Excited 0^+, $T = 2$ states in ^{12}Be, ^{12}C, and ^{12}O

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We present predictions of a relatively simple model of the low-lying 0^+ states in ^{12}Be, their predicted energy splitting in ^{12}O, their cross-section ratios in ^{10}Be(t, p) and ^{14}C(p, t) and their decay widths. Comparison is made with predictions using earlier wave functions of Barker.

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A brief glance at the low-lying energy levels of ^{11}Be [1] makes it clear that ^{12}Be should have two low-lying 0^+ states that are mixtures of p-shell [2] and ^{10}Be $\times (sd)^2$ configurations. Barker [3] computed the expected positions of three 0^+ states and some of their properties. He obtained an energy splitting of 2.35 MeV for the first two, with the majority of the s^2 component in the second state, even though he stated "these numbers should not be taken too seriously." His lowest two 0^+ states are not primarily admixtures of the lowest p-shell and lowest $(sd)^2$ 0^+ states. They have considerably different d^2/s^2 ratios and opposite relative phases between d^2 and s^2 [3]. His lowest has 32% s^2, 29% d^2, whereas the second has 67% s^2 but only 10% d^2. His states resemble orthogonal mixtures of s^2 and Ap^2 + Bd^2, with his third 0^+ being then predominantly Bp^2 – Ad^2.

The first excited 0^+ state in ^{12}Be was recently discovered at an excitation energy of 2.24 MeV [4]. (An early 0^+ candidate [5,6] at 2.7 MeV turned out [7] to have $J^\pi = 1^-$.) We discuss the properties of this 0^+state and estimate its location in ^{12}O. We also estimate relative cross sections for $2n$ stripping and pickup.

A $1p$ shell-model calculation [2] produces one low-lying 0^+ state in ^{12}Be and a 2^+ state 4.37 MeV above it. Coupling two sd-shell neutrons to a ^{10}Be core produces two low-lying 0^+ states and two 2^+ states, all primarily involving only the $2s1/2$ and $1d5/2$ orbitals. With two-body matrix elements from LSF [8] and single-particle energies from ^{11}Be (Table I), these two $(sd)^2$ 0^+ states have absolute energies [relative to the physical ^{12}Be (g.s.)] calculated to be 0.20 and 4.35 MeV (Table II). The first is predicted to be very strong in ^{10}Be(t, p), the second much less so. The corresponding second $(sd)^2$ 0^+ states in ^{14,16}C have small (t, p) cross sections [9]. Experimental results from the ^{10}Be (t, p) reaction [5] confirm that ^{12}Be(g.s.) contains significant sd-shell admixtures, a conclusion reached earlier in connection with β decay [10] and $2^+ \rightarrow$ g.s. γ decay [11].

Because the Coulomb energy shift is very sensitive to the $2s1/2$ occupancy, fitting the mass difference between ^{12}Be and ^{12}O ground states allows a determination of this $(2s1/2)^2$ component in the two states (assumed equal, if isospin invariance holds). Such a calculation [12] gave 53% s^2, with the remaining 47% split among p-shell and d^2 components. Because the calculation is much less sensitive to the relative amounts of these admixtures, the Coulomb-energy calculation provided no further insight into the d^2/p^2 ratio. However, it is extremely likely that the physical g.s. is primarily a mixture of the p-shell ground state and the lower of the two $(sd)^2$ 0^+ states mentioned above, which is calculated to contain 22% d^2, 78% s^2. If the physical ground state has the same d^2/s^2 ratio, we would then have 53% s^2, 15% d^2, and 32% p shell. These components are listed in Table III, where we compare with the results of the earlier calculation by Barker. [3].

The second $(sd)^2$ 0^+ state is far away, is orthogonal to the first $(sd)^2$ 0^+ state, and is expected to have a very small mixing matrix element with the p-shell 0^+ state. Thus, we expect the second observed 0^+ level to be the orthogonal admixture to the ground state, viz. 32% $(sd)^2$ and the remainder p shell. In (t, p), these amplitudes interfere destructively (they are constructive in the ground state). Distorted-wave cross sections [5] for $(sd)^2$ transfer are about seven times those for the p-shell amplitudes of CK for ^{10}Be to ^{12}Be, leading to a predicted $0^{+\prime}$/ground state ratio of 0.008. It is thus not surprising that this second 0^+ state was not observed in (t, p).

By contrast, Barker has more s^2 component in the excited 0^+ state than in the ground state. Using his wave functions for the excited 0^+ state, the predicted $0^{+\prime}$/ground state (t, p) cross-section ratio is 0.10 , i.e., the excited state should have been strong enough to have been observed, though still considerably weaker than the ground state.

The energy shift from ^{12}Be to ^{12}O is also very different with the two sets of wave functions. Because Barker has more s^2 in the upper state, the splitting between the two 0^+ states in ^{12}O will be significantly smaller than in ^{12}Be. We have computed this energy difference with our wave functions and those of Barker. Results are also listed in Table III. We see that the expected location of the second 0^+ state in ^{12}O is considerably different in the two models. With a splitting of 2.24 MeV in ^{12}Be, our wave functions provide a splitting of 1.95 MeV in ^{12}O, whereas using Barker's wave functions yields 1.19 MeV splitting in ^{12}O. This difference is easily understood, because it is the upper of the two that has the most s^2 strength in Barker's calculation.

If we accept the result [12] that the ^{12}Be-^{12}O ground-state energy difference requires about $53 \pm 3\%$ s^2, with little

TABLE I. Hamiltonian for low-lying $(sd)^2$ states in ^{12}Be.

J^π	Matrix element	Value (MeV)[a]
0^+	$V(s^2, s^2)$	−1.54
	$V(s^2, d^2)$	−1.72
	$V(d^2, d^2)$	−2.78
2^+	$V(ds, ds)$	−0.59
	$V(ds, d^2)$	−0.59
	$V(d^2, d^2)$	−1.02
$1/2^+$	s.p.e.	−0.503[b]
$5/2^+$	s.p.e.	+1.275[b]

[a]From LSF (Constrained II), Ref. [8].
[b]From ^{11}Be, Ref. [1].

TABLE II. Calculated energies (MeV) and wave functions of three lowest 0^+ and 2^+ states in ^{12}Be.

J^π	Config.	Eigenvalue	E_x[a]	Wave fn.	
0^+				s^2	d^2
	$(sd)^2$	−3.47[b]	0.20	0.78	0.22
	p shell	28.69[c]	1.10	0	0
	$(sd)^2$	0.68[b]	4.35	0.22	0.78
2^+				ds	d^2
	$(sd)^2$	−0.04[b]	3.63	0.88	0.12
	p shell	33.06[c]	5.46	0	0
	$(sd)^2$	1.75[b]	5.42	0.12	0.88

[a]Relative to physical ^{12}Be(g.s.).
[b]Relative to ^{10}Be+2n (present).
[c]Relative to ^{12}C(g.s.) (Ref [2]).

dependence on the p^2/d^2 ratio, we can look elsewhere for more information on that ratio. A recent breakup experiment [13] extracted a $d5/2$ spectroscopic factor of 0.48 ± 0.06 for ^{12}Be(g.s.), implying 0.24 ± 0.03 for the d^2 probability in that state. If this value is correct we would then have $1 - 0.53 - 0.24 = 0.23 \pm 0.05$ for the p-shell component—in excellent agreement with the value 0.22 ± 0.04 implied by the $p1/2$ spectroscopic factor of 0.44 ± 0.08 in Ref. [13]. This mixture of s^2, d^2, and p-shell components results in the same ^{12}Be-^{12}O energy difference as our earlier mixture, to within 2.5 keV. And, changing the d^2/s^2 ratio from 0.2/0.8 to 0.3/0.7 reduces the $(sd)^2$ (t, p) cross section by only 1.4%.

In ^{14}C$(p,t)^{12}$C [14], two states were observed in the excitation-energy region where $T = 2$ states should begin. The lower of the two, at 27.57(3) (later refined to 27.5950(24) [15]) MeV, is almost certainly the double analog of ^{12}Be (g.s.). For the second one, at 29.63(5) MeV, the data do not distinguish between 0^+ and 2^+ but slightly favor 0^+. The first 2^+ state in ^{12}Be is primarily of $(sd)^2$ character. It is strong in (t, p), whereas the p-shell 2^+ is calculated to be significantly weaker (only 5% of the observed strength). The p-shell 2^+ state is also much further away. Of course, some p-shell admixtures are undoubtedly present. The g.s. of ^{14}C does contain some $(sd)^2$ admixture—estimated to be about 12% from ^{12}C(t, p) [16]. But, those nucleons are coupled to $J^\pi = 0^+$ and hence would not contribute to the 2^+ state. [Any ^{12}C$(2^+) \times (sd)^2_{2+}$ component in ^{14}C (g.s.) is too small to contemplate.] The p-shell 2^+, $T = 2$ state in ^{12}C is expected to be quite strong in (p, t). Even though the majority of the CK 2^+ strength lies considerably higher, only about 25% of the p-shell 2^+ state mixed into the physical 2^+ state could explain the observed magnitude of the (p, t) cross section, without any need for any 0_2^+ contribution. We have no reliable estimate of the amount of this mixing. The near degeneracy of the second $(sd)^2\,2^+$ state and the p-shell one (see Table II) could complicate the mixing. It is possible that both 0^+ and 2^+ states are being populated. If we fit the 29.63-MeV angular distribution with a mixture of a smooth curve drawn through the 0^+ 27.57-MeV angular distribution and the $L = 2$ curve displayed with the data in Ref. [14], a reasonable fit is obtained, with $\sigma(2^+) = 3.8 \pm 2.0\mu$b/sr at the average of the first two angles and $\sigma(0_2^+) = 3.6 \pm 1.0\mu$b/sr at $\theta \sim 30°$–$35°$. This 2^+ yield would correspond to about 19 ± 9% of the CK 2^+ state mixed into the physical state. Some of the ∼200-keV width [14] could then come from overlapping levels rather than natural width. The decay branching ratios would then be difficult to untangle because both states would contribute to the decay.

In (p, t) the two 0^+ states can be populated in direct $2n$ pickup from both components of ^{14}C (g.s.) (see Fig. 1), but both paths involve p-shell transfer, as we now demonstrate. For simplicity, think of $2p$ pickup to ^{12}Be, rather than $2n$ pickup

TABLE III. Calculated and measured properties of first two 0^+ states in ^{12}Be.

Calc.	Ref.	State	Wave-function intensities			Splitting in ^{12}O (MeV)	Cross-section ratio	
			s^2	d^2	p shell		^{10}Be(t, p)	^{14}C(p, t)
	Present[a]	g.s.	0.53	0.15	0.32			
		$0^{+\prime}$	0.17	0.05	0.68	1.95	0.008	∼0.5
	Barker[b]	g.s.	0.325	0.292	0.384			
		$0^{+\prime}$	0.67	0.10	0.23	1.19[c]	0.10[c]	∼0.07[c]
Exp.						Unknown	Very small[d]	∼0.3–0.5[e]

[a]Reference [12].
[b]Reference [3].
[c]Our calculation with Barker's wave function.
[d]Reference [5].
[e]Reference [14] and present.

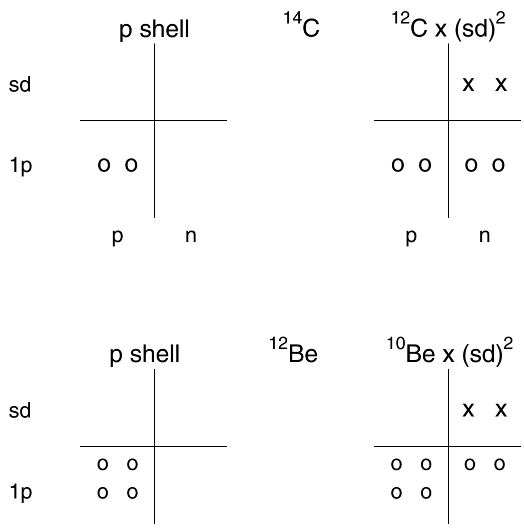

FIG. 1. Depiction of shell-model occupancies of two largest components in ground states of ^{14}C and ^{12}Be. Circles denote holes; x's denote particles.

to the $T = 2$ states of ^{12}C; the nuclear structure is the same. (We assume the wave-function admixtures for the 0^+, $T = 2$ states in ^{12}C are the same as in ^{12}Be.) Then, with

$$^{14}\mathrm{C(g.s.)} = (1\text{-}\varepsilon^2)^{1/2}\left(^{14}\mathrm{C_{CK}}\right) + \varepsilon\left(^{12}\mathrm{C_{CK}}\right) \times (sd)^2,$$

$$^{12}\mathrm{Be(g.s.)} = \alpha\,^{10}\mathrm{Be_{CK}} \times (sd)^2 + \beta\,^{12}\mathrm{Be_{CK}},$$

and

$$^{12}\mathrm{Be}(0^{+\prime}) = -\beta\,^{10}\mathrm{Be_{CK}} \times (sd)^2 + \alpha\,^{12}\mathrm{Be_{CK}},$$

the $2p$ transfer amplitudes for ^{14}C to ^{12}Be are

$$\begin{aligned}\mathrm{A(g.s.)} = \beta(1\text{-}\varepsilon^2)^{1/2}\mathrm{A}(^{14}\mathrm{C} \rightarrow {}^{12}\mathrm{Be})_{\mathrm{CK}} \\ + \alpha\varepsilon\mathrm{A}(^{12}\mathrm{C} \rightarrow {}^{10}\mathrm{Be})_{\mathrm{CK}},\end{aligned}$$

and

$$\begin{aligned}\mathrm{A}(0^{+\prime}) = \alpha(1-\varepsilon^2)^{1/2}\mathrm{A}(^{14}\mathrm{C} \rightarrow {}^{12}\mathrm{Be})_{\mathrm{CK}} \\ - \beta\varepsilon\mathrm{A}(^{12}\mathrm{C} \rightarrow {}^{10}\mathrm{Be})_{\mathrm{CK}}.\end{aligned}$$

If the proton structures in ^{14}C and ^{12}C are similar, and likewise for p-shell ^{12}Be and ^{10}Be, then the two pure amplitudes will be comparable. In Ref. [2], their ratio is 1.24. All the pickup is from the $1p$ shell. Any other small amplitudes are likely to be constructive for the ground state, mixed for the excited state, slightly favoring the ground state. The experimental (p, t) ratio of about 0.3–0.5 is well within expectations (Table III) for this simple model. With our wave functions, our ground-state amplitude is the *sum* of two terms, each of which contains as factors "large" times "small," whereas for the excited state the amplitude is the *difference* of two terms, one of which is "large" times "large" and one that is "small" times "small." For that reason, small changes in wave functions will not dramatically change our predicted ratio. We thus conclude that the ^{14}C (p, t) reaction probably populates the second $0^+ T = 2$ state. Some contribution of the 2^+, $T = 2$ state is also likely.

Barker estimated the (p, t) cross sections by taking 80 μb/sr as "typical" of the p shell and then multiplying by the percentage of p^2 in his wave functions, getting an expected ratio of about 0.5 or 2.0 with two different sets of calculations. Because two-nucleon transfer involves a coherent sum of amplitudes (see above), a small amount of core excitation in the ^{14}C (g.s.) can change the cross section significantly, even though (as demonstrated above) it still involves only p-shell pickup. With the ^{14}C (g.s.) used above, Barker's wave functions would predict a (p, t) ratio of about 0.07, considerably less than the ratio observed. With Barker's wave functions, the ground-state amplitude is the *sum* of "large" times "large" and "small" times "small," whereas the excited-state amplitude is the *difference* of "large" times "small" and "small" times "large." Reasonable changes in his wave functions will keep the ratio small. If any of the observed cross section is to the 0_2^+ state, however, it should have a strong isospin-allowed decay branch (see Table IV) to the $1/2^+ T = 3/2$ state of ^{11}B. We compute the expected width from the expression

$\Gamma = \mathrm{C}^2 S \Gamma_{\mathrm{sp}}$, where $\mathrm{C}^2 = 1/2$ here, and S is twice the s^2 occupancy in Table III. The result is a proton width of about 800 keV with Barker's wave function, and about 200 keV with ours, to be compared with the experimental upper limit [14] of $\leqslant$200 keV. And Ref. [14] states that only about 40% of the observed p decays are to the $T = 3/2$ states of ^{11}B. [The widths for decay to the $1/2^-$state are only upper limits because the p-shell component is not pure $(p1/2)^2$]. If 2^+ and 0^+ are populated with comparable cross sections, then the total decay is from both and it is possible that all this decay to $T = 3/2$ states could come from the 0^+. So, from the (p, t) data, it is possible that the 0^+ has a width near 200 keV and decays mostly to the $1/2^+$, $T = 3/2$ state. But, if it is populated at all in (p, t), the 800-keV width seems unlikely.

TABLE IV. Isospin-allowed p decays of ^{12}C(0_2^+, $T = 2$).

Final state	E_x (MeV)	(12.557)	12.916
	J^π	$1/2^+$	$1/2^-$
	ℓ	0	1
	E_p (MeV)	1.12	0.76
	Γ_{sp}	~1.2 MeV	85 keV
$\Gamma_{\mathrm{calc.}}$	Present	~200 keV	<58 keV
	Barker[a]	~800 keV	<20 keV
$\Gamma_{\mathrm{exp.}}$[b]		⩽200 keV[c]	

[a]Our calculations with Barker's wave function [3].
[b]Reference [14]. Probably contains both 0_2^+ and 2^+ (see text).
[c]This is Γ_{tot}; less than about 40% of the decay is to $T = 3/2$ states in ^{11}B (Ref. [14]).

TABLE V. Calculated properties of the third 0^+ state in ^{12}Be.

Reference	E_x(MeV)	Wave function			(t, p) ratio
		s^2	d^2	p shell	
Present	4.8	0.22	0.78	~0	1.4%
Barker[a]	8.5	0.005	0.61	0.38	1.3%[b]

[a]Reference [3].
[b]Our calculation using Barker's wave function.

The nature of the third 0^+ state is also quite different in the two models. In our approximation it is predominantly the second $(sd)^2$ 0^+ state, whereas Barker's is nearly all p^2 and d^2. His third 0^+ level is 8.46 MeV above the first. Ours is computed to be 4.15 MeV above the position of $(sd)^2 0_1^+$ before the latter mixes with the CK g.s. to form the physical ground state. With the observed separation of the two lowest 0^+ states and our wave functions, it is a simple matter to calculate this unmixed location, which turns out to be 0.7 MeV above the physical ground state—implying our third 0^+ level should lie near 4.8 MeV. The properties of this third 0^+ state of ^{12}Be in the two models are summarized in Table V. Perhaps surprisingly, the predicted (t, p) cross sections (which are quite small) are nearly identical. The biggest differences are in the excitation energy and in the expected neutron-decay properties.

In conclusion, we have presented predictions of a relatively simple model of the low-lying 0^+ states in ^{12}Be, their predicted energy splitting in ^{12}O, their cross-section ratio in ^{10}Be(t, p) and ^{14}C(p, t) and their decay widths. Comparison has been made with predictions using earlier wave functions of Barker. In all cases for which the experimental quantity is known, agreement is better with our wave functions. This is especially true for the decay width of the second 0^+state.

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