Deformation parameter for diffuse density

K. Hagino,[1] N. W. Lwin,[1] and M. Yamagami[2]
[1]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
[2]*Radioactive Isotope Physics Laboratory, RIKEN, Wako, Saitama 351-0198, Japan*
(Received 20 April 2006; published 27 July 2006)

In extracting deformation parameters from multipole moments for deformed nuclei, one commonly uses the formulas which are based on a sharp-cut density distribution. We discuss a possible ambiguity for this procedure and clarify the role of diffuseness parameter of the density distribution. For this purpose, we use a deformed Woods-Saxon density as well as a density distribution obtained from the self-consistent relativistic mean-field (RMF) model. We show that the formula using a root-mean-square radius instead of a sharp-cut radius requires a large correction due to a finite surface diffuseness parameter even for stable nuclei. An implication to neutron-rich nuclei is also discussed.

DOI: 10.1103/PhysRevC.74.017310 PACS number(s): 21.10.Ky, 21.10.Gv, 21.60.Jz, 27.30.+t

Deformation of a density distribution in the intrinsic frame is one of the most important concepts in nuclear physics. It is characterized by a finite value of intrinsic quadrupole moment Q, but a more intuitive quantity is a deformation parameter β, which removes the trivial dependence on the radius of a nucleus from the quadrupole moment. In order to extract the deformation parameter from the quadrupole moment, one often uses the formula which is obtained by assuming a sharp-cut density distribution [1,2]. To partly account for the deviation of density distribution from a sharp-cut function, sometimes one also uses a formula in which the sharp-cut radius is replaced by a root-mean-square radius (with a trivial constant factor) [3–5]. In general, the latter formula is believed to be better than the former when the surface diffuseness parameter of density distribution is large as in neutron-rich nuclei.

In this paper, we systematically investigate whether that is the case. To this end, we first use a deformed Woods-Saxon density. This model has an advantage in that the deformation parameter is given as an input parameter. This enables us to check whether the commonly used formulas lead to the correct value of deformation parameter. We also derive the correction to the formulas due to a finite value of surface diffuseness parameter. We then discuss the deformation parameter of neutron-rich Mg isotopes using the self-consistent relativistic mean-field (RMF) model.

Let us begin with the definition for the intrinsic quadrupole moment,

$$Q = \sqrt{\frac{16\pi}{5}} \int d\boldsymbol{r}\, \rho(\boldsymbol{r})\, r^2 Y_{20}(\theta). \tag{1}$$

When the density $\rho(\boldsymbol{r})$ has a sharp edge, that is,

$$\rho(\boldsymbol{r}) = \rho_0\, \theta(R(\theta) - r), \tag{2}$$

with $\rho_0 = 3A/(4\pi R_0^3)$, A being the mass number of a nucleus, and

$$R(\theta) = R_0(1 + \beta\, Y_{20}(\theta)), \tag{3}$$

the quadrupole moment Q is evaluated as

$$Q = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} A R_0^2 \beta, \tag{4}$$

to the first order of deformation parameter β. One often takes $R_0 = 1.2\, A^{1/3}$ fm for the sharp-cut radius R_0 [2]. For a sharp-cut density (2), the root-mean-square radius is calculated as

$$\langle r^2 \rangle = \frac{\int r^2 \rho(\boldsymbol{r})\, d\boldsymbol{r}}{\int \rho(\boldsymbol{r})\, d\boldsymbol{r}} \sim \frac{3}{5} R_0^2, \tag{5}$$

again to the leading order of β. Therefore, the relationship between the quadrupole moment and the deformation parameter given by Eq. (4) can be also written as [3–5]

$$Q = \sqrt{\frac{16\pi}{5}} \frac{5}{4\pi} A\beta\, \langle r^2 \rangle. \tag{6}$$

The effect of finite surface diffuseness of density distribution can be accounted for using a deformed Woods-Saxon density

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + e^{(r - R_0 - R_0\beta Y_{20}(\theta))/a}}. \tag{7}$$

In order to derive the correction term to Eqs. (4) and (6), we expand Eq. (7) with respect to the deformation parameter β and keep only the first order term, that is,

$$\rho(\boldsymbol{r}) \sim \rho_0(r) - \frac{d\rho_0}{dr} R_0 \beta Y_{20}(\theta), \tag{8}$$

where

$$\rho_0(r) = \frac{\rho_0}{1 + e^{(r - R_0)/a}}. \tag{9}$$

Notice that the angle dependence of the surface diffuseness parameter a [6–8] as well as the dependence of R_0 on the deformation parameter due to the volume conservation [1] does not appear in Eq. (8) since they are higher order terms of β. With the density given by Eq. (8), the quadrupole moment Q is calculated as

$$Q = -\sqrt{\frac{16\pi}{5}} R_0 \beta \int_0^\infty r^4 dr\, \frac{d\rho_0}{dr}. \tag{10}$$

Since $d\rho_0/dr$ has a finite value only in a small region near the nuclear surface $r \sim R_0$, we expand r^4 around $r = R_0$. Following the same procedure as in Refs. [9,10], we find

$$Q \sim \sqrt{\frac{16\pi}{5}} R_0^5 \rho_0 \beta \left(1 + 2\pi^2 \frac{a^2}{R_0^2}\right), \tag{11}$$

to the order of $(a/R_0)^2$. One can eliminate the dependence on ρ_0 in Eq. (11) using the condition for the normalization

$$A = \int d\boldsymbol{r}\, \rho(\boldsymbol{r}) \sim \frac{4\pi}{3} R_0^3 \rho_0 \left(1 + \pi^2 \frac{a^2}{R_0^2}\right). \tag{12}$$

This yields

$$Q \sim \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} A R_0^2 \beta \left(1 + \pi^2 \frac{a^2}{R_0^2}\right). \tag{13}$$

Furthermore, one can use the relation

$$\langle r^2 \rangle \sim \frac{3}{5} R_0^2 \left(1 + \frac{7\pi^2}{3} \frac{a^2}{R_0^2}\right), \tag{14}$$

to eliminate the dependence on R_0, leading to

$$Q = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} A\beta \left(\frac{5}{3}\langle r^2 \rangle - \frac{4}{3}\pi^2 a^2\right). \tag{15}$$

A similar formula can be found also in Ref. [6].

We now investigate the performance of Eqs. (4), (6), and (15) using realistic density distributions. To this end, we first compute the quadrupole moment (1) using a deformed Woods-Saxon density, (8). Following Ref. [15], we choose $R_0 = 1.31A^{1/3} - 0.84$ fm in Eq. (9). This value was obtained by fitting to theoretical as well as to experimental density distributions for a number of nuclei with the Woods-Saxon shape [15]. For a deformation parameter β in Eq. (8), we choose $\beta = 0.3$. Once the quadrupole moment is obtained, we can use Eqs. (4), (6), and (15) to obtain an approximate value for the deformation parameter, β. If the formulas worked perfectly, they would lead to $\beta = 0.3$ as in the original density distribution.

Figure 1 shows the deformation parameters obtained in this way as a function of diffuseness parameter a in the density distribution. The upper panel is for $A = 40$, while the lower panel for $A = 238$. The dashed line is obtained with Eq. (6) using the root-mean-square radius. We see that this formula significantly underestimates the deformation parameter, especially for the lighter system, $A = 40$, except when the diffuseness parameter a is close to zero. Surprisingly, the formula does not seem to work even for stable nuclei around $a \sim 0.55$ fm. When one takes into account the surface diffuseness correction with Eq. (15), one obtains a reasonable agreement with the exact value of the deformation parameter, as shown by the solid line. The dot-dashed line is obtained by using Eq. (4) with $R_0 = 1.2\, A^{1/3}$ fm. We see that this formula provides a reasonable value of the deformation parameter if the surface diffuseness is around 0.55 fm as in stable nuclei, although the deviation from the exact value becomes large when the surface diffuseness is around 1.0 fm.

Let us now discuss the deformation parameters obtained with self-consistent RMF calculations. In self-consistent

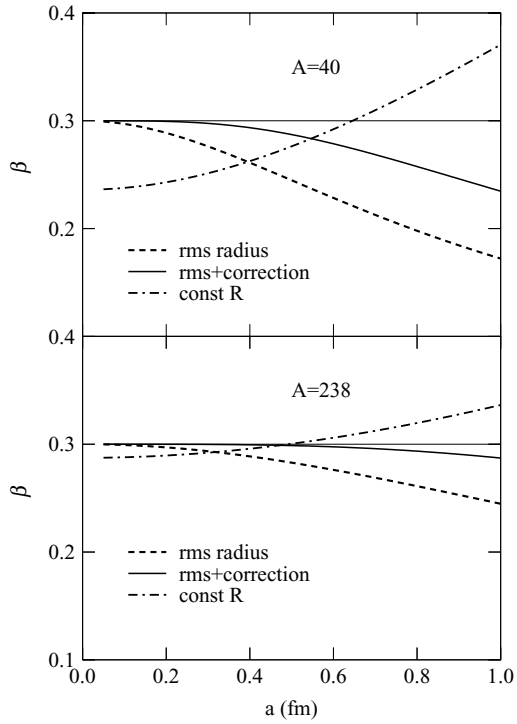

FIG. 1. Deformation parameters obtained with several formulas as a function of surface diffuseness parameter a in the density distribution. The deformation parameter is set to be $\beta = 0.3$ in the deformed Woods-Saxon density. The upper panel is for $A = 40$, while the lower panel for $A = 238$. The dot-dashed line is obtained with Eq. (4) assuming a sharp-cut density, while the dashed line with Eq. (6) using the root-mean-square radius. The solid line is obtained with Eq. (15), including the surface diffuseness correction.

mean-field calculations, another complication arises in extracting the deformation parameter β from the quadrupole moment Q because of a finite value of higher-multipole deformation parameters, such as β_4. In this paper, where we discuss only the linear order formulas, we disregard this problem since those contributions appear as higher order terms with respect to the deformation parameters.

The upper panel of Fig. 2 shows the deformation parameters for Mg isotopes. The filled circles, the open circles, and the open triangles are obtained with Eqs. (4), (6), and (15), respectively. In this calculation, we use the NL3 parameter set [11] and assume the axial symmetry. We use the computer code RMFAXIAL [2], which solves the RMF equations using the harmonic oscillator expansion method. We employ the constant gap approach with the pairing gap given in Refs. [12,13]. In order to estimate the diffuseness parameter a, we expand the density distribution into multipoles (see Appendix B of Ref. [14]),

$$\rho(r, \theta) = \rho_0(r) + \rho_2(r) P_2(\cos\theta) + \rho_4(r) P_4(\cos\theta) + \cdots, \tag{16}$$

and fit the monopole density $\rho_0(r)$ with the Woods-Saxon shape, Eq. (9). The diffuseness parameter thus obtained is shown in the lower panel of Fig. 2. Using the deformed Woods-Saxon density (7), we have confirmed that this procedure

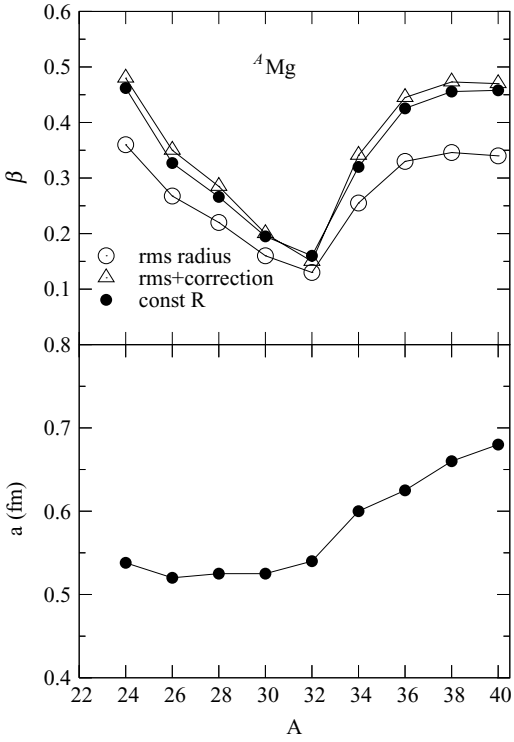

FIG. 2. Upper panel: Deformation parameters for Mg isotopes extracted from the relativistic mean-field (RMF) density distribution. The open triangles, the open circles, and the filled circles are obtained with Eqs. (4), (6), and (15), respectively. Lower panel: The diffuseness parameter for each nucleus obtained by fitting the RMF density to the Woods-Saxon shape.

works well unless the deformation parameter is unphysically large. In Ref. [16], the Woods-Saxon parameters were determined so as to reproduce the calculated radial moments, $\langle r \rangle$ and $\langle r^2 \rangle$, obtained with spherical self-consistent Hartree-Fock densities. We have confirmed that this procedure also leads to almost the same value of surface diffuseness parameter as in Fig. 2. One finds that the deformation parameter estimated with Eq. (6) (the open circles) is considerably smaller than that estimated with Eq. (15) which includes the surface diffuseness effect (the open triangles), in accordance with the study with the deformed Woods-Saxon density. One also finds that the deformation parameters estimated with Eq. (4) are close to those with Eq. (15) (the filled circles), although it might simply be accidental. We have checked that our conclusions remain the same also for Sr isotopes.

Figure 3 shows the potential energy surface for ^{40}Mg nucleus as a function of deformation parameter β, obtained with constrained RMF calculations. The solid and the dashed lines are obtained with Eqs. (4) and (6), respectively. Since Eq. (6) underestimates the deformation parameter, the distance between the prolate and oblate minima is also underestimated. The energy surface which takes into account the surface diffuseness correction is denoted by the filled circles. As we showed in Fig. 2, it is similar to the energy surface obtained with Eq. (4).

In attempting to extract the deformation parameter from an experimental quadrupole moment, an interesting question

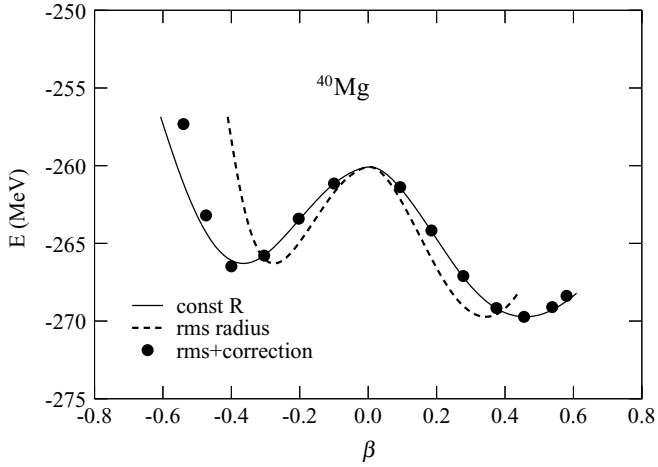

FIG. 3. Potential energy surface for ^{40}Mg. The solid line, the dashed line, and the filled circles are obtained with Eqs. (4), (6), and (15), respectively.

is how to estimate the surface diffuseness parameter a from experimental data. Unfortunately, the root-mean-square radius alone does not determine the radius and the surface diffuseness parameters, R_0 and a, simultaneously. One possible prescription is to assume $R_0 = 1.31A^{1/3} - 0.84$ fm as in Ref. [15], and use Eq. (14) to estimate the surface diffuseness parameter a from the root-mean-square radius. In fact, our RMF calculations show that the R_0 parameter estimated from the fitting to the monopole density is well parametrized by this function up to the drip line nucleus, at least for the Mg isotopes presented in Fig. 2. One can then use Eq. (13) to estimate the deformation parameter β from the quadrupole moment Q. This procedure may be important in discussing the deformation parameter of neutron-rich nuclei, where the surface diffuseness parameter is expected to be significantly larger than that of stable nuclei [16]. We mention that the deformation parameter is more useful than the quadrupole moment in discussing the deformation property, despite that it is inherently a model dependent quantity. One reason for this is that the distorded wave Born approximation (DWBA) and the coupled-channels calculations, which have been often used to describe inelastic scattering, require a coupling strength as an input which is intimately related to the deformation parameter.

In summary, we discussed the role of surface diffuseness parameter of density distribution in converting the quadrupole moment to the deformation parameter. For this purpose, we used both the deformed Woods-Saxon and the RMF density distributions. We showed that the widely used linear order formula with root-mean-square radius significantly underestimates the deformation parameter. After including the surface diffuseness correction, the resultant deformation parameters were found to be close to those estimated with the linear order formula with a sharp-cut radius.

The deformation property of neutron-rich nuclei has systematically been investigated recently in Ref. [3] using the Skyrme-Hartree-Fock-Bogoliubov method. See also Refs. [17–20]. The present consideration will be important when one discusses the deformation properties of such

neutron-rich nuclei, where the surface diffuseness parameter is expected to be large. In particular, when one draws a two dimensional potential energy surface spanned by proton and neutron deformation parameters, β_p and β_n, it may look considerably different depending on which formula one employs in estimating the deformation parameters. Such studies are now in progress, and we will report on them in a separate paper [21]. Also, it will be an interesting future problem to extend the formula derived in this paper by including the higher order terms of deformation parameter β. This will involve the contribution from hexadecapole deformation, the angle dependent surface diffuseness parameter and the condition for volume conservation.

We thank H. Sagawa for useful discussions. This work was supported by the Grant-in-Aid for Scientific Research, Contract No. 16740139 from the Japanese Ministry of Education, Culture, Sports, Science, and Technology.

[1] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer-Verlag, New York, 1980).
[2] P. Ring, Y. K. Gambhir, and G. A. Lalazissis, Comput. Phys. Commun. **105**, 77 (1997); Y. K. Gambhir, P. Ring, and A. Thimet, Ann. Phys. (NY) **198**, 132 (1990).
[3] M. V. Stoitsov, J. Dobaczewski, W. Nazarewicz, S. Pittel, and D. J. Dean, Phys. Rev. C **68**, 054312 (2003).
[4] H. Molique, J. Dobaczewski, and J. Dudek, Phys. Rev. C **61**, 044304 (2000).
[5] M. Yamagami and K. Matsuyanagi, Nucl. Phys. **A672**, 123 (2000).
[6] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, Eqs. (4–188c) and (4–191).
[7] I. Hamamoto and X. Z. Zhang, Phys. Rev. C **52**, R2326 (1995).
[8] N. Tajima, S. Takahara, and N. Onishi, Nucl. Phys. **A603**, 23 (1996).
[9] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1, p. 160.
[10] F. Reif, *Fundamentals of Statistical and Thermal Physics* (McGraw Hill, New York, 1965), p. 394.
[11] G. A. Lalazissis, J.Konig, and P. Ring, Phys. Rev. C **55**, 540 (1997)
[12] P. Möller and J. R. Nix, At. Data Nucl. Data Tables **39**, 225 (1988).
[13] D. G. Madland and J. R. Nix, Nucl. Phys. **A476**, 1 (1989).
[14] D. Vautherin, Phys. Rev. C **7**, 296 (1973)
[15] L. C. Chamon, B. V. Carlson, L. R. Gasques, D. Pereira, C. De Conti, M. A. G. Alvarez, M. S. Hussein, M. A. Candido Ribeiro, E. S. Rossi Jr., and C. P. Silva, Phys. Rev. C **66**, 014610 (2002).
[16] J. Dobaczewski, I. Hamamoto, W. Nazarewicz, and J. A. Sheikh, Phys. Rev. Lett. **72**, 981 (1994).
[17] J. Terasaki, H. Flocard, P.-H. Heenen, and P. Bonche, Nucl. Phys. **A621**, 706 (1997).
[18] T. Misu, W. Nazarewicz, and S. Åberg, Nucl. Phys. **A614**, 44 (1997).
[19] I. Hamamoto, Phys. Rev. C **69**, 041306(R) (2004); **72**, 024301 (2005).
[20] K. Yoshida and K. Hagino, Phys. Rev. C **72**, 064311 (2005).
[21] N. W. Lwin, K. Hagino, and N. Takigawa, to be published.