

Pionic color transparency

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We use a semiclassical approximation to investigate the effects of color transparency on pion electroproduction reactions. The resulting reduced nuclear interactions produce significant, but not dominating, differences with the results of conventional distorted-wave, Glauber-type treatments at kinematics accessible to Jefferson Laboratory. Nuclear effects that could mimic the influence of color transparency are also discussed.

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Color transparency is the reduction of initial or final state interactions in high-momentum-transfer coherent processes that occurs if a projectile or ejectile propagates in the nucleus as a small color singlet object. (See the reviews: Refs. [1–3]. In such situations, the effects of emitted gluons are canceled [4–6] in a manner analogous to the propagation of a small electric dipole moment through an electrodynamic medium [7] so that the small color singlet behaves as a point-like configuration (PLC). The PLC is a component of a hadron and evolves to its full size in a characteristic hadronic time of the order of 1 fm/c. However, produced at sufficiently high energies, time dilation effects allow a PLC to propagate through the entire nucleus before expanding. In that case, the production cross sections will be larger than those computed using the standard distorted wave or Glauber treatment.

An electroproduction ($e, A, e'\pi X$) experiment at JLAB [8] attempting to measure pion transparency in nuclei has completed running and is currently under analysis. The pion electroproduction cross section is measured for various values of Q^2 and p_π between 1 and 5 GeV and $|t| < 0.5 \text{ GeV}^2$. The experiment was done at parallel kinematics so that $p_\pi = \sqrt{Q^2 + v^2} - k$ where k is the magnitude of the three-momentum imparted to the target. Targets used in the experiment include ^1H , ^2H , ^{12}C , ^{64}Cu , and ^{197}Au . From this data it may be possible to extract the Q^2 , p_π , k and A dependence of any observed color transparency.

The state of pion transparency theory has been quiet for some time. Stimulated by the new experiment, our purpose is to display the results of earlier theory [9] using specific kinematics and nuclear targets that allows us to discuss the feasibility of observing color transparency in electroproduction reactions as well as other nuclear effects. Papers on pionic transparency [10] have appeared since the time of Ref. [9]. The new feature of the present work is the evaluation at the specific experimental kinematics of Ref. [8]. We also include the effects of expansion absent from Ref. [10]. We do not incorporate the possible interference effects between point-like and blob-like configurations that could produce oscillations in color transparency that are included in Ref. [10].

The final state interactions of semiexclusive nuclear reactions can be described quite well by Glauber-type calculations. If a PLC is created inside the nucleus and subsequently evolves

into a physical particle its final state interactions will be modified and transparency will result.

If one is summing over all nuclear final states, the nuclear transparency can be defined as the ratio of a model calculation of a nuclear cross section (including the effects of FSI's to the (A times the) cross section produced by a free nucleon target. The use of this ratio allows one to assess the influence of FSI's without having a detailed knowledge of the reaction dynamics.

A semiclassical formula [11] has been developed to compute nuclear transparency for situations in which the kinematics of the outgoing pion are known precisely, but the cross section involves a sum over all of the excited nuclear states [12]. The strength of the final state interactions depends on an emission probability computed using the eikonal approximation and an effective interaction that parametrizes the variation of the final state interactions as the ejectile propagates through the nucleus. If the particle produced inside the nucleus is a PLC which then expands into the observed final state, the interaction with the nuclear matter deviates from that of Glauber-type calculations (Fig. 1).

The semiclassical formula for pion transparency in the reaction ($e, e'A \rightarrow \pi^+ X$) involves only a single integral over the path of the outgoing pion,

$$T = \frac{A_{\text{eff}}}{A} = \frac{1}{A} \int d^3r \rho(r) \exp \left[- \int_z^\infty dz' \sigma_{\text{eff}}(z' - z, p_\pi) \rho(r') \right]. \quad (1)$$

The nuclear density $\rho(r)$ is of Woods-Saxon form with radius parameter $R = 1.1 \text{ fm } A^{1/3}$ and diffuseness $a = 0.54 \text{ fm}$, and is normalized to A and the effects of final state interaction is contained in the effective interaction, σ_{eff} . The effective interaction contains two parts, one for $z' - z$ less than a length l_c describing the interaction of the expanding PLC, another, for larger values of $z' - z$ describing the final state interaction of the physical particle. The effective interaction for the PLC is

$$\sigma_{\text{eff}}(z, p_\pi) = \sigma_{\pi N}(p_\pi) \left[\left(\frac{n^2 \langle k_r^2 \rangle}{Q^2} \left(1 - \frac{z}{l_c} \right) + \frac{z}{l_c} \right) \times \theta(l_c - z) + \theta(z - l_c) \right]. \quad (2)$$

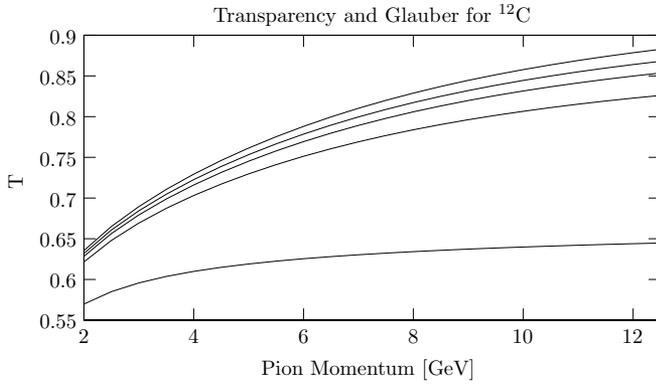


FIG. 1. Transparency and Glauber-like calculation for a ^{12}C target and pion momentum up to 12 GeV. The top curve is for color transparency calculated with $\sigma_{\text{PLC}} = 0$. The next three curves are for $\sigma_{\text{PLC}} \propto 1/Q^2$ evaluated at $Q^2 = 10, 5$ and 2.5 GeV^2 , respectively. $\Delta M^2 = 0.7 \text{ GeV}^2$.

The prediction that the interaction of the PLC will be approximately proportional to the propagation distance z for $z < l_c$ is called the quantum diffusion model. For $z = 0$ in Eq. (2) the cross section for the initially-produced PLC is identified.

$$\sigma_{\text{PLC}} \equiv \sigma_{\pi N}(p_\pi) \frac{n^2 \langle k_t^2 \rangle}{Q^2}. \quad (3)$$

For the pion, $n = 2$ and $\langle k_t^2 \rangle^{1/2} \simeq 0.35 \text{ GeV}$ [11]. The coherence length, l_c sets the time scale for the PLC to evolve and determines the probability that a particle experiences reduced PLC interactions before leaving the nuclear matter. For propagation distances $z > l_c$ the PLC interaction is that of a standard final state approximation with $\sigma_{\text{eff}} \simeq \sigma_{\pi N}(p_\pi)$, and is that of a typical Glauber-like calculation. We take the values of $\sigma_{\pi N}(p_\pi)$ from the Particle Data Group parametrization [13]. In the limit $l_c = 0$ a PLC is not created and the calculation reduces to a Glauber-like calculation.

The transparency is known [11,14] to have a strong dependence on the unknown parameter l_c . The best current estimate for l_c is $2p_h/\Delta M^2$, with ΔM given by the lowest lying Regge partner. Current data do not constrain the value of l_c for pions. In our calculations we use $\Delta M = 0.7 \text{ GeV}^2$ which leads to a prediction of transparency at currently accessible energies, but also show results for $\Delta M^2 = 1.4 \text{ GeV}^2$. This larger value corresponds to a very short expansion time $ct_{\text{exp}} = 1/\Delta M = 0.16 \text{ fm}$ in the rest frame, and so might be expected to provide a reasonable estimate of a lower limit on the effects of color transparency.

The first step is to assess the size of T and its dependence on the PLC cross section of Eq. (3) for ^{12}C . As shown in Fig. 2, the effects of color transparency increase T over its value as obtained from a Glauber calculation. There is generally a small variation with Q^2 . The Glauber result $T \approx 0.6$ is similar numerically to the value measured for protons in $(e, e'p)$ with ^{12}C , and might be expected to be larger than that because the πN cross section is about $2/3$ the NN cross section. However, the proton experiments are set up so that there is essentially only one final nuclear state for each knocked out proton. As a result the transparency is computed using a formula different

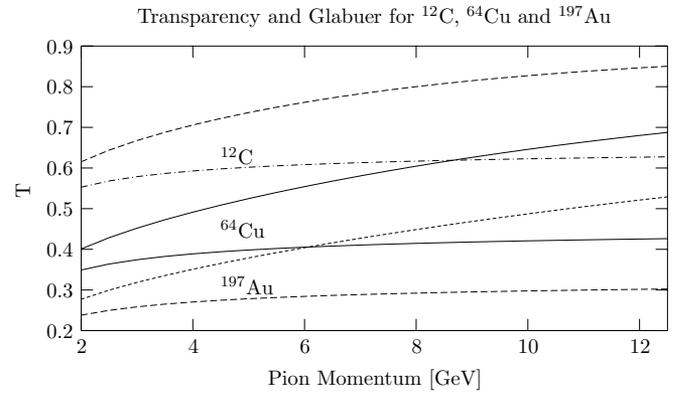


FIG. 2. Transparency (upper curve) and Glauber (lower curve) calculation for ^{12}C , ^{64}Cu , and ^{197}Au for pion momentum up to 12.5 GeV. Color transparency is calculated with σ_{PLC} evaluated at 10 GeV^2 , and $\Delta M^2 = 0.7 \text{ GeV}^2$. Greater enhancement is to be found for larger nuclei.

than Eq. (1). Analyzing the two formulas shows that the chance for protons to be produced near the edge of the nucleus (where there is little absorption) is larger than that for pions.

The A dependence is displayed in Fig. 2. For any fixed range of the momentum of the outgoing pion, increasing the size of the target nucleus increases the relative increase in the value of T . The dependence on coherence length is shown in Fig. 3. About half (but not all) of the effects of transparency are removed by decreasing the coherence length.

The experiment recently completed at JLAB measured pion electroproduction cross sections at specific kinematics. The kinematics are listed in Table I and the evaluations of T are shown in Fig. 3. The relevant parameters in transparency are the pion momentum, Q^2 and the size of the nucleus. For measurements of different pion momenta and different nuclear targets at a given Q^2 one can define an enhancement factor which can be predicted from our semiclassical approximation. It is important to emphasize here that the momenta of the pions k in the kinematics of the JLAB experiment are large. Hence this process cannot be thought of as a knockout of the pion from the meson field of the nucleon, but rather

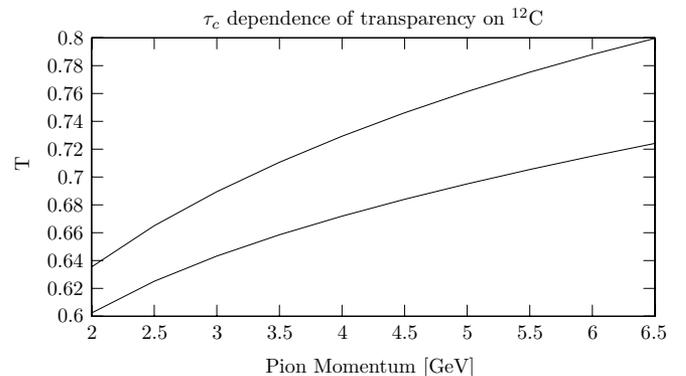


FIG. 3. The effects of varying the coherence length, $l_c = 2p_\pi/\Delta M^2$, on the transparency. The upper curve is obtained using $\Delta M^2 = 0.7 \text{ GeV}^2$. The lower curve is obtained using $\Delta M^2 = 1.4 \text{ GeV}^2$.

TABLE I. Kinematics of Ref. [8].

Q^2	ν	k	p_π
1.83	2.046	0.56	1.877
1	2.414	0.23	2.38
2.15	2.276	0.66	2.074
1.83	2.901	0.39	2.823
1.2	3.286	0.29	3.257
2.15	3.276	0.46	3.187
3	3.48	0.58	3.315
3	3.576	0.62	3.418
4	4.096	0.78	3.857
4	4.126	0.71	3.897
4	4.235	0.75	4.016
4	4.732	0.58	4.571
4.8	4.645	0.88	4.361
5	4.822	0.79	4.54

as a case of the hard process ($\gamma^*N \rightarrow \pi N$) off a nucleon bound in the nucleus which is governed at large Q^2 by the QCD factorization theorem [15] which states that in this limit the process is dominated by the PLC configurations of the produced $q\bar{q}$ pair.

Since the pion momenta are very large compared to the characteristic scale of the nuclear momenta one expects nuclear modifications to the reaction to be small. There is still a need to treat the modifications due to the excitation energy of the residual nucleus and the Fermi motion of the initial nucleon. If the four-momentum of the struck bound nucleon is given by $(m_N - E, \mathbf{p}_N)$, the change in the momentum of the emitted pion is $\approx -2.5E - p_{N3}$, where 3 represents the photon direction. This effect shifts the peak momentum of the produced pion by about 50 MeV/c with a spread of about 200 MeV/c. The acceptance of the experiment is broad enough to measure pions within this momentum spread [8]. A second effect is the enhancement of the scattering by forward moving nucleons due to the energy dependence of the elementary cross section. These effects are mostly important for comparing deuteron results to those of heavier nuclei. Comparing the

deuteron to heavier nuclei requires accurate modeling of the experimental cuts. For $A \geq 12$ the average excitation energy and average momentum become a rather weak function of A and so one expects that the effects discussed above will mostly cancel in the ratio of heavy nuclei vs carbon.

In the following T is plotted vs k , the magnitude of the three-momentum exchanged with the target. For parallel kinematics, the magnitude of the pion three-momentum is $p_\pi = |\vec{q}| - k$. The usual color transparency prediction is for greater transparency with greater final state momentum. Thus the chosen value of k strongly influences the computed values of T . For the experimental data one can look at the greatest ratio of transparencies for the collected data and look for any variation in this ratio between the different targets.

There seems to be enough variation with Q^2 and A to allow identification of color transparency for the current beam energies. However, the use of a higher energy beam planned for JLAB12 would greatly extend the scope of the study as well as the possibility of obtaining a clean observation of the effects of color transparency.

There are other significant effects that depend on the value of k that are not included in the present calculation. The nuclear EMC effect tells us that nuclear interactions act to suppress the effects of quarks moving with large Bjorken x . Therefore the chance for a pion of large momentum, k , to exist in a nucleus could depend on the nuclear density and differ strongly between the deuteron and heavier nuclei. However, the EMC effect is similar for all nuclei with mass as great as ^{12}C . Therefore comparisons between heavier nuclei could be more useful in extracting information about color transparency than comparing between heavy nuclei and the deuteron.

Figure 4 shows a clear pattern in the Q^2 , A and k (three-momentum transfer to the nucleus) dependence of the transparency T . Use of the A -dependence could allow the identification of color transparency at the current energies of JLAB. The use of a higher energy beam expected at JLAB12 would significantly enhance the probability of observe color transparency in pion electroproduction experiments. If the dependence of the cross section on k does not vary greatly between different nuclei than an enhancement of the cross

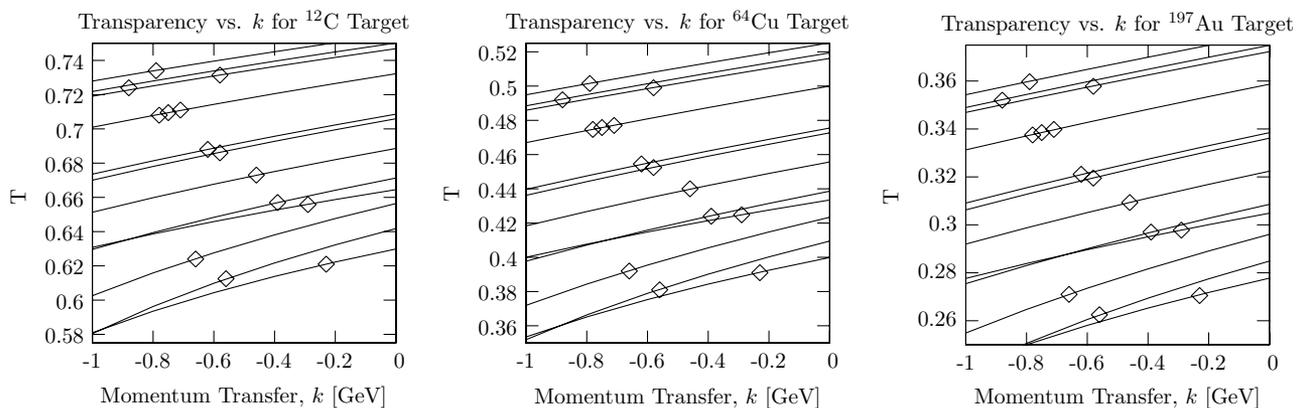


FIG. 4. Plot of transparency vs k for a given Q^2 and $|\vec{q}|$ at values measured in the experiment at JLAB. The curves, from bottom to top correspond to the values of Q^2 and ν listed in Table I as a function of k . The diamonds show the results for the experimentally chosen values of k (and therefore p_π). For a given Q^2 and ν at parallel kinematics $p_\pi = |\vec{q}| - k$.

section for a given k will be a clear signal of color transparency. We may expect that this variation will be similar for heavy nuclei but not for light nuclei. The ratios of cross sections for heavy nuclei can therefore provide more reliable information

than considering ratios with the deuteron cross section. Any understanding of transparency can not be complete without an understanding of the k dependence of the $(e, e'\pi)$ reaction. This will be pursued elsewhere.

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