  $\theta = 125^\circ$ . Figure 1 shows the ratio of the Doppler corrected  $\gamma$ -spectra at 100 and 125 MeV for various angles. The 100 MeV spectra are normalized (up to 10%) to make the ratio about 1.0 at  $E_{\gamma}$  = 5 MeV. The  $\gamma$ -ray fold gates used for 100 MeV and 125 MeV are 6–14 and 7–14, respectively [11]. The average temperature  $\langle T \rangle$  and average angular momentum  $\langle J \rangle$  corresponding to these folds for  $E_{Si} = 100 \text{ MeV}$  are 1.12 MeV and 22 $\hbar$ , while for  $E_{Si} = 125$  MeV, those are 1.29 MeV and 30 $\hbar$ . At these high folds (high angular momentum) the attenuation in anisotropy should be weak, or in other words enhanced. The observed isotropy at 100 MeV beam energy could be due to a spherical shape. However, the extracted strength functions from the data has been described due to prolate shape. The solid lines in Fig. 1 show the results of the orientation fluctuation calculation for a prolate nucleus with the deformation  $\beta = 0.3$ . This is so

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FIG. 1. Angular distribution of high energy  $\gamma$ -rays at  $E_{\rm Si} = 100$  MeV and  $E_{\rm Si} = 125$  MeV. Solid line shows the orientation fluctuation calculations for a prolate nucleus with the deformation  $\beta = 0.3$ .

chosen because the values of  $\beta$  extracted from the GDR data for various *T* and *J*, lies in the range of ~0.2 to 0.4 [11]. The present data at 100 MeV are thus consistent with the orientation fluctuation calculation. There is a discrepancy between the forward angle data and the calculation at 125 MeV. One of the reasons for this discrepancy can be due to the contribution of preequilibrium  $\gamma$ -rays at 125 MeV. If we add the odd order Legendre polynomial, i.e.,  $a_1 E_{\gamma} P_1(\cos \theta)$  then the spectra at  $E_{\text{Si}} = 125$  MeV can be explained. At  $E_{\gamma} = 7$  MeV, where there is a maximum deviation, the required value of  $a_1$  is ~0.1, and the change in Y(125°) is found to be ~5%. At higher energies this change will be even less. Hence, it is not expected to affect the extracted GDR parameters.

#### **III. SHAPE FLUCTUATION ANALYSIS**

The fluctuations in the nuclear shape, characterized by the shape parameters  $\beta$  and  $\gamma$ , decide the angle integrated  $\gamma$ -spectrum. In a simplistic approach, the shape fluctuations are described by Gaussian distributions in  $\beta$  and  $\gamma$ . The energy dependent average  $\gamma$ -ray strength function is calculated by integrating the strength function for a given  $\beta$ ,  $\gamma$  over these Gaussian distributions with a Monte Carlo procedure. The averaging is done using a volume element  $\beta^4 \sin 3\gamma d\beta d\gamma$ . For a given set of  $(\beta, \gamma)$ , the GDR energies of different components are given by [2]

$$E_k = E_0 \exp\left[-\sqrt{\frac{5}{4\pi}}\beta \cos\left(\gamma - \frac{2\pi k}{3}\right)\right],\qquad(3)$$

where k = 1, 2, 3 correspond to the three principle axes in the intrinsic frame and  $E_0$  is the intrinsic energy of the dipole vibration. The component widths are related [9] as

$$\frac{\Gamma_k}{\Gamma_0} = \left(\frac{E_k}{E_0}\right)^{\delta},\tag{4}$$

where  $\Gamma_0$  is the intrinsic width.

TABLE I. GDR width and effective deformation parameter  $\beta$  from the Monte Carlo procedure. Also given is the first variance  $\sigma_{\beta}$  in  $\beta$ .

$\Gamma_D(MeV)$	β	$\sigma_{eta}$
$5.75 \pm 0.50$	0.10	0.03
$7.30 \pm 0.20$	0.20	0.07
$8.00\pm0.30$	0.24	0.04
$8.20\pm0.10$	0.24	0.04
$8.30\pm0.10$	0.25	0.07
$9.70\pm0.50$	0.33	0.05
$9.35 \pm 0.05$	0.30	0.10
$9.65 \pm 0.35$	0.32	0.08
$11.1 \pm 0.90$	0.39	0.09
$11.6\pm0.20$	0.40	0.13

The parameters  $E_0$ ,  $\Gamma_0$  and  $\delta$  are supposed to be constants for a given nucleus. The values of  $E_0$  and  $\Gamma_0$  are taken from the the ground state data [15] as 17.5 MeV and 5 MeV, respectively. The value of  $\delta$  is fixed at 1.5. While the mean  $(\beta_m)$  and the FWHM  $(\Delta\beta)$  for the distribution in  $\beta$  are varied in steps of 0.1, the mean  $(\gamma_m)$  and the FWHM  $(\Delta \gamma)$  for the distribution in  $\gamma$  are varied in steps of 10°. In order to extract the width from the average GDR strength function, the following procedure is adopted. A three component Lorentzian strength distribution, characterised by energy (E1, E2, E3) and width parameters ( $\Gamma$ 1,  $\Gamma$ 2,  $\Gamma$ 3) are generated and then compared with the average GDR strength function from Monte Carlo procedure. The width for the average GDR strength function is defined as the FWHM of the three component strength function. The effective quadrupole deformation  $\beta$  is obtained from the averaging of  $\beta$  over many trials (typically  $10 \times 10^4$ ). The sets of GDR width and effective  $\beta$  from the Monte Carlo procedure for  $10 \times 10^4$  trials is shown in Table I. For a given set of  $\beta_m$ ,  $\Delta\beta$ ,  $\gamma_m$  and  $\Delta\gamma$ , effective  $\beta$  has a distribution. The first variance  $\sigma_{\beta}$  is given in Table I. Also different sets of  $\beta_m$ ,  $\Delta\beta$ ,  $\gamma_m$  and  $\Delta\gamma$  corresponding to same effective  $\beta$  gives slightly different  $\Gamma$  values, which are included by defining errors in  $\Gamma$ . It should be noted that in this analysis, there are many solutions for  $\gamma$  ranging from 0° to 60° for a given  $\beta$ . Figure 2 shows the plot of effective  $\beta$  as a function of  $\Gamma$  from the shape fluctuation analysis. A functional relation between effective  $\beta$  and  $\Gamma$ 

$$\beta = m\Gamma + b, \tag{5}$$



FIG. 2. Effective  $\beta$  as a function of  $\Gamma$ .



FIG. 3.  $\beta$  as a function of  $\langle J \rangle$  and  $\langle T \rangle$ . Solid line is a guideline for the  $\beta$  values from Eq. (5).

with  $m \sim 0.05$  and  $b \sim -0.02$  is obtained. As discussed in Ref. [11], the data are presented as a function of  $\langle T \rangle$  and  $\langle J \rangle$  over the various  $\gamma$ -decay steps. The GDR widths from these data corresponding to various  $\langle T \rangle$  and  $\langle J \rangle$  are used in this equation to obtain the values of  $\beta$  from shape fluctuation analysis.

Figure 3 shows the extracted deformation parameter  $\beta$  [11] as a function of  $\langle J \rangle$  and  $\langle T \rangle$ . Also shown is effective  $\beta$  from the shape fluctuation calculations. Figure 3(b) shows that the shape fluctuations increase with the increase in the temperature. The data reasonably agrees with the fluctuation analysis except for the low values of *T*.

The above mentioned procedure is a very simplistic approach towards the shape fluctuation analysis. A more detailed procedure would be to use the average  $\gamma$ -ray strength function (calculated by the above mentioned Monte Carlo procedure) in the CASCADE [16] calculations, then calculate the  $\gamma$ -spectrum and fold it with the detector response function to compare with the measured  $\gamma$ -spectra (discussed later). An even more advanced analysis would be to calculate the potential energy surfaces for the nuclei at  $A \sim 85$  at the relevant T, J, to obtain the strength function and use that in the statistical model code CASCADE.

A systematic analysis of the GDR width as a function of T, J, and A has been done by Kusnezov *et al.* [17]. The experimental results are compared with the theoretical calculations in nuclei ranging from  $A \sim 45$  to 208. The calculations include thermal shape fluctuations using both the Nilsson-Strutinsky and the liquid drop free energy surfaces. The resultant phenomenological formula to describe the global dependence of the GDR width on T, J and A is parametrized as

$$\Gamma(T, J, A) = \Gamma(T, J = 0, A) \left[ L\left(\frac{J}{A^{5/6}}\right) \right]^{4/[(T/T_0)+3]},$$
(6)

$$\Gamma(T, J = 0, A) = \Gamma_0(A) + c(A)\ln(1 + T/T_0), \quad (7)$$

where  $L(\xi) = 1 + 1.8[1 + \exp(1.3 - \xi)/0.2]^{-1}$  and c(A) = 6.45 - A/100. The value of  $\Gamma_0$  is taken as the measured ground state GDR width and the reference temperature ( $T_0$ ) is assumed to be 1 MeV. The experimentally extracted data from Ref. [11] is compared with the above phenomenological formula [Eq. (6)] in Fig. 4. The value of  $\Gamma_0$  used is 3.8 MeV.





FIG. 4. Comparison of experimental widths with the phenomenological width [Eqs. (6) and (7)]. (a) shows the ratio of experimental width to theoretical scaled  $\Gamma(T, J = 0, A)$  vs  $\xi = J/A^{5/6}$ . The solid line is the scaling function. (b) shows the *T* dependence of the GDR width. Solid line is a guideline.

As can be seen from Figs. 4(a) and 4(b), the formula describes well the behavior of width as a function of J, while it fails to explain the T dependence. In Ref. [18] the experimental widths are plotted against the compound nucleus spin, while in this paper the experimental widths are plotted against the average spin. It has also been shown in Ref. [18] that the phenomenological model is not applicable for all masses. It would be interesting to carry out measurements at higher J in this mass region, to verify the phenomenological formula.

Kusnezov and Ormand [19] have proposed a simple functional relation between the average deformation parameter  $\beta$  and the experimental width  $\Gamma$  based on liquid-drop parametrization as

$$\beta = a' \frac{\Gamma(J, T, A) - \Gamma_0}{E_0} + c', \tag{8}$$

where  $E_0$  and  $\Gamma_0$  are the ground state energy and width. The values of a' = 0.8 and c' = 0.12 are obtained from Ref. [17]. The nuclei around  $A \sim 85$ , have ground state widths ranging from 3.5 to 5.5 MeV [15]. The values of  $E_0$  and  $\Gamma_0$  used are 17 MeV and 3.8 MeV. The  $\Gamma(J, T, A)$  values are taken from Ref. [11]. The comparison of the  $\beta$  values from the Ref. [11] and those from Eq. (8) is shown in Fig. 5. The values of



FIG. 5.  $\beta$  as a function of  $\langle J \rangle$  and  $\langle T \rangle$ . The solid line is a guideline for the values of  $\beta$ , calculated from Eq. (8).



FIG. 6. Divided plots of the measured  $\gamma$ -spectra in the system <sup>28</sup>Si+<sup>58</sup>Ni. The fits are from the shape fluctuation calculations with the best choices for the relevant parameters.

 $\beta$  predicted from the above relation reosonably agrees with the present results.

## IV. EXTRACTION OF THE COLLISIONAL DAMPING WIDTH

As discussed earlier, the calculated average strength function is used in the statistical model code to extract the collisional damping width. The prescription for the nuclear level density used in this work is that of Ref. [20] with the asymptotic liquid drop value for the NLD fixed at  $A/7.5 \text{ MeV}^{-1}$  [21]. A sum rule strength of 100% is used in all the calculations. The calculated  $\gamma$ -spectrum is folded with the detector response function to compare with the experimental  $\gamma$ -spectra in a chi-square minimisation procedure. The values of  $E_0$  and  $\Gamma_0$  are varied over the ranges of 16 to 18.5 MeV and 3 to 9 MeV, respectively, in steps of 0.2 MeV. The value of  $\delta$  is fixed at 1.8. The values of  $\beta_m$  and  $\Delta\beta$  are varied in steps of 0.1 and  $\gamma_m$  and  $\Delta \gamma$  in steps of 10°, until the best fit to the data is obtained for each fold window. Figure 6 shows the divided plots of measured  $\gamma$ -spectra. The continuous lines are shape fluctuation calculations with best choices for the relevant parameters.

Figure 7 shows the value of  $\Gamma_0$  derived from the shape fluctuation analysis. The fold gated  $\gamma$ -spectra at 100 MeV cannot be explained with a constant value of  $\Gamma_0$ , while  $\Gamma_0$ does not show any explicit variation with fold at 125 MeV. A constant  $E_0 \sim 17.5$  MeV explains the fold gated spectra. The value of  $\beta_m$  and  $\Delta\beta$  is found to be 0.1 and 0.2, respectively. The value of effective  $\beta$  obtained is  $0.25 \pm 0.07$ . In this analysis also, there are many solutions for  $\gamma$ , which gives a good fit to the data. Similar observation was reported in Ref. [22].



FIG. 7. Extraction of collisional damping width as a function of fold.

Further, we report here also exclusive temperature and angular momentum dependence of  $\Gamma_0$ . The extraction of  $\langle T \rangle$  and  $\langle J \rangle$ of the states on which GDR is built, for various  $\gamma$ -multiplicity windows is reported in Ref. [11]. The corresponding standard deviations ( $\sigma$ ) for temperature and angular momentum are ~0.28 MeV and ~7 $\hbar$ .  $\Gamma_0$  is found to be independent of  $\langle J \rangle$  (Fig. 8(a)), while showing a mild *T* dependence above  $\langle T \rangle \sim 1.3$  MeV (Fig. 8(b)). Below, a very small value of  $\Gamma_0 \sim$ 3.8 MeV is found. Similar results are also reported in <sup>59-63</sup>Cu, <sup>120</sup>Sn, <sup>179</sup>Au and <sup>208</sup>Pb nuclei [17,23,24] and seems to be a general feature of all nuclei as indicated in Ref. [25].

#### V. SUMMARY

In summary, a simplistic orientation and shape fluctuation analysis are compared with the data and they reasonably agree for the system <sup>28</sup>Si+<sup>58</sup>Ni at E(<sup>28</sup>Si) = 100 and 125 MeV. The extraction of  $\Gamma_0$  under the shape fluctuation model analysis is also reported for this system. A constant value of  $\Gamma_0$  cannot reproduce the measured  $\gamma$ -spectra.  $\Gamma_0$  is found to be independent of  $\langle J \rangle$ . Above  $\langle T \rangle \sim 1.3$  MeV,  $\Gamma_0$  shows a mild *T* dependence. Below, a very small value of  $\Gamma_0 \sim$ 3.8 MeV is found. A detailed calculation of the potential energy surfaces for these nuclei would be very interesting.



FIG. 8. Extraction of collisional damping width as a function of (a) average angular momentum and (b) average temperature.

At this low temperature, spin dependence of level density can also play an important role. Since the system studied lies away from the nearby stable nuclei, it will be interesting to make some more exclusive GDR measurements in the near-by mass region, and see if at low temperature similar results are observed.

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